ARTh[1qo

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUhjcb	D\cbY
GhYd\Ub]Y^6]g\cd	GHYd\Ub]Y"V]g\cd4 i bgk "YXi "Ui		5ddc]bha Ybh	
			j]U [.] Ya U]`	

GWkcc~7cbhJWh~abZcfa Uh]cb

GVXcc``cZhY`5fhg`UbX`AYX]U

Fcca ' %&ž@YjY`' žFcVYfhK YVghYf6i]`X]b[fl %(Ł

D\cbY.'f\$&E'-',)'(,)*

9a U]` gUa 4 i bgk "YXi "Ui

K YVq]hY. k k k "Ufhq"i bak "YXi "Ui #qUa

 $\label{eq:hamiltonian} $$HY'GWKcc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]hjcbU``7ighcX]Ubgžh\Y'6YXY[U'`fPYbg]b[hcb'WJadigŁž'; UX][U'`fP]hmUbX'5fh\/'8Yg][b'7UadigYgŁ'UbX'h\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cf\W'5\WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`c\WJhYX''$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

±b 'th]g'Wci fgY'mci 'k]``YI Ua]bY'th Y'dfUWjW'cZh Y'g\cfhghcfm''7 cbg]XYf]b['U'j Uf]YhmcZa cXYfb]ghUbX WcbhYa dcfUfmYI Ua d'Ygž'mci 'k]``YI d'cfY'th Y'df]bWjd'Yg'th Uh'\Uj Y'g\UdYX'th Y'XYj Y'cda YbhcZh\]g'Zcfa gi W\"Ug'th Y'i gY'cZglfYUa 'cZWcbgWjci gbYggžYd]d\Ubmžth Y'Z bWjcb'cZWca dfYgg]cb'UbX'th Y'f]gY'cZ WcbhYa dcfUfm'ZjfghdYfgcb'thYgtja cb]Yg"'±b'UXX]tjcbž'k Y'k]```cc_'Uh'hYW\b]ei Yg'UbX'glfUthY[]Yg'Wfi WjU`tc'th Y gi WWygg'cZh\Y'g\cfh'Zcfa 'UbX'i gYZ `'tc'mci f'XYj Y`cda YbhUg'k f]thYfgž']bWi X]b['dc]bhcZj]Yk ž'bUffUtjj Y XYg][bž'th\Y'XYj Y`cda YbhcZj c]W''UbX'th\Y'i gY'cZYWcbca mUbX'gma Vc`]ga "

H\fci [\ci hih\Y 'Wti fgY'k Y'k]'``Wtbg]XYf'h\Y']bZi YbW'cZbYk 'hYW\bc`c[mcb`\ck 'k Y'hY```UbX'hfUbga]h ghtf]Yg. 'k Y'```h\]b_'UVci hih\Y'fY`Uh]cbg\]d 'VYhk YYb'h\Y'g\cfhightfmUbX'h\Y'V`c[ž'h\Y'WtbhYa dcfUfm XYj Y`cda YbhcZh\Y']fghdYfgcb'Wta d`YI 'UbX'h\Y'hfUbga]gg]cb'cZh\Y'g\cfhightfmUbX'h\Y'\Z]fghdYfgcb'Wta d`YI 'UbX'h\Y'hfUbga]gg]cb'cZh\Y'g\cfhightfmH\fci [\`gc\Y]U``a YX]U" K f]h]b['YI Yf\YgYg'k]```VY'dfcj]XYX'UbX'Yj YfmcbY'k]```\Uj Y'h\Y'W\UbW'hc'k cf_g\cd'h\Y]f'k f]h]b['UbX dUfh]\YdUhY']b'h\Y'X]g\W gg]cb'cZ\WYUh]j Y'k cf_"'7 cbg]XYfUh]cb'k]```U`gc'VY'[]j Yb'hc'j c\Wh]cbU``XYj Y`cda Ybh' 5\Wt\fX]b[`mz'mci 'k]```[U]b'j U`i UV`Y'g_]``g']b'YX]h]b[`UbX']b'h\Y'`UhYf'dUfhcZh\Y'\tei fgY'k Y'k]```X]g\W gg'h\Y dfygYbhUh]cb'UbX'gi Va]gg]cb'cZmci f'k cf_'Zcf'di V`]\Wh]cb"

7 ci fgY @YUfb]b[Ci hWca Yg

%" 9I d`U]b'h\Y'df]a UfmY`Ya Ybhg'cZg\cfh'Z]\\f\g\b''\\&" 5ddfU]gY'mci f'ck b'\\\Y'\Uhj Y'k cf_'UbX'h\Y'\\\\Y'\\\Y\Uhj Y'k cf_'cZch\Yfg" ' " 7 ca dcgY'k f]hhYb'hYl hg']b'9b[`]g\"

HYUWX]b[GhfUhY[]Yg

5ggYgga Ybh

 $5 \text{ 'fYUgcbUV'Y'UhhYa dh'a i gh'VY'a UXY'hc'Z '`mWca d'YhY'U```UggYgga Ybhgž'ch\Yfk]gY'U'ZU]``[fUXY'a UmVYUk UfXYX''$

5ggYgga Ybh HUg_g

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[CihWca Yg 5ggYggYX
A]X!GYa YghYf"G\cfh'9ggUm	(\$1	&%#\$' #&\$&%`%%\$\$`DA	%ž [*] &
A Uʻcfʻ7fYUhjjYʻK cf_ʻ	* \$1	&(#\$(#&\$&%`%% \$\$`DA	%ž`&ž`'

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% A]X!GYa YghYf G\cfh9ggUm

 $\textbf{GhUfhXUhY}. \ \texttt{Bch5dd']WUV'Y}$

8YHJ]`g.

%) \$\$!k cfX`YggUm'`Hi fb]h]b`gi Va]gg]cb"`: YYXVUW_j]U`@A G

5XX]h]cbU``XYhJ]`g.

 $\label{eq:continuous} D`YUgY`gYY`A\ ccX`Y`Zcf`Z\ ff\ Yf`XYfU]`g`UbX`fYgci\ fV\ g$

5 X X] h] c b U` ` X Y h U] ` g.

5ggYgga Ybh7f]hYf]U

.

• : `ck #YUX]b['W&\YfYbW'.'hfUbg]h]cbg'VYhk YYb'gYbhYbW'g'UbX'dUfU[fUd\gža cj Ya Ybh'k]h\]b'h\Y d]YW

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

- Wci fgY ci h]bY
- k YY_`mfYUX]b[g
-]bhYfbYhg]hYg'fY`Yj Ubhhc'h\Y'Wci fgY
- Ui X]c 'UbX'j]XYc 'a UhYf]U`

FYWca a YbXYX FYgci fWyg

5 "k YY_mgYhfYUX]b[g'UbX'fY`UhYX'a UhYf]U`'UfY'Uj U]'UV'Y'cb'A ccX'Y

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g Hi fb]h]b Gi Va]gg]cb

⊋mci 'YbWci bhYf'U'dfcV`Ya 'k \Yb'UhhYa dh]b['hc'gi Va]h'mci f'Ugg][ba Ybh'h\fci [\'Hi fb]h]bž'd`YUgY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/