

# **MDIA5021**

5Xj Yfh]g]b[ 'UbX'7fYUh]j ]hm

HYfa 'C bY''##'&\$&\$

## 7ci fgY'Cj Yfj ]Yk

### Ghuzz 7 cbhuWh8 Yhu]`g

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
>Ua YgʻK ][[]bg	0 0	5ddc]bha Ybhgʻa Um VYʻa UXYʻVmYa U]`	Fa&'\$ KYVghYf^6`X[	

#### GWkcc~7cbhJWh~abZcfa Uh]cb

Fcca " &ž Yj Y`" FcVYfhK YVghYf 6i ]X]b[

D\cbY. f\$&L'-',) '(,) \*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##gUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fth' '8Yg][b'7UadigYgŁ'UbX'thY'B[ibbUkU`dYcd'Y'fbigtfU]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZthY'`UbXg'k\YfY'YUW\cong WJadig'cZIBGK']g`cWJtYX''$ 

## 7ci fgY 8YHJ] g

### 7 fYX]hDc]bhg \* \*

### Gi a a UfmcZh\Y 7ci fgY

H\Y'Wci fqY'ZcW'qYq'cb'h\Y'WfYUhj Y'UqdYWfq'cZUXj Yfhlq]b['UbX'W'ffYbh'hfYbXq']b'UXj Yfhlq]b["

Mci 'k ]```YUfb \ck 'h\Y'UXj Yffqq]b[ 'dfcW/gg'k cf\_g'ZcW q]b[ ž']b 'dUffqW `Ufž'cb 'h\Y'Zc``ck ]b[ 'UgdYWfg.

! '⇒XYbhjZjWuhjcb'cZ'Wcaai b]Wuhjcb'WkU``Yb[Yg'UbX'cddcfhi b]hjYg. 'fYgdcbX]b['hc'U'W]Ybh'Vf]YZ/ ! '7 fYUhjj Y'ghfUhY[]Yg'UbX'dfUWjWYg'Zcf'[YbYfUhjb[']XYUgžWcbWYdhXYg][bžghcfmhY``]b[žYachjcbU`'UddYU`g UbX'd]hWk]b['Zcf'WJadU][bg/ ! '7 UadU][b'Yj U`i Uhjcb'aYhkcXg"

H\Y`fYei]fYa Ybhg`Zcf`UbX`]a dUWicZUXj Yfh]g]b[`UWfcgg`a i`h]d`Y`d`UhZcfa g`UbX`UggcW]UhYX`fYei]fYa Ybhg Zcf`X][]hU``]bhYfUWh]j ]hmž`Wc``UVcfUh]cb`UbX`Wc!dfcXi Wf]cb`k]```VY`Ya d\Ug]nYX"

### 7 ci fqY @YUfb]b[ 'Ci hMca Yq

%" 5dd`miWfYUhjj Y`ghfUhY[]Yg`UWfcgg`U`VfcUX`fUb[Y`cZUXj Yfhjg]b[`a YX]U &" 8Yj Y`cd`U`Wf]hJWU`i bXYfghUbX]b[`cZUXj Yfhjg]b[`cVYWfj Yg ' " 7 f]hjei Y`UbX`fYj ]gY`mci f`k cf\_ ( " K cf\_YZZYWfj Y`mk ]h\`ZYYXVUW\_`UbX`dfYgYbh'mci f`]XYUg`k ]h\`WcbZJXYbWf

### HYUW ]b[ GhfUhY[]Yg

Ghi XYbhgʻk]``VYʻfYei]fYX`hcʻUhhYbX`Uʻhk cʻ\ci fʻgYa]bUfʻYUWX`k YY\_"ʻGYa]bUfgʻUfYʻXYg][bYX`hcʻ]bhfcXi W\U``ghi XYbhgʻ]bʻUʻWcbg]ghYbhik Umhcʻh\YʻfY`Yj Ubhih\Ycf]Ygž'dfUWNWgʻUbX`hcc`g"'H\YmfY`m\YUj]`mcb YI Ua d`YgʻcZWcbhYa dcfUfmUXj Yfh]g]b[ 'hcʻdfUW]WJ``mXYa cbghfUhYʻh\Yʻf\Ycf]Ygʻ]bʻdfUW]WYʻ]bʻfYU` UXj Yfh]q|b[ 'q]hi Uh]cbg"

