ARTS2122

DYfZcfa UbWY DfcXi Wgcb

HYfa 'C bY''##'&\$&%

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
AY[ʿAia ZcfX	a "a i a ZcfX4 i bgk "YXi "Ui	6m'Uddc]bha Ybh	Fcca '‱ž	-',)'(,*)
			K YVghYf 6 X[

Hi hcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUhjcb	D\cbY
5bh\YU'K]``]Ua g f8]fYWcfŁ	-	8U]`m']b"fY\YUfgU` UbX"Vm'Uddc]bha Ybh		-',) (, *' fW#c A Uf_ A]hWXY``Ł
AUf_`A]hWXY`` fDfcXiWMjcb AUbU[YfŁ	-	8U]`m']b"fY\YUfgU` UbX"Vm'Uddc]bha Ybh	∉`AmYfg GhX]c	- ' ,) '(, *'
DUi `A UHNYk g f8Yg][bYfŁ	ĕ ĕ	8U]`m']b'fY\YUfgU` UbX'Vm'Uddc]bha Ybh	∉`AmYfg GhiX]c	-',)`)'+,

GWkcc~7cbhJWh~abZcfa Uh]cb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca " %&ž@Yj Y`" žFcVYffiK YVgfYf'6i]`X]b['f) %(Ł

D\cbY.'f\$&Ł'-',)'(,)*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubgž h,Y'6YXY[U'`fPYbg]b[tcb'WJa di gŁž'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7Ua di gYgŁ'UbX'th,Y'B[i bbUk U` dYcd'Y'fbi gtfU`]Ub'8YZYbW': cfW'5WJXYa m]b'7UbVYffUŁ'cZth,Y'`UbXg'k \YfY'YUW\.WJa di g'cZl BGK ']g `cWJtYX''$

g\UdY`cZh\Y`Z]bU``dYfZcfa YX`dfcXi Whžmci f`gi [[Ygh]cbg`UbX`dYfgdYWh]j Yg`cb`h\Y`Ybei]fmUfY fY`]YX`i dcb`Uh'U```h]a Yg"`H\Uh']gžh\Y`Wci fgY`\Y`dg`mci `hc``YUfb`Vch\`hc`hU_Y`X]fYWh]cb`Zfca `U dfcZYgg]cbU``dfUWh]h]cbYfžUbX`hc``YUX`UbX`]b]h]UhY`XYW]g]cbg`UbX`UWh]cbg"``DUfhcZmci f`hUg_`]g`hcbUj][UhY`h\]g`Wca d`YI `dfcWYgg`k]h\`[YbYfcg]miUbX`]a U[]bUh]cb"

 $H\g Wc i fg Y c ZZ Y fg U f U f V f V c d d c f h b] m k] h \] b h \ Y 65] b H \ Y U f Y U b X D$

b\$

5ggYgga Ybh

Q YfY'mci 'WUb'ci h]bY'UbmfY'Yj Ubh]bZcfa Uh]cb'h\Uhk Ug'bch]bWi XYX']b'5 ≠ G'Vi ha Umdfcj Y'\Y`dZ ``Zcf mci f'ghi XYbhg": cf'Yl Ua d'Yžmci 'a][\hdfcj]XY'XYhU]`g'cb'h\Y'fYZYfYbW]b['gmghYa ž`]b_g'hc'dfYj]ci g ghi XYbhYl Ya d'Ufg'cf'h\Y'XYg][bUhYX'k YY_`]b'h\Y'Wci fgY'h\Uhmci 'k]``X]gWi gg'h\Y'UggYgga YbhUh`Yb[h\" = dcfhUbhmž'h\]g'gYWf]cb']g'Ub'UfYU'Zcf'mci 'hc'dfcj]XY']bZcfa Uh]cb'h\UhXcYg'bch[c'h\fci [\'h\Y'Uddfcj YX [cj YfbUbWYghfi Whi fY"Q

5ggYgga YbhHUg_g

5ggYgga Ybh'HJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[CihWca Yg 5ggYggYX
7 cblf]Vi ljcb hc DfcXi Wjcb	*\$1	: fca 'h\Y'Z fghfY\YUfgU``cb % #\$%%%hc'h\Y'Z bU` dYfZcfa UbWY"	%ž &ž'
FYZYW¶jY`>cifbU`	' \$1	\$, #\$' #&\$&%'%%) - 'DA	%ž'&ž'' ž'*
DfcXi Wjcb'6'c['	%\$I	6ma]Xb][\hcb`YUW\cZh\Y XUmg`mci `\Uj Y`VYYb U``cWUhYX`hc`VY`GhU[Y A UbU[Yf"	&ž') ž'*

