

MDIA2005

5Xj Yfh]g]b[. H\Y 7 fYUh]j Y 8]a Ybg]cbg

HYfa 'C bY''##'&\$&\$

7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

7cbj Ybcfg

B Ua Y	9a U]`		

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y'7ci fgY

Gi VYWh5fYU. ? WW[S

5 Xj Yfrjgjb[']g'U'WfYUhjj Yž'UbX'Uhhja Ygž'Wcbffcj YfgjU''Wca a i b]WUhjj Y'[YbfY'h\Uhid'Umg'U'WfbffU''fc'Y']b Wcbgi a Yf'W 'hi fY"'H\]g'Wci fgY'k]'''W\U''Yb[Y'mci 'hc'Wcbg]XYf'bchicb`mh\Y'WfYUhjj Y'X]a Ybg]cbg'cZ UXj Yfrjg]b['Vi hU'gc'hc Wcbg]XYf'Ugi ghU]bUV'Y 'gcW]U''fc'Y'Zcf'UXj Yfrjg]b["'Nci 'k]'''VY'W\U''Yb[YX'hc XYj Y`cd'Ub'i bXYfghUbX]b['cZh\Y'fY`Uhjcbg\]d'VYhk YYb'UXj Yfrjg]b["UbX'WfYUhjj]hm'UbX'ZcWi g'cb'\ck 'hc XYj Y`cd']bbcj Uhjj Y'gc'i hjcbg'hc'WcbhYa dcfUfmUbX'gcW]U''Wca a i b]WUhjcb'dfcV'Ya g"'H\Y'Wci fgY'k]'''\Uj Y U'g][b]ZJWUbhUdd']YX'Wca dcbYbh'mci 'k]'''ZcWi g'cb'XYj Y`cd]b['h\Y'WfYUhjj Y'X]a Ybg]cbg'Zcf'Ub'UXj Yfrjg]b[WUa dU][b'fY`UhYX'hc'U'WcbhYa dcfUfmgcW]U'']ggi Y'UbX'Yb[U[Y']a U[]bUhjj Y`mk]h\'U[YbWmVf]YZgžYI Ua]bY\ck ']XYUg'UfY'WfYUhjj Y`mYI dfYggYX'i g]b['k cfXg'UbX']a U[Yg'UbX'XYj Y`cd'UXj Yfrjg]b['WcbWrdhgždfYdUfY U'WfYUhj Y'd]hW\žUbX'YI d`cfY'h\Y'ghcfm'hY'`]b['UbX'Wcdmk f]hjb['X]a Ybg]cbg'cZUXj Yfrjg]b["

7 ci fgY @YUfb]b['Ci hWca Yg

- %" 7 f]h]ei Y`WfYUh]j Y`UXj Yfh]g]b[`]b`U`fUb[Y`cZWca a YfW]U`ž'Wcbgi a Yf`UbX`gcW]U``WcbhYl hg
- &" 7 f]hei Y'h\Y'fc`Y'cZUXj Yfh]a]b[']b'fY`Uh]cb'hc'gcW|YhmhcXUm
- ' "8Ya cbglfUhY'_bck`YX[Y'cZUbX'UV]`]lmihc'Udd`mWfYUh]jY'dfUWfJWYg'hc'Wcaaib]WUh]cb']ggiYg'UbX dfcV`Yag
- (" =XYbh]ZmU'hUf[Yh'Ui X]YbW''UbX'_Ym]bg][\hg'h\fci [\'fYgYUfW\
-) " A UbU[Y'h\Y'XYj Y`cda YbhcZ_Ym]bg][\hg'h\fci [\'h\Y'WfYUh]j Y'dfcWfgg

HYUWX]b['GhfUhY[]Yg'

FUNCbUY.

H\Y`Wci fgY`]g`XYg][bYX`rc`]blfcXi WY`ghi XYbhg`rc`ghfUhY[]WUbX`WfYUh]j Y`UddfcUWXYg`rc`UXj Yfh]g]b[`]b fUd]X`mWXUb[]b['a YX]U`WcbhYl hg"`H\Y`Wci fgY`]g`U`gc`XYg][bYX`rc``]b_`rc`rh\Y`l B`GK `ghfUhY[]Wdf]cf]lmicZ gi ghU]bUV]`]mm\fci [\`Ub`Ya d\Ug]g`cb`UXj Yfh]g]b[`Wca a i b]WUh]cb`r\Uhigi ddcfhg`gcW]U`ž`W``hi fU`Ybj]fcba YbhU``gi ghU]bUV]`]mm'

HYUW]b['GhfUhY[]Yq.

