

MUSC4706

Aig]W9bgYaV`Y

HYfa 'C bY''##'&\$&\$

7ci fgY'Cj Yfj]Yk

Ghuzz 7 cbhuWh8 Yhu]`g

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
A Ubc`YhY'A cfU	a "a cfUh4 i bgk "YXi "Ui	,	FcVYfh KYVghYf`‰(\$&`-' ,) *, %%

GWkcc~7cbhJWh~abZcfa Uh]cb

Fcca " &ž Yj Y`" FcVYfhK YVghYf 6i]X]b[

D\cbY. f\$&L'-',) '(,) *

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##gUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]tjcbU``7igtcX]Ubgz'h\Y'6YXY[U'`fPYbg]b[tcb'WJadigŁz'; UX][U'`fP]tmUbX'5fti/'8Yg][b'7UadigYgŁ'UbX't\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`cWJhYX''$

7ci fgY⁻8YHJ]`g

7 fYX]h'Dc]bhg '* '

Gi a a UfmcZh\Y 7ci fgY

 $H\]g']g'U'dfUWI)WU''a i g]WWci fgY'fh Ufic ZZYfg'mci 'Ub'cddcfhi b]lmhc'dUff]WJdUhY']b'fk c'gY'ZlgY'YWYX YbgYa V'Yg'''H\Y'A i g]WDfc[fUa 'cZZYfg'U'k]XY'Wkc]WY'cZga U'`'UbX''`Uf[Y'YbgYa V'Yg'fh Uh']bWi XY'WUgg]WU'ž$

5ggYgga Ybh

5ggYgga Ybh'HUg_g

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
9bgYa V`Y`5`7cblf]Vi ljcb`UbX DUfH7\YW_]b[' \$I	Bch5dd`]WUV`Y	%
9bgYa V`Y`6`7cblf]Vi ljcb`UbX DUftl7\YW_]b[' \$ı	Bch5dd`]WUV`Y	%
A U]b DYfZcfa UbWY	&\$I	Bch5dd`]WUV`Y	%ž&
7 ca dufuhji Y Yj U`i Uhjcb	&\$ı	6mh\Y`YbX`cZkYY_`%\$	1

5ggYgga Ybh8YhJ]`g

5ggYgga Ybh% 9bgYa V`Y 5 7cbff]Vi h]cb UbX DUfH7\YW_]b[

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHU]`g. `Ghi XYbhgfidUfh]WdUh]cb`UbX`Wcbhf]Vi h]cb`hc`YbgYa V`Y`fY\YUfgU`g`k]```VY`UggYggYX`Zcfa Uh]j Y`mž`Vm h\Y`YbgYa V`Y`X]fYWcf`fY[i``Uf`ma cb]hcf]b['ghi XYbhg`]b`fY\YUfgU`g`hc`Ybgi fY`h\Uh`YUfb]b[']g`HU_]b[d`UWY"`Gi a a Uh]j Y`UggYgga Ybh`]g`UXa]b]ghYfYX`h\fci [\`U'dUfh\W\Y\W_]b['hYghUhih\Y`YbX`cZh\Y gYgg]cb": YYXVU\W_.`8]fYWhUbX`cb[c]b['ZYYXVU\W_Xi f]b['fY\YUfgU`g`UbX`UZhYf`U`a U]b`dYfZcfa UbW\"

5XX]h]cbU`XYhJ]`g.

 $H\Y'dUfftWXYW_]b['hYghi gi U``m'tU_Yg'd`UWY']b' 'hXY'k YY_'UZhYf'hXY'Z]bU``fY\YUfgU'#di V`]WdYfZcfa UbWY fK YY_''%$`cf'ff%ft" 8 YtU]`g'cZh\Y'dUfftWXYW_]b['hYghik]```VY'Ubbci bWYX'''K \YfY'dcgg]V`Yž_YYd'U'WcdmcZU`` a i g]WdUffg#gWcfYg'fY\YUfgYX'Xi f]b['h\Y'hYfa 'UbX'Vf]b['h\Ya 'hc'h\Y'dUfftWXYW_]b['hYgh'$

5ggYgga Ybh& 9bgYa V`Y 6 7cblf]Vi lijcb UbX DUfH7\YW]b[

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHU]`g. `Ghi XYbhgfidUfh]WdUh]cb`UbX`Wcbhf]Vi h]cb`hc`YbgYa V`Y`fY\YUfgU`g`k]```VY`UggYggYX`Zcfa Uh]j Y`mž`Vm h\Y`YbgYa V`Y`X]fYWcf`fY[i``Uf`ma cb]hcf]b[`ghi XYbhg`]b`fY\YUfgU`g`hc`Ybgi fY`h\Uh`YUfb]b[`]g`hU_]b[d`UW\"`Gi a a Uh]j Y`UggYgga Ybh`]g`UXa]b]ghYfYX`h\fci [\`U'dUfh\W\Y_]b[`hYghUhh\Y`YbX`cZh\Y gYgg]cb": YYXVU_`8]fY\\HUbX`cb[c]b[`ZYYXVU_Xi f]b[`fY\YUfgU`g``UbX`UZhYf`U`a U]b`dYfZcfa Ub\\"

5XX]h]cbU`XYhU]`g.