H\Y`gYa]bUf`ZcfaUh`]g`UXcdhYX`hc`VYhhYf`g]ai`UhY`h\Y`U[YbWhiYldYf]YbW\žk]h\`h]aY`gdYbh`Vf]YZ]b[ghiXYbhg`cb`kYY\_`mUgg][baYbhg`UbX`UggYgg]b[`h\Y`YZZYW]jYbYgg`cZh\Y]f`WfYUh]jY`fYgdcbgYg"`H\YgaU``Yf`[fcidg`[]jY`h]aY`Zcf`ZYYXVUW\_`UbX`fYZYW]cb`cb`fY`YjUbhdf]bW]d`Yg`UbX`h\Ycfm'`GhiXYbhdUfn]w]dUh]cb`k]```VY`fYei]fYX`h\fci[\cihgYa]bUfg"

7 fYUhji Y'g\_]``g'UfY'VYghXYj Y`cdYX'h\fci [\'h\Y]f'WcbghUbhUdd`]WUhjcb"'H\YfYZcfYž'k YY\_`mVf]YZg'UfY XYg][bYX'hc'dfcj]XY'ghi XYbhg'k]h\'Ub'cddcfhi b]hmhc'X]gWcj Yf'UbX'XYj Y`cd'h\Y]f'ck b'WfYUhjj Y'dfcV'Ya!gc`j]b['ghm`Yg'UbX'g\_]``g"

Ghi XYbhgʻUfYʻUʻgcʻYbWci fU[YXʻhcʻUWijiYʻmWcʻ`YVViUgʻa Ubmd]YWʻgʻcZUXj Yffig]b[ʻWca a i b]WVlijcbʻUgʻh\Ym WUbʻZfca ʻUʻX]j YfgYʻfUb[YʻcZgci fWʻgžʻhcʻ\_YYdʻUVfYUghicZWcbhYa dcfUfmWJa dU][bgʻUbXʻUXj Yffig]b[ʻ]ggi YgUbXʻYb\UbWʻh\Y]fʻdfUWijWU`i gYʻcZh\Yʻh\Ycf]Ygž'dfcWfggYgʻUbXʻhccʻgʻWcj YfYX`]bʻh\Y`Wci fgY"

## 5ggYgga Ybh

H\Y'UggYgga Ybhg'k ]``'VY'ci h]bYX']b'XYhU]`'cb'A ccX`Y'UbX']b'GYa ]bUf'%"

### 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hJg_	K Y][ \h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
Cb`]bY`fYZYWNjjY`kf]hjb[`HUg_`	)	&, #\$&#&\$&\$`\$, .\$\$`DA</td><td>%ž&</td></tr><tr><td>7 f]h]WU`'UbU`mg]g'</td><td>(\$1</td><td>\$'#\$(#&\$&\$`\$,.\$\$`DA</td><td>%ž&Ž Ž(</td></tr><tr><td>Dcfh7c`]c'</td><td>) \$1</td><td>%+#\$(#&\$&\$`\$, .\$\$`DA</td><td>%ž&Ž Ž(</td></tr><tr><td>D]HW.</td><td>)  </td><td>Bch5dd`]WUV`Y</td><td>&ž(</td></tr></tbody></table>	

### 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% Cb`]bY fYZYWh]j Y k f]h]b[ 'HJg\_

GhUfhXUhY. Bch5dd`]WUV`Y

**8YHJ]`g**. `&\$\$`k cfXg"`±bX]j ]Xi U``HJg\_"`A ccX`Y`Zcfi a "`DUgg# U]``f**)** ı `dUgg/&ı `ZJ]`/\$ı `bcb!gi Va ]gg]cbŁ" : YYXVUW\_`j ]U`@A G"`

5XX]h]cbU`XYhJ]`g.

FYZYf'hc'AccX`Y'Zcf'XYhU]`g"

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh'& 7f]h]WU`UbU`mg]g

GhUfhXUhY. Bch5dd`]WUV`Y

**8YHJ]`g.** :: ci f`f(\text{!k f]HYb`X]UfmYblf]Yg`cZUddfcl "`) \$\$`k cfXg`YUW\/&\$\$\$`k cfXg`]b`lcHJ`"9ei ]j U`Ybhlc`U

hJg\_":: YYXVUW\_ij ]U'@A G"

5XX]h]cbU`XYhU]`g.