5ggYgga Ybh

H\]g^]g^dfcj]XYX^Ug^U^[YbYfU^^[i]XY^!^d^YUgY^gYY^A ccX^Y^Zcf^UggYgga YbhXYfU]^"

BchY. 5gg][ba Ybh&'UbX'' 'a i gh'VY'gi Va]hhYX']b cfXYf hc dUgg h\]g Wci fgY"

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
Cb`]bY`fYZYWNjjY`kf]hjb[`HUg_`)	\$&#\$' #&\$&\$`\$, .\$\$`DA</td><td>%ž&</td></tr><tr><td>7 f]h]WU`5bU`mg]g'</td><td>()।</td><td>&\$#\$' #&\$&\$`\$, .\$\$`DA</td><td>%ž&Ž Ž(ž)</td></tr><tr><td>Dcfh7c`]c'hUg_'</td><td>()।</td><td>%+#\$(#&\$&\$`\$, .\$\$`DA</td><td>%Ž Ž(Ž)</td></tr><tr><td>DfYgYblUljcb⁻</td><td>) </td><td>Bch5dd`]WUV`Y</td><td>' Ž)</td></tr></tbody></table>	

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% Cb`]bY fYZYWNjj Y k f]hjb['HJg_

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. '&\$\$"K cfXg

 $\label{eq:higher_problem} \mbox{Hi fb]h]b "gYhh]b[. "H\]g"]g"bchU"Hi fb]h]b"Ugg][\mbox{ba Ybh}$

5ggYgga Ybh'& '7f]h]WU` '5bU`mg]g

GhUfh'XUhY.

@Yb[h\.'&\$\$\$

8 YHJ]`g. `&\$\$\$`k cfXgž`]bX]j]Xi U``HJg_. `(`k f]HhYb`X]UfmYbHjfYg"`: YYXVUW_`j]U`@A G9ei]j U`YbhcZa U*`*cf YggUm`h\YcfYhJWJ``fYZYWf]cbg`cb`h\Y`Zci f`a U]b`hcd]Wg`]b`h\Y`Wci fgY"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' Hi & fb \end{tabular} $$ h' \end{tabular}$

5ggYgga Ybh' . 'DcfhZc`]c 'HJg_

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.B5

8YHJ]`g. 'H\fYY`ff Ł'WfYUfj Y`UXj Yffjg]b['d]YWYg`fc`VY`gi Va]ffYX`Ug`Wca dcbYbfg`cZf\Y`dcffZc`]c"'±bX]j]Xi U` fUg_"': YYXVUW_j]U`@A G" Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5ggYgga Ybh (. DfYgYbhUh]cb

GhUfh'XUhY.

@Yb[h\.B5

 $8YHU]^*g. ^*DfYdUfY^*UbX^*i bXYfHU_Y^*U^*\&!' ^*a]bi ^HY^*Y^*Yj Uhcf^*d]^HW.'^*cZcbY^*dcfhZc^*]c^*UXj Yfh]g]b[^*d]YWY^*^*bX]j]Xi U^*HUg_ž^*]b!W^*Ugg^*') !dYfWYbhdUgg#Uj]^*'f] i ^*dUgg/&i ^*ZU]^*/$i ^*bcb!gi Va]gg]cb!'': YYXVUW_*j]U^*@A G$

5XX]h]cbU``XYhU]`g.

5 https://bxubwww.Fyei]fya Ybhg

7ci fgY GWX YXi `Y

J]Yk 'WUgg'ha YhUV'Y

H]a YtUV`Y

8 UHY	HmdY	7 cbhYbh		
K YY_'%'%+': YVfi Ufm!'&% : YVfi Ufm	@YWhifY	⇒hfcXi Whjcb hc Wca a YfWjU`WfYUhjj]hmUbX U WfYUhjj Y WcbWfdh'		
		FYZYf`hc`AccX`Y`Zcf`Zlfh\Yf`XYhU]`g"		
	Gh X]c	K \Uha U_YgʻUʻ[ccXʻUX"ʻ±bhfcXi Wh]cbʻhcʻh\Y WfYUh]jYʻVf]YZʻʻ		
		Mci k]``fYW]jYmcifZfghWYYUhjjYvYf]YZghckcf_cbZcfmcifdcfhZc`]c"		
		FYZYfihoiAccXiYiZcfiZifh\YfiXYhU]jg"		
	C b`]bY'5W j j] h m	FYZYf'hc'AccX`Y'Zcf'Z fh\Yf'XYHJ]`g"		
K YY_'&.'&(': YVfi Ufm!'&, : YVfi Ufm	@YWn fY	7 fYUh]j Y'h\]b_]b['UbX'WfYUh]j Y'dfUWh]WY"'		
		FYZYfihoi'A ccXiYiZcfiZifh\YfiXYhU]`g"		
	Gh X]c	6f]b['mcif']b]h]U`WfYUh]jY'WcbWfdhg'Zcf'h\Y'Z]fghdcfhZc`]c'W]b]W		
		FYZYfihoiAccXiYiZcfiZifh\YfiXYHJ]`g"		
	Cb`]bY'5WMjj]hm	FYZYfihciAccXiYiZcfiZifh\YfiXYhU]`g"		
K YY_'' .`&'A UfWX'!'* A UfWX	@YWhifY	A U_]b[a YUb]b["		
		FYZYf'hc'AccX`Y'Zcf'Zlfh\Yf'XYhU]`g"		
	Gh X]c	H\Y`gYWcbX`dcfhZc`]c`W]b]W		
		FYZYf'ho'AccX`Y'Zcf'Z fh\Yf'XYhU]`g"		
	C b`]bY'5Whjj]hm	FYZYf'hc'AccX`Y'Zcf'Z fh\Yf'XYhU]`g"		
K YY_^(.^-^A UfVX^!^% A UfVX	@YWn fY	l bXYfgHJbX]b['dYcd`Yži bXYfgHJbX]b[']bg][\h''		
		FYZYfihoi'AccXiYiZcfiZlfh\YfiXYhU]`g"		
	Gh X]c	H\Y`h\]fX`dcfhZc`]c`W]b]W'		
		FYZYfihoiAccXiYiZcfiZifh\YfiXYhU]jg"		
	C b`]bY'5Whjj]hm	FYZYf'hc'AccX`Y'Zcf'Z fh\Yf'XYHJ]`g"		
K YY_^).^%*^A UfWX^!^&\$ A UfWX	@YWhifY	HY``]b['dck YfZ ``VfUbX'ghcf]Yg"'		
		FYZYf`hc`AccX`Y`Zcf`Zlfh\Yf`XYhU]`g"		