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% 7cblf]Vi h]cb hc DfcXi Wh]cb

GhUfhXUhY. Bch5dd`]WUV`Y

8 YW] g.

8YjY`cdYX'h\fci [\ci hfY\YUfgU`dfcWfgg UbX di V`]WdYfZcfa UbWfg"

 $: \mathsf{cfa}\ \mathsf{Uhj}\ \mathsf{Y}^\mathsf{T}\mathsf{UbX}^\mathsf{T}\mathsf{gi}\ \mathsf{a}\ \mathsf{a}\ \mathsf{Uhj}\ \mathsf{Y}^\mathsf{T}\mathsf{Z}^\mathsf{Y}\mathsf{Y}\mathsf{X}\mathsf{V}\mathsf{U}\mathsf{W}^\mathsf{T}\mathsf{j}\ \mathsf{Y}\mathsf{X}^\mathsf{T}\mathsf{k}\ \mathsf{jh}\mathsf{k}\,\mathsf{jb}^\mathsf{T}\ \mathsf{'k}\ \mathsf{Y}\mathsf{Y}_\mathsf{g}^\mathsf{T}\mathsf{c}\mathsf{Z}\mathsf{Z}\!\mathsf{j}\mathsf{b}\mathsf{U}^\mathsf{T}\mathsf{d}\mathsf{Y}\mathsf{f}\mathsf{Z}\mathsf{c}\mathsf{f}\mathsf{a}\ \mathsf{Ub}\mathsf{W}^\mathsf{T}^\mathsf{T}\mathsf{k}\,\mathsf{J}\mathsf{k$

5XX]h]cbU`XYhU]`q.

K \Yb'UggYgg]b['mci f'Wcbff]Vi hlcbz'mci f'8]fYWcf'UbX'7cbj Ybcf'k]``'c]bhmVY'`cc]b['Uh

UdWSf[hWSbbdaSUZ. \ck 'ZJf'mci 'XYj Y`cd'cf][]bU`]XYUgž'UbX'HJ_Y'f]g_g'-fYZYWMYX']bž'Zcf
Yl Ua d`Yž'mci f'Wca a UbX'cZmci f'ck b'fc`Y']b'fY`Uh]cb'hc'h\Y'k \c`Y'k cf_/fY`Uh]cb'hc'ch\Yf
dYfZcfa Yfg/mci f'h]a]b['UbX'gYbgY'cZgdUh]U`]mm'UV]`]mihc'Wca a i b]WUhY'hYl h'Wca a UbX'cZ
cVYWgž'a UhYf]U`g"



h\fci [\"h\Y"`c[Vcc_"""ffYdfcXi WYX"k]h\"dYfa]gg]cbŁ"

⊋mci `WUb`\Ub[`]b'h\YfYžfYWcfX'h\Y`Wf]gYgž'hi fb]b[`dc]bhgž'gi fdf]gYgž'ei Ygh]cbgž'UbX`ZJ]`i fYgž'mci `k]``\Uj Y`gca Yh\]b[`a i W\`a i W\`f]W\Yf`]b`A UfW\"`-ZU``Y`gY`ZJ]`gž'hfmih\YgY`hk c`ghfUhY[]Yg.

%" WfYUhY 'h\ fYY'ei Ygh]cbg'Uhih\ Y'YbX'cZh\ Y'XUmih\ UhifYZYWhk\ \Uhi]g'i f[Ybhcf'Uf]g]b['Zcf'mci "'H\Ym a][\hiVY'gdYW]ZWhc'gca Yh\]b['mci 'UfY'Xc]b[žcf'a cfY'[YbYfU''ei Ygh]cbg''Uvci ha U_]b['h\YUhfY/ &" fYhi fb'hc'U'dfYj]ci g'XUm'UbX''cj Yf!k f]hY''mci f'Ybhfmk]h\ 'bYk 'cVgYfj Uh]cbg'''Wci 'g\ci 'X'U'gc'XUhY UbmbYk 'Ybhf]Yg"

$H\Y'FYZYWh]cb$

F Uh\Yf'h\Ub`\UbX]b[i]b'h\Y'Ybh]fY'>ci fbU``hc'h\Y'7 ci fgY'7 cbj Ybcfž'h\]g'UggYgga YbhUg_g'mci 'hc'gY`YWi_YmgYWl]cbg"'H\YgY'a][\h'VY'h\Y'Wf]gYg'UbX'ei Ygh]cbg'a Ybh]cbYX'UVcj Yž'cf'h\Yma][\h']bj c'j Y gca Yh\]b['Y'gY'U[U]b"' & 'UXX]h]cbž'Uh`YUgh'cbY'Ybhfmg\ci `X'fYgdcbX'hc'cbY'cZh\Y'gYhfYUX]b[g'fbb A ccX`YŁ"

>cifbU`ŽFYZYWh]cb'1FYZYWh]jY'>cifbU`

H\YfY UfY k c Wa dabybhg Zafigi Va]gg]ab.