FYZYf'hc'AccX`Y'Zcf'XYhU]`g"

5ggYgga Ybh(. D]hW

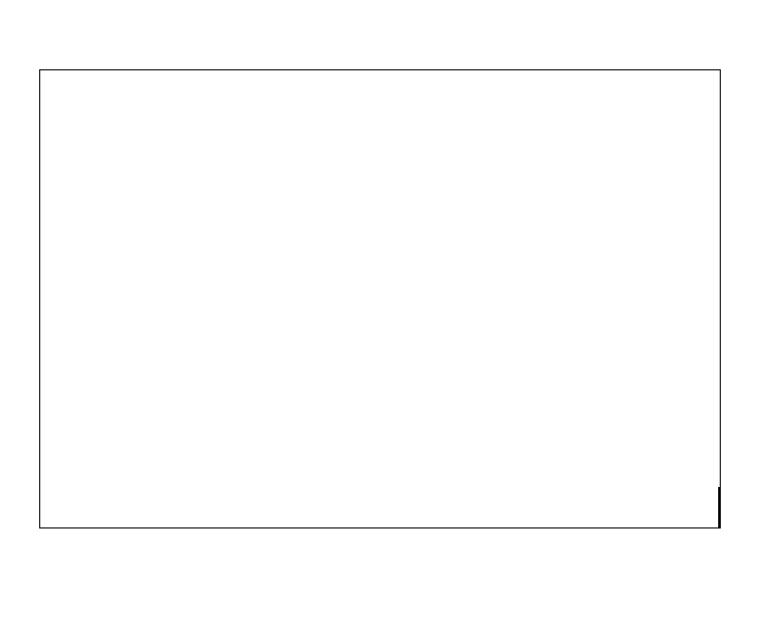
GhUfh XUhY.

**8YHJ]**`g. `8Y`]j Yf`U`' !a ]bi HY 'Y`Yj Uhcf`d]hW.'`cb`cbY`dcfhZc`]c`UXj Yfh]g]b[ `d]YWY") !dYfWYbhf) ı `dUgg/&ı ZJ]`/\$ı `bcb!gi Va ]gg]cbٱx]j ]Xi U``HUg\_ž`]b`WUgg"`DUgg# U]`: YYXVUW\_j ]U`@A G

5XX]h]cbU`XYhU]`g.

±b"k YY\_"%\$"gYa ]bUfg"

 $\label{eq:higher} \mbox{Hi fb]h]b "gYhh]b[ . "H\]g"]g"bch"U"Hi fb]h]b"Ugg][ ba Ybh$ 



### FYgci fW/g

### DfYgWf]VYX F Ygci fWYg

6UffmžD"'f&\$%\* Ł"'H\Y'5Xj Yfnjg]b[ '7cb\Wdh6cc\_ž' fX'9X]njcbž@cbXcbž'H\Ua Yg'/ '<i Xgcb"

### FYWca a YbXYX FYgci fW/g

5]hWk]gcbž>"f&\$\$(\text{E"7i hhjb["9X[Y'5Xj Yffhjg]b[\text{z'&bX'9X]hjcb""G]b[UdcfY.\*DYUfgcb'<U``"
5 hgh]Y\text{z'} fck\text{z'} YUb\text{z'} fx\text{z'} 5Xj Yffhjg]b["7 fYUhj Y."GhfUhY[m\text{z'} 7 cdm\text{z'} UbX'8 Yg][b\text{z'}(h\ YX]hjcb\text{z'}
@cg'5b[Y'Yg."GU[Y'Di V']\Whjcbg"
<Y[Uffm\text{z'} - Y[Uffm\text{z'} - Y[Uffmcb'5Xj Yffhjg]b["'@cbXcb."H\Ua Yg'UbX'<i Xgcb"
F ca Ub\text{z'} - Y A UUg\text{z'} - Y[Uffmcb'5Xj YffhjgY.\*K \Uhik cf\_g\text{z'k \UhiXcYgb'h!"UbX'k \m\text{m'z'} fX'9X]hjcb"
@cbXcb.'<c[Ub
Gi ``]j Ub\text{z''} f\text{k\$\$, \text{E"}< Ym\text{z'} K \]dd`Y\text{z'Gei YYnY'H\]g. 5; i ]XY'hc'7 fYUhjb["; fYUh5Xj Yffhjg]b[\text{z'} fX
9X]hjcb"'< cVc Yb\text{z''} 8 > .'>c\b'K ]\text{Ym/} Gcbg\text{z'} \text{\uniffy}

