

MDIA3011

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K YVg]hY. <u>\hdg.##gUa "Ufhg"i bgk "YXi "Ui</u>

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7UadigYgŁ'UbX'thY'B[ibbUkU`dYcd'Y'fbigtfU]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZthY'`UbXg'k\YfY'YUW\cong WJadig'cZlBGK']g`cWJtYX''$

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H\]g`Wci fgY`U`gc`\Ug`U`%\ci f`k YV`Wca dcbYbhk\]W\`k]```Wcbg]ghcZWci fgY`fYUX]b[gž'k]h\`YUW\`ghi XYbh W\ccg]b[`U`gdYW]Z]WfYUX]b[`hc`fYdcfhcb`hc`h\Y`WUgg"`H\YgY`k]```bchVY`UggYggYX`Vi hUfY`Wca di `gcfmhc Wca d`YhY`h\]g`Wca dcbYbh'

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GhUfh'XUhY. "%+# #&\$

8YHJ]`g. "%\$\$\$ k cfXg": YYXVUW_j]U'Hi fb]h]b"

5XX]h]cbU`XYhU]`g.

DfYgYbhUhcbg`k]```VY`]b`K YY_g') `UbX`*ž'k]h\`k YY_'hc`VY`U``c\WhYX`]b`ZfghgYa]bUf"

K f]hh/b fydcfhk]``VY Xi Y') 5df]`ž') da "

: i ```XYHJ]`g`k]```VY`cb`A ccX`Y"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhf]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\"Hi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b gJa]"Uf]hmfYdcfhg" \end{tabular}$

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8 YHJ]`g.

5 https://bxubwww.Fyei]fya Ybhg

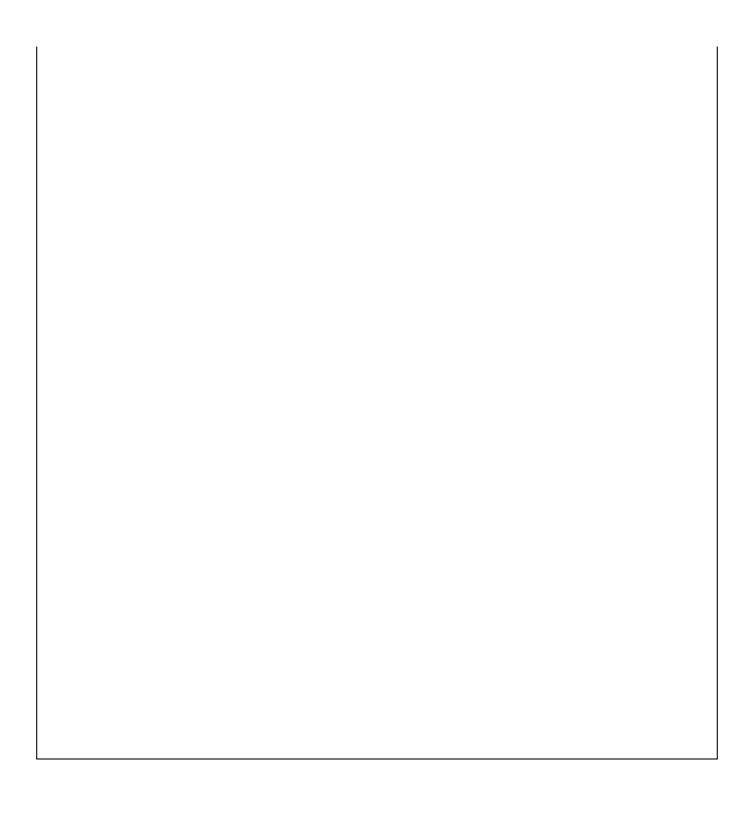
 $Ghi XYbhg`UfY`fYei]fYX`hc`UhthYbX`, $1 \ `cZWUggYg`UbX`UhthYbXUbWY`k]```VY`hU_Yb`]b`h\Y`gYa]bUf''$