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

H\Y'UXjYffq]b['WcbWdhVcc_Ž6Uffmž'D"'f&\$%*Ł

FYWca a YbXYX FYgci fWyg

D`YUgY fYZYf hc A ccX`Y Zcf gci fWYg"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\Y`Wci fgY`Wcbj Ybcf`UbX`hi hcfg`k Y`Wca Y`]bZcfa U``ZYYXVUW_`UVci hh\Y`Wci fgY`h\fci [\ci hh\Y`gYa YghYf": i fh\Yfa cfYž'ghi XYbhg`UfY`YbWci fU[YX`hc`dUfh]W]dUhY`]b`cb`]bY`ZYYXVUW_`gi fj Ymg``UhYf`]b`h\Y`gYa YghYf"`5b Ya U]``k]``VY`gYbhhc`U```ghi XYbhg`UXj]g]b[`k\Ybž'k\YfYž'UbX`\ck `hc`dfcj]XY`ZYYXVUW_"

D`YUgY'hU_Y'h\Y'h]a Y'hc'dfcj]XY'ZYYXVUW_"'=D`fYWYbhmYUfg'h\Y'Wci fgY`\Ug'VYYb'W\Ub[YX'Ug'U'fYgi `hcZ ghi XYbh]bdi h': cf'Yl Ua d`Yž'h\Y'Wci fgY'UggYgga Ybh\Ug'VYYb'W\Ub[YX'Zfca 'U'[fci d'Ugg][ba Ybhhc'Ub]bX]j]Xi U``WfYUh]j Y'dcfhZc`]c'Ugg][ba Ybh'

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Zmci YbWcYjbhYffbWidfc'kKYai dkdxxbWthYa dhjb['hc'gi Va]hmci f'Ugg][Y]M_YWbhbafkb[ih'Hi fb]hjbžd'YUgh Z``Gi ddchU hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',)'''' %cf'Ya U]``h\Ya 'cb'Y| hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui ""Gi ddcfh\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft''=Zmci UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]hjb'mci 'a UmUdd'm'Zcf'Ub'Y| hYbg]cbžVi hmci a i ghfYhJ]b'mci f'hjW_Yhbi a VYf'Zfca '9| hYfbU``Gi ddcfhfU`cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Yl hYbg]cb'Udd`]WJhjcb''=Zmci 'Ya U]``9| hYfbU``Gi ddcfhmci 'k]``

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfX]h'

=\uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU`
ghfi Whi fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`ž'UbX']bZcfa Uh]cb'k]h\ci h\UW_bck `YX[Ya Ybh''H\]g'U`gc
Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY 'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h\UhYX]hUbX'hc
d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yž'k]h\ci h\Uddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI_BGK_`cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b['UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[`7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Gh XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``ha Ybu[Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`h]a Y`a UbU[Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'h]a Y`Zcf fYgYUfW\žXfUZh]b[`UbX`dfcdYf`fYZYfYbW]b[`cZgci fWfg`]b`dfYdUf]b[`U``UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@G9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK "'9@G9`]gXYg][bYX`hc`]bhfcXiVW`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcifqhiXm"

7 ca d'Yhlb['h\Y'9@&9'hi hcf]U`'UbX'ei]n'k]`'YbUV'Y'mci 'hc.

 UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\W\`Zcf'U\WXYa]\Wk f]hjb[`UbX`ch\Yf UggYgga YbhiHUg_g • i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5WUXYa]W = DZcfa Uh]cb

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '<YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUh]cb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg'cf'a]gUXj Ybhi fY/
- gh XYbhYei]hmUbX X]gUV]`]hm/
- ÜbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Ufng"i bgk "YXi "Ui #W ffYbhghi XYbhg#UWUXYa]W]bZcfa Uhlcb#dfchcWc`g![i]XY`]bYg#

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