 $H\Y'dUfftWXYW_]b['hYghi gi U``m'tU_Yg'd`UW'']b''hXY'k YY_'UZhYf''hXY'Z]bU``fY\YUfgU'*#di V`]WdYfZcfa UbW' fK YY_'%$`cf''f%#L''8YtU]`g'cZh\Y'dUfftWXYW_]b['hYghik]```VY'Ubbci bW'X'''K \YfY'dcgg]V`Yž_YYd'U'WcdmcZU`` a i g]WdUffg#gWcfYg'fY\YUfgYX'Xi f]b['h\Y'hYfa 'UbX'Vf]b['h\Ya 'hc'h\Y'dUfftWXYW_]b['hYgh'$

5ggYgga Ybh' . A U]b DYfZcfa UbWY

GMUfhXUMfBch5dd`]WUV`Y

 $8\ YHJ]^cg. \ Ghi \ XYbhg`UfY`fYei \]fYX`hc`dUfh]W]dUhY`]b`h\Y`a U]b`dYfZcfa \ UbWYg`cZh\Y]f`W\cgYb`YbgYa V`Ygž'k \]W\k \]``VY`UggYggYX`Vmh\Y`YbgYa V`Y`X]fYWhcf": \YYXVUW_. `[]j \Yb`]b`k f]h]b[``UZhYf`h\Y`a U]b`dYfZcfa \UbWY"`H\]g`]g h\Y`Z]bU``UggYgga YbhHUg_"$

5XX]h]cbU`XYhU]`g.

9UWX YbgYa V`Y`A U]b`DYfZcfa UbWY`]g`U`[fci d`UWf]j]lmiUbX`Wcblf]Vi hYg`%\$i `hc`h\Y`Wci fgY`a Uf_`f&\$i `]b hchJ`t"9UWX Di V`]WDYfZcfa UbWY`k]``VY`UggYggYX`Vmih\Y`YbgYa V`Y`X]fYWcf`UbX#cf`Ubch\Yf`YI Ua]bYf" Ghi XYbhg`g\ci `X`VY`Uk UfY`h\Uhih\Y`A U]b`DYfZcfa UbWY`UbX`gca Y`fY\YUfgU``h]a Y`Zcf`YUWX`YbgYa V`Y`a Um hU_Y`d`UWY`ci hq]XY`h\Y`fY[i`Uf`qWXYXi`YX`k YY_`mfY\YUfgU``h]a Y"

5ggYgga Ybh(. 7a V& gga Ybh(.

UW

li fb]h]bʻgYhh]b[.`H\]gʻUgg][ba Ybh']gʻgi Va]hhYXʻh\fci [\`Hi fb]h]bʻUbXʻghi XYbhgʻXcʻbch'gYYʻHi fb]h]b]a]`Uf]hmifYdcfhg")

5 https://bxubwww.Fyei]fYaYbhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`m'YbWc'i fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk'``YWhfY'fY'WcfX]b[g''' } \\$

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'hja YHUV'Y

H]a YtUV`Y

8UhY	HmdY	7 cbhYbh
K YY_'%'%+': YVfi Ufm!'&% : YVfi Ufm		7 ci fgY`gW\YXi `Y`]bZcfa Uh]cb`j Uf]Yg`XYdYbX]b[`cb h\Y`YbgYa V`Yg`W\cgYb"

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

Bch'Udd`]WUV`Y"

FYWca a YbXYX FYgci fWYg

Bch'Udd`]WUV`Y"

7ci fgY '9j U`i Uh]cb 'UbX '8Yj Y`cda Ybh

 $8]fYWhUbX`cb[c]b[`ZYYXVUW_Xif]b[`fY\YUfgU`g`UbX`UZhYf`U`aU]b`dYfZcfaUbWY"$

Gi Va |gg|cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybht\fci [\ 'Hi fb]hJbž'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-',) '' ' ' %cf'Ya U] 'h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'\\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ" Image:

UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]hJb'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WUh]cb" Image: Ya U]''9! hYfbU''Gi ddcfhmci 'k]''

Ui hca UhJWJ''mfYW'] Y'U'hJW_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'hc'gdYWJZWJ''mUg_'Zcf'cbY"

Hi fb]hJb'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mj]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUhj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±i\WbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiW\YUh]b[ihciU\W]XYbhU`m\Wcdm]b[iZfca iUigci f\Wik]h\ci hiU\w_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[. '7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]Y 'a cghmfYhU]b]b['h\Y'cf][]bU`ghi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`žUbX']bZcfa Uh]cb'k]h\ci hUW_bck`YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k\YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'UbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk'k\c`Yžk]h\ci h'Uddfcdf]UhY'fYZYfYbW]b['"

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ffgY`Zd`U[]\Uf]ga "Ł gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

• DUfUd\fug]b[z'gi a a Uf]q]b[z'YggUmk f]hb['UbX'ha Y'a UbU[Ya Ybh