

ARTS2061

7 cbhYa dcfUfm5ddfcUWXYg hc 7]bYa U

HYfa 'C bY''##'&\$&%

7ci fgY Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
8f'>cX]'6fcc_g		Hi YgXUmg'%\$!%% UbX'ch\Yf'h]a Yg'Vm Uddc]bha Ybh	&' %< 'K YVghYf 6i]`X]b[f* %&£' - \$*) , \$&,

Hi hcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
>cX] ⁻ 6fcc_g		Hi YgXUmgʻ‰!‱Ua UbXʻch\Yfʻh]a YgʻVm Uddc]bha Ybh		f* ‰Ł - \$*), \$&,

GWXcc`7cbHJWhi±bZcfa Uhjcb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca " &ž@Yj Y`" žFcVYfhK YVghYf'6i]`X]b['f] %(Ł

D\cbY. 'f\$&Ł'-',) '(,) *

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWKcc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck``YX[Y'h\Y'HfUX]hjcbU``7i ghcX]Ubgž h\Y'6YXY[U'`fPYbg]b[hcb'WUadigŁz'; UX][U'`fP]hmUbX'5fh\/\'8Yg][b'7UadigYgŁ'UbX'h\Y'B[i bbUkU`dYcd'Y'fbi ghfU`]Ub'8YZYbW'': cfW'5WUXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WUadig'cZl BGK ']g`cWJhYX''$

7ci fgY⁻8YHJ]`g

7fYX]hDc]bhg**

Gi a a UfmcZ'h\Y'7ci fgY

 $\label{thm:linear} $$H\g''\d'' = fgY''\d'' = fgY''\d$

5ggYgga Ybh

: i ```XYhJ]`g`UVci h`YUW\`UggYgga Ybh'hUg_ž`]bWi X]b['hcd]Wg#i Ygh]cbgž`fYgci fWfgž': 5E g`UbX`a Uf_]b[Wf]hYf]U`UfY`dfcj]XYX`cb`h\Y`Wci fgY`A ccX`Y`UbX`UfY`X]gWi ggYX`]b`WUgg"

5ggYgga Ybh HUg_g

5ggYgga Ybh'HJg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
FYdcfh	(\$1	% #\$' #&\$&%`%%\$\$`DA	%ž [*] &
F YgYUfWX '9ggUm'	*\$1	\$' #\$) #&\$&%'%% \$\$"DA	%ž.' ž.(

5ggYgga Ybh8YhJ]`g

5ggYgga Ybh% F Ydcfh

@Yb[h\. *\%\$\$*k cfXg

8 YW] g.

₽X]j]Xi U`

%\$\$\$`k cfXg`d`i g`gi ddcff\b['j]gi U``a UhYf]U`"

 $\label{eq:local_$

: YYXVUW_'j]U'@A G

5XX]h]cbU`XYhU]`g.

GYY'A ccX`Y'Zcf`XYhJ]`YX`]bZcfa Uh]cb`UVci hh\]g`UggYgga YbhhUg_"

Gi Va]gg]cb bch/g. k f]h/Yb gi Va]gg]cb j]U Hi fb]h/b

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h XYbhg"Xc "bchgYY"Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h XYbhg XYbhg XYbhg"Xc "bchgYY"Hi fb]h XYbhg XYbhg XYbhg XYbhg"Xc "bchgYY"Hi fb]h XYbhg XYbhg$