%" >ci fbU``GY`Z\5ggYgga Ybh: cfa "'H\]g`]g`BCH'; F5898`Vi hfYei]fYX'hc`dUgg'h\Y`UggYgga Ybh'` D`YUgY`gi Va]hih\]g`]b`\UfX`WcdmZcfa 'Vm-'" \$Ua 'hc`A Uf_'A]hW\Y``cb`A cbXUm%: YVfi Ufm &\$&%"H\]g`Zcfa ']g`Uj U]`UV`Y`]b`h\Y`5ggYgga YbhgYWf]cb`]b`A ccX`Y"``Mci f`7 cbj Ybcf`k]``dfcj]XY ZYYXVUW_`k]h\]b`%k YY_`cZgi Va]gg]cb"

&

GiaaUhjj Y ZYYXVUW_fYWY]j YX k]h\]b '' 'k YY_g cZgi Va]gg]cb"

5XX]h]cbU`XYhU]`g.

Mci 'k]```VY'U``cWUh'X'' 'cf'a cfY'XUmg'h\fci [\ci hih\Y'fY\YUfgU``UbX'dfcXi Wljcb'Zcf'k\]W\ 'mci 'k]```VY GHJ[Y'A UbU[Yffij'F Ydcfhž k\]W\ 'g\ci 'X'VY'Ub'cf[Ub]gYX'UbX'dfUWljWU``fYWcfX'cZYI YfWlgYg'UbX'UWlj]hjYgž'_YmX]gW gg]cb'dc]bfgž hYW\b]WU``bchYgž'UbX'UWljcb'dc]bfg"'fl b`]_Y'mci f'>ci fbU``]hXcYg'bchWcbHJ]b'mci f'dYfgcbU``fYZYWljcbg"Ł CbY'cZh\Y'_YmZ bWljcbg'cZh\Y'fYdcfh]g'nc'dfcj]XY'Ub'UWW fUhY'UbX'i gYZ ``S[VWZ_W_a[dWZcf'YUW\dUff]W]dUbh']b'h\Y'dfcXi Wljcb'Ug'nc'k\Uhh\Y'dYfZcfa UbW'YbgYa V'Y'\Ug'XcbY'UbX'X]gW ggYX'cb'cbY'XUm cZfY\YUfgU``cf'dYfZcfa UbW'UbX#cf'k\Uh]hbYYXg'nc'dfYdUfY'cf'WcffYWhZcf'h\Y'bYI hXUm''C b'h\Y'XUmih\Uh mci 'UfY'GhU[Y'A UbU[YfZmci 'k]``'U`gc'bYYX'nc'Uff]j Y'U``]hhY'YUf'mž'W\YW_h\Y'dfYj]ci g'XUmfij'fYdcfhz'UbX\Y'd'dfYdUfY'h\Y'gdUWY''

 $\label{eq:continuity} $$H\setminus]g^UggYggaYb^UWb^cfX]b[\ 'hc''h\Y'Zc``ck\]b[\ 'Wf]hYf]U.$$

- WUf]mrUbX'YZZYWfj YbYgg'cZgi a a UfmcZ_YmYl YfWjgYg'UbX'X]gW gg]cbg
- XYHJ]`'UbX'Wca d`YhYbYgg'cZ'hYW\b]WU`'bchYg'UbX'UWfjcb'dc]bhg

9 UWX 'c Zmci f'Ybhf]Yg'k]```VY 'gi Va]hhYX 'hc'Ub'cb`]bY 'XcW a Ybhj]g]V`Y 'hc'U```Wci fgY 'ghUZZUbX'dUfh]W]dUbhg'' Mci 'k]``'VY ']bZcfa YX'UVci hih\]g'cb`]bY 'ZJW[]hmcb'h\Y'Z[fghXUm'\Mci 'k]``fYWY]j Y'Zcfa Uh]j Y'ZYYXVUW_