### 7 ci fgY '9j U`i Uh]cb 'UbX '8 Yj Y`cda Ybh

#JZcfa U`ZYYXVUW\_'UVci hih\Y`Wci fgY`]g`k Y`Wca YX`UbX`YbWci fU[YX`h\fci [\ci hih\Y`gYa YghYf`Vmih\Y`Wci fgY Wcbj YbYf`UbX`hi hcfg": YYXVUW\_'UbX`Yj U`i Uhjcb`]g`i gYX`hc`]a dfcj Y`h\Y`Wci fgY`YUW\`gYa YghYf": cf Yl Ua d`Yž`U`[fci d'dfc YWh\Ug`VYYb`fYa cj YX`Zfca 'h\Y`UggYgga YbhihUg\_g`]b`&\$% 'VUgYX`cb`ghi XYbh ZYYXVUW\_']b`&\$% "'Ghi XYbhq`UfY`YbWci fU[YX`hc`dUfh]WidUhY`]b`cb`]bY`gi fj Ymq``UhYf`]b`h\Y`gYa YghYf":5b

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va ]hmci f'Ugg][ ba Ybht\fci [\'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-', ) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''#mci
UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'YI hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW'a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbWY'hc'gi ddcfhmci f'YI hYbg]cb'Udd']WUh]cb'''#mci 'Ya U]''9I hYfbU''Gi ddcfhmci 'k]''
Ui hca Uh]WJ''mfYWY']j Y'U'h]W\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'hc'gdYW]ZWJ''mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]``'VY'gHJhYX'cb'mci f'\\ci fgY'g'A ccX`Y'g]hY'k ]h\ U`hYfbUh]j Y'gi Va ]gg]cb'XYhJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va ]hUgq][ ba Ybha ccXiY

### 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'=hWJb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhU`mWcdm]b[ 'Zfca `U`gci fWY'k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]uf]ga ]bhch\YZc``ck]b[ WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfX]h'

=\uddfcdf]UhY 'dUfUd\fUg]b[ .'7\Ub[ ]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b[ 'h\Y'cf][ ]bU`
ghfi Whi fY 'UbX#cf'dfc[ fYgg]cb'cZ]XYUg'cZh\Y'cf][ ]bU`ž'UbX']bZcfa Uh]cb'k ]h\ci h\UW\_bck `YX[ Ya Ybh''H\]g'U`gc
Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY 'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci h\UhYX]hUbX'hc
d]YW]b[ 'hc[ Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yž'k ]h\ci h\Uddfcdf]UhY'fYZYfYbW]b[ "

7 c``i g]cb.'k cf\_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf\_'Ug'U'dYfgcb'g']bX]j ]Xi U``k cf\_"'7 c``i g]cb'U`gc ]bWi XYg'dfcj ]X]b['mci f'k cf\_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa ]WhUg\_ž'ghYU']b['cf'UWei]f]b[ Ubch\Yf'dYfgcb'g'UWUXYa ]Wk cf\_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf\_'cf'gYY\_]b[ dUma YbhZcf'Wca d'Yh]b['UWUXYa ]Wk cf\_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW\_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`\_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\W\n]cb`ffgY`Z\d`U[]\Uf]ga "Ł`gi Va]\n]b[`mci f`ck b`k cf\_z`]b`k \c`Y`cf`]b`dUffz`k \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\nYX`Zcf`Ubch\Yf`UggYgga Ybhcf`\\cf\\text{i fgY`UhI\_BGK\_`cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb[ 'dfUWnWyg.

- DUfUd\fUg]b[žgi a a Uf]g]b[žYggUmk f]h]b[ 'UbX h]a Y a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhtf]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j ]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[ `7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Gh XYbhg`UfY`U`gc`fYa ]bXYX`h\UhWUfYZ ``ha Ybu[ Ya Ybh]g`Ub`]a dcfhUbhdUfhcZgh XmUbX`cbY`cZh\Y ]XYbh]Z]YX`WUi gYg`cZd`U[ ]Uf]ga `]g`dccf`h]a Y`a UbU[ Ya Ybh'`Gh XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'h]a Y`Zcf fYgYUfW\žXfUZh]b[ `UbX`dfcdYf`fYZYfYbW]b[ `cZgci fWfg`]b`dfYdUf]b[ `U``UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@G9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK "'9@G9`]gXYg][bYX`hc`]bhfcXiVW`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcifqhiXm"

7 ca d'Yhlb[ 'h\Y'9@&9'hi hcf]U`'UbX'ei ]n'k ]`'YbUV'Y'mci 'hc.

 UbU`mgY`hcd]\Mgž'd`Ub`fYgdcbgYg`UbX`cf[Ub]gY`fYgYUf\W\`Zcf'U\WXYa]\Wk f]hjb[`UbX`ch\Yf UggYgga YbhiHUg\_g