5ggYgga Ybh& FYgYUfW 9ggUm

@Yb[h\. &' \$\$ k cfXg

8 YW] g.

₽X]i [Xi U`

&' \$\$ k cfXg

: YYXVUW<u>_</u>'j]U'@A G

H\]g^]g^h\Y^Z]bU`^UggYgga Ybh

 $\hbox{Gi Va]} gg] \hbox{cb'bch'} g. \hbox{'D'YUgY'gYY'Wt'i fgY'a ccX'Y} \\$

	GWYYb]b[Eg[fUSeWaX>ahWS`VEZS_WfbUbY';]`cc`mž'&\$%(Ł
K YY_').'%) 'A UfWX'!'% A UfWX	Hi lcf]U`	hcd]W(.`YId`cf]b[`dfUW]VWg`cZUddfcdf]Uh]b[`UbX`fYdifdcg]b[`dfY!YI]gh]b[`UiX]cj]giU``aUhYf]U``fWUgYgh X]Yg`UbX`XYVUhYE"
	FYUX]b[9ggYbh]U`fYUX]b[fdfYdUfUh]cbZcf`YWhffY/hcd]W)Ł
		7 Uff][mžA Y[Ubˈf&\$%&Ł"'"F Y!ghU[]b[ˈh\Y W]bYa U. BekUZał:]`a 'GdYWfUhcfg\]džUbX'h\Y F YXi bXUbhBYk 'F Ya U_Yž" GWfWWb]]b/[f£WY BSeffGdYW]U``]ggi Y'"I bh]a Y`m7]bYa U"Łžj c`"', "
		8YHJ]`gʻUVci hgi dd`Ya YbHJfmfYUX]b['UbX'fYgci fWycb'A ccX`Y"
	@YWhifY	@YWhifY'). FYa U_Ygž±bhYfa YX]U`]hmUbX h\Y'"5ZhYf`]j Yg" cZ7]bYa U
	GWfYYb]b[fcb`]bYŁBekUZa`flig'JUb'GUbhž'%-,Ł'UbX``Dgm\\\c fb`ZfYX'<]h\\\ \\c\W_Z*%*\$Ł
K YY_'*.'&&'A UfWX'!'&* A UfWX	< ca Yk cf_	F YgYUfWX 'K YY 'GYY'A ccX`Y'Zcf'XYHJ]`g
K YY_'+.'&-'A UfVX'!'& 5df]`	Hi lcf]U`	8] by Wyggglab food for any lyfxyx [i] bixy by [cgy wb Xidfo [U"Yf]YgifgYY'AccXYL
^d\\\$V[`Y/bd\\bSd\$f[a` Xac	H WHURGUMYIS[V	9ggYbh]U`fYUX]b[`fdfYdUfUh]cb`Zcf``YWhifY`UbX

	@YWhi fY	@YWhifY'+.'5Wcigh]Wg'UbX'gdUWY.'7]bYaU]b'Ub'9IdUbXYX':]Y`X'
	GWfYYb]b[fC b`]bYŁ'9 [dZaaV f7 Y`]bY'GW]Ua a Už'&\$%(Ł'fH67 Ł
K YY_', .') '5df]`'!'- '5df]`	C b`]bY 5 W]j]hm	Hi hcf]U`'WUggYg`k]```fi b`Ug`cb`]bY`UWfj]hJYg'/ gh XYbhWcbgi `hUhJcbg`Xi Y`hc`di V`]W\c`]XUmh\]g k YY_`fgYY`A ccX`Y`Zcf`XYHJ]`gŁ'
	F YUX]b[9ggYbhjU`fYUX]b[`fdfYdUfUhjcb`Zcf``YWhifY`UbX hcd]W, Ł
		; fUbhž7Uh\Yf]bY'f&\$\$\$\!'`~kkk"UihYif"Vca3"~` EUdWW (\$'f%\L
		A UmYfz'Gcd\]Y'f&\$%* Ł'Ba {f[US^3` [_ S'e, fZW@W 8W_ [` [ef 5 [` W_ Sz'@cbXcb.`="6"'HUi f]g'fYI WYfdhgŁ UbX'YY'A ccX'Y'Zcf'gi dd'Ya YbhUfmfYUX]b[g
	@YWn fY	@YWhifY',5i hYif]ga]b WcbhYa dcfUfm W]bYa U'fdUfh%L/ WUgY ghi Xm'fdUfh&L
	GWfYYb]b[fbb`]bYŁ8[eZ FS`] fbbXfYU'5fbc`Xž'&\$\$-ŁWW
K_YY_'''&'5df]`'!''%* 5df]`	Hi hcf]U`	9ggUmik f]h]b['k cf_g\cd'/ 'WUgY 'ghi Xm'UbU`mg]g
	F YUX]b[9ggYbh]U`fYUX]b[fdfYdUfUh]cbZcf``YWhffY`UbXhcd]W-Ł
		91 W/fdhg'Zfca 'EbWUfSUg/Sd6[Y[fS^7 XWUfe, 59; S`V 5a`fW_ bacSck 5[`W_ SVm?f]ghYb'K\]ggY``f8i _Y I D'&\$%(Ł
		8YHJ]g'UVci hgi dd'Ya YbhJfmfYUX]b[g'UbX fYgci fWg'cb'A ccX'Y"
	@YWhi fY	@YWhifY - GdYWHUW `UfYZZYWhg UbX h\Y fY`Uh]cbg\]d VYhk YYb bUffUh]j Y UbX gdYWHUWY]b WcbhYa dcfUfmV`cW_Vi ghYf W]bYa U
K YY_'%\$.'% '5df]`'!'&' 5df]`	Hi hcf]U`	H\Y`fY`Unjcbg\]dfgEVYhkYYb`bUffUhjjY`UbX gdYWfUWY`]b`WcbhYadcfUfmV`cW_VighYf`WjbYaU'/ YggUmkf]njb[`kcf_g\cd`&`
	FYUX]b[fYUX]b['Zcf``YWi fY'/ 'hcd]W%.
		GWcbWfz>YZZfYmf&\$\$+Ł" "=blfcXiVMjcbz"]bEWSIW 3df[efe, 5[W_SSffZW?SdY[eaXFSefWEfkWS V Ba1{f[Ue"8if\Ua:8i_YIb]jYfg]lmDfYgg"