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'h]a YhUV'Y

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8 UhY	HmdY	7 cbhYbh
K_YY'%_'%+': YVfi_Ufm!_'&%	GYa]bUf	Ai`hjacXU`jhmžWcaaib]WUhjcb'UbX'aYX]U"
: YVfi Ufm		H\]g`k YY_`Zcfa g`h\Y`Zci bXUh]cb`cZh\Y`Wci fgY.`]b k \UhgYbgY`]g`Wca a i b]WUh]cb`fXYg][bYXf3`K \m gh`Xma i `h]a cXU``hYI hg3`K \UhhYI hg`YI UWhmk]```k Y gh`Xm3` <ck`k]``k 3<="" td="" y`gh`xmh\ya=""></ck`k]``k>
		GYhfYUX]b[.:h\]g:]g:h\Y:a]b]a i a 'fYUX]b['hc'Xc']b dfYdUfUh]cb'Zcf'YUW\'k YY_'UbX'h\YgY'k]``VY Uj U]`UV`Y'cb'A ccX`Y"'D`YUgY'fYUX'TW&dWh\Y gYa]bUf"'5'k]XY'fUb[Y'cZUXX]h]cbU`'fYUX]b[g'k]```VY dcghYX'cb'A ccX`Y"'Gca Y'cZh\YgY'k]``VY'Uj U]`UV`Y cb'A ccX`Y'UbX'gca Y'mci'k]```\Uj Y'hc'\i bh'Zcf'
		%"`?fYggž; "`f&\$%) Ł"`8Yg][b.`h\Y`f\Yhcf]WU``k cf_`cZg\Ud]b[`h\Y`gYa]ch]Wk cf`X"`±b`±b`5"`5fW\Yf`UbX`96fYi Yf`f9XgŁ`A i `h]a cXU`]hm]b`K f]h]b[.`h\Y`gHUhY`cZh\Y`Ufh]b`h\YcfmžaYh\cXc`c[mUbX`dYXU[c[mdd%' %1%)&
		&"`?fYggž; "`UbX'j Ub`@YYi k Ybž'H"`f&\$\$%£"` ?g^f[_aVS^V[eUag&W fZW?aVWeS`V?W[SaX 5a`fW_ba&&k5ag`[USf[a`"`@cbXcb.'5fbc`X" O'\UdhYf'%cb`m`\bfcXi W]cbž'dd%&'Q
		' "'j Ub'@YYi k YbžH"'f&\$%\&"'A i `h]a cXU`]mž']b'>" G]a dgcb'f9X"\&FZWDagf\WYWZS` VTaa] aX3bb\[\W >[`Yg[ef[UezFci hYX[Yz'@cbXcb'UbX'BYk 'Mcf_z'dd **,!*, &"
K YY_`&.`&(`: YVfi Ufm!`&, : YVfi Ufm	GYa]bUf	5```h\Y`[ccX`V]hgžDUfh'5.`gYa]ch]W fYgcifWYg`fgiW\`Ug`Wc`cifžgcibXžhYlhifYžg\UdY """"Ł
		GYhF YUX]b[.
		%"JUb'@YYikYbžH"'f&\$%\L"FZW>S`YgSYWaX 5a'agd, 3`[`fobYgUf[a`"'@cbXcb.'FcihYX[Y O7\UdhYf'%cb`m'+blfcXiWh]cb/dd'%1,Q
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 $\label{eq:linear_continuity} $$ Mci^k] $$ ``bYYX'hc'fYUX'k]XY'mUbX'XYYd'm']b'h\Y'Wci^fgY'hc'a YYhh\Y'Wci^fgY''Wci^fgY''VUfb]b['ci^hWca Ygž'UbX'hc'Xck Y``'''GYhfYUX]b['g'k]```U```VY'Uj U]`UV'Y'cb'A ccX'Yž'Ug'k Y``'Ug'U'``Uf[Y'bi a VYf'cZh\Y'cdh]cbU``fYUX]b['g''=kci^X'VY'a cfY'h\Ub'\Uddmhc'[]j Y'Z fh\Yf'gi [[Ygh]cbg'Zcf'fYUX]b['g']b'`UbmgdYW]ZWhcd]WUfYUg''$

FYWca a YbXYX FYgci fW/g

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

=i gY'U'Wca V]bUhcb'cZZcfa U``UbX']bZcfa U``Yj U`i Uhcb'hc'a cb]hcf'UbX']a dfcj Y'h\]g'Wci fgY"'A 8=5' \$\% k Ug']bHcXi WYX']b'&\$\% žk]h\'U'dUfhJW `Ufmdcg]hj Y'fYgdcbgYžYgdYWJU``mhc'h\Y'fYU``k cfX'Udd`]\WJhjcbg UbX'[i Ygh`YWf fY"'Gca Y'gh XYbhg']bX]\WJhYX'h\Ymk UbhYX'a cfY'\WJf]hm]b'gca Y'cZh\Y'\Wcb\W'dhgžgc']b'&\$\\$ f]hk Ug'bchHJi [\h']b'\&\$\& E'=\Uj Y'ghfYUa `]bYX'h\Y'hcd]\Wg"'=k Y'\Wca Y'mci f'\ZYYX\U\W_U'UhUbmdc]\bhžY]h\Yf \Zfa U``mh\fci [\\'X]fY\Wh\Ybc a Y bhcf'Ya U]'"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

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Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybht\fci [\ 'Hi fb]hJbž'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-',) '' ' ' %cf'Ya U] 'h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'\\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ" Image:

UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]hJb'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WJhJcb" Image: Ya U]''9! hYfbU''Gi ddcfhmci 'k]''

Ui hca UhJWJ''mfYW'] Y'U'hJW_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'hc'gdYWJZWJ''mUg_'Zcf'cbY"

Hi fb]hJb'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

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5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfX]h'

=\uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU`
ghfi Whi fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`ž'UbX']bZcfa Uh]cb'k]h\ci h\UW_bck `YX[Ya Ybh''H\]g'U`gc
Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY 'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h\UhYX]hUbX'hc
d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yž'k]h\ci h\Uddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI_BGK_`cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b['UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[`7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Gh XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``ha Ybu[Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`h]a Y`a UbU[Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'h]a Y`Zcf fYgYUfW\žXfUZh]b[`UbX`dfcdYf`fYZYfYbW]b[`cZgci fWfg`]b`dfYdUf]b[`U``UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@G9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK "'9@G9`]gXYg][bYX`hc`]bhfcXiVW`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcifqhiXm"

7 ca d'Yhlb['h\Y'9@&9'hi hcf]U`'UbX'ei]n'k]`'YbUV'Y'mci 'hc.

 UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\W\`Zcf'U\WXYa]\Wk f]hjb[`UbX`ch\Yf UggYgga YbhhUg_g • i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5 WUXYa]W = b Zcfa Uh cb.

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- I BGK '< YU`h\ 'UbX'GUZYhmdc`]\\\Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
- gh XYbhYei]lmUbX X]gUV]`]lm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY

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