	GWcbWfz">YZfYmff%-) t""HfUg\]b["h\Y"5WUXYa m HUghYz"91 Wfggz"UbX"Ub"9a Yf[]b["Dc`]h]Wg"cZ 7]bYa Uh]WGhmYz"EUdWW jc`" *"
	8 YfU]`g`UVci figi dd`Ya YbfUfmfYUX]b[g`UbX fYgci fWg`cb`A ccX`Y"
@YWhi fY	@YWhifY'%\$7]bYaU'Uh'h\Y'aUf[]bg.:Wi`h W]bYaU'UbX'h\Y'ffi\YUhf]WU`'YIdYf]YbWYfi
GWfYYb]b['7h[^6 W\$V \$ fGUa 'F U]a]ž'% , +Ł'
	fcb!WUa di gʻUb#cf'cb`]bY`gWfYYb]b[`Zcf'Z bU`kYY_Ł

F Ygci fWyg

DfYgWf]VYX F Ygci fWfg

9ggYbh]U`fYUX]b[.

 $\label{thm:continuous} $$HY'gYhk YY_mfYUX]b[g'Zcf'h\]g'Wci fgY'UfY'Uj U]'UV'Y'h\fci [\ensuremath{\mathchar`e}]U'A ccX'Y'!!'d'YUgY'a U_Y'gi fY mci 'Vf]b['Y'YWfcb]Wcf'\UfX'Wcd]Yg'cZh\Y'fY'Yj UbhfYUX]b[g'hc'WUgg'YUW\'k YY_'gc'h\Uh'h\Ym'WUb'VY dfcdYf'mX]gW ggYX'UbX'XYVUh'X'''$

Cb`]bY`WcifgY`fYgcifWY.'5FHG&\$*%]g`cb`AccX`Y—d`YUgY`Wcbgi`h']hfY[i`Ufm'Zcf`]adcftUbh']bZcfaUh]cbUVcihh\Y`WcifgY"

9UW\ 'k YY_'g'hcd]W\Ug']hg'ck b'gYWfcb']b'A ccX`Y"'\ \D'h\YgY'gYWfcbg'mci 'k]``\ZbX.

- 5b']blfcXi Wjcb'hc'h\Y'hcd]WVY]b['Yl Ua]bYX/
- Gi [[YghYX Z fh\Yf fYUX]b[
- @]b_g hc fY Yj Ubhik YVg]hYg
- 5 WWY gg 'hc "YWh fY 'fYWcfX]b[g
- Hi hcf]U`dfYdUfUh]cb']bZcfa Uh]cb

FYWca a YbXYX FYgci fWYg

FYWca a YbXYX VUW[fci bX fYUX]b[.

- 8 "B "F cXck]W_'H\Y'J]fh U''@]Z''cZ:]`a ''f@cbXcb'UbX'7 Ua Vf]X[Yž'A Ugg". '< UfJ UfX'I Dž'&\$\$+Ł"'5b]a dcftUbhUbX'_YmhYI h]b'WcbHYa dcfUfm'Z]`a 'h\Ycfmž'UbX'U'_YmhYI h'Zcf'h\]g'Wci fgY"'9! 'Vcc_Uj U]`UV'Y'h\fci [\'h\Y']VfUfmž']b_'cb'A ccX'Y
- ; `mb'8Uj]gž? Um'8]W_]bgcbž@gU'DUht]ž'UbX'5a mJ]``UfY'cž:]`a 'Ghi X]Yg.'5'; `cVU``±bffcXi Wf]cb fBYk 'Mcf_'UbX'@cbXcb.'F ci h`YX[Yž&\$%) Ł"'H\]g'fYW\bhidi V`]WUh]cb']g'U'i gYZ ``fYgci fW\'Vch\'Zcf h\cgY'bYk 'hc':]`a 'Ghi X]Yg'UbX'Zcf'h\cgY'k \c'UfY']a a YfgYX']b'h\Y'Z]Y`X"'Ghfcb[`mfYWca a YbXYX'!! YVcc_'Uj U]`UV`Y'h\fci [\'h\Y'`]VfUfmž`]b_'cb'A ccX`Y

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

7 ci fgY'a UhYf]U'g'UbX'Wci fgY'ghfi Whi fY'f]bWi X]b['h\Y'fYUX]b[gž'gWfYYb]b[gž'cfXYf]b['cZhcd]Wg'UbX'hcd]Wgž UbX'h\Y'Zcfa g'cZUggYgga Ybhz'UfY'fY[i `Uf`mfYj]gYX'VUgYX'cb'ghi XYbhZYYXVUW_'Ug'k Y```Ug'bYk 'X]fYWf]cbg UbX'bYk 'di V`]WUh]cbg']b'h\Y'Z]Y`X": cf'h\]g'fYUgcbž'mci f'h\ci [\hg'cb'h\Y'Wci fgY'UfY'dUfh]W'`Uf'mi gYZ ``UbX i U'i UV'Y'UbX'k Y'k]``VY'Uq_]b['mci 'hc'fYZYWhcb'mci f'`YUfb]b[']b'h\Y'Wci fqY'Xi f]b['h\Y'qYqq]cb"

7 cbhYa dcfUfm5ddfcUWXYg'hc'7]bYa U\Ug'VYYb'fYj]gYX'Zc``ck]b['ZYYXVUW_Wc``YWhYX'Zfca 'ghi XYbhg']b dfYj]ci g'mYUfg'fVch\'h\fci [\'h\Y'75H9=UbX'a m9l dYf]YbWY'ghi XYbhZYYXVUW_UbX'h\fci [\'ch\Yf ZYYXVUW_E''DfYj]ci g'ghi XYbhg']b'h\Y'Wci fgY'\Uj Y'dfcj]XYX'dcg]hj Y'ZYYXVUW_UVci hVch\'h\Y'VfYUXh\'cZ Zj'a 'UbX'ch\Yf'gWfYYb!VUgYX'a UhYf]U`Yl Ua]bYX']b'h\Y'Wci fgY''=YbWci fU[Y'mci 'hc'[]j Y'ZYYXVUW_cb'h\Y

Wci fgYžbchcb`mih\fci [\'h\Y'Zcfa l	J``ZYYXVUW <u>_</u> `dfc\W*gg`V	i h'U`gc`]b'WUgg'X]gW	gg]cbgʻXi f]b[ˈh\YˈgYa Ygl	Yf"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g Hi fb]h]b Gi Va]gg]cb

⊋mci 'YbWci bhYf'U'dfcV`Ya 'k \Yb'UhhYa dh]b['hc'gi Va]h'mci f'Ugg][ba Ybh'h\fci [\'Hi fb]h]bž'd`YUgY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/