5 http://bxubww.F.Yei]fYa Ybhq

5 thY b X U b WY 'U h U `` 'f Y \ Y U f g U `g 'U b X 'd Y f Zc fa U b WY g '] g 'a U b X U h c f m] b 'h \] g 'W c i f g Y 'U g 'h \ Y m U f Y X] f Y W n U g g c W U h `Y U f g U `g 'U b C i t W c a Y g " \ \ Z U g h X Y b h X [`e fa S f f W V i [f Z a g Y a a V U S g e W U h `Y U g h X Y b h X '('k Y Y _ g 'c Z f Y \ Y U f g U `g 'U b X '% \$ i 'c Z h Y 'D f c X i W] c b 'K Y Y _ `U b X 'h Y 'D Y f Z c fa U b W 'K Y Y _ ž h \] g k] `` V Y 'W c i b h Y X 'U g 'U b c b ! g i V a] g g] c b c Z 5 g g Y g g a Y b h % f I c b f f] V i h c b h c 'D f c X i W] c b . * \$ i \ L "

Mci f'dfcZ/gg]cbU`]ga ']g`Uggi a YX`]b 'h\]g`Wci fgY"`Mci 'UfY`YI dYWfYX`hc 'hi fb`i d`cb!nja Yž'Yj Yfminja Yž dfYdUfYX`UbX`Wca a]htYX`hc 'Wcbhf]Vi hY`hc 'U`hYUa 'dfc YWf'`'=Zmci 'UfY`]``žfi bb]b[``UhY`cf`Zcf`gca Y`[ccX fYUgcb`]h]g`]a dcgg]V`Y`Zcf`mci 'hc 'UhHYbX`U`fY\YUfgU`ž]h]g`mci f`fYgdcbg]V]`]mhc 'WcbhUWh\Y`7 ci fgY 7 cbj Ybcf`fA Y['A i a ZcfXŁ'UbX`DfcXi Whcb'A UbU[Yf`fA Uf_'A]h\XY``Ł'Ugʻgccb`Ugʻdcgg]V`Y`UbX`hc`dfcj]XY fY`Yj UbhXcW a YbhUhjcb`YI d`U]b]b['h\Y`fYUgcb"`=Zmci 'WbbchUhYbX`U`dYfZcfa UbWY`h\YgY`ghUZa Ya VYfg a i ghVY`WcbhUWhYX`]a a YX]UhY`m'``: U]`i fY`hc`Xc`gc`k]```VY`hfYUhYX`Ugʻbcb!UhHYbXUbWY"BchZ]WUhjcb`cZ UVgYbWY`]g`U`gc`Ub`]a dcfhUbhWci fhYgmhc`mci f`ZY``ck 'WUghUbX`WfYk 'a Ya VYfg"

7cj]X!% <YU`h\/ GUZYhm=bZcfa Uh]cb

\hdg.#kkk"Ufhg"i bgk "YXi "Ui #gUa #MfYUh]j Y!dfUWh]WY#Yga Y!h]a VYfm!WfYUh]j Y!dfUWh]WY!`UV#Mcj]X!gUZY!dfUWh]WYg

7 ci fqY GWX YXi `Y

J]Yk Wugg'ha Yhuv'Y

H]a YHJV`Y

8UhY	HmdY	7 cbh/bh
K YY_'%'%) ': YVfi Ufm!'%- : YVfi Ufm		H\]g`WcifgY`fibg`Zfca`h\Y`ZfghXUmcZfY\YUfgU`ž%, >UbiUfm&\$&%zhc'h\Y`ZbU``giaaUhjY`ZYYXVUW_ acaYbhcb'%)`AUfW\"``+h\L_Yg`d`UWY`aU]b`mdf]cf`hc h\Y`WcaaYbWaYbhcZHYfa'%"``GYY'h\Y`7cifgY 7cbjYbYfž8f`AY[`AiaZcfX fa"aiaZcfX4ibgk"YXi"UiŁUg`kY```Ug`AccX`Y`Zcf h\Y`fY\YUfgU``UbX'dfcXiWf]cb`gW\YXi`Y"``
		Gh XYbhg g\ci `X bchY h\UhfY\YUfgU`g fi b '\$Ua!).'\$da `A cbXUmhc: f]XUmZcfh\Y ``UghZci f k YY_g cZh\Y Gi a a Yf 6fYU_f% #\$%#&% &&&&\Z UbX h\Yma i ghU`gc `VY 'Uj U]`UV`Y`).'\$!%\$.'\$da A cbXUm!: f]XUm]b K YY_%cZH%ZcfhYW\b]WU` fY\YUfgU`g 'UbX').'\$!%\$.'\$da `A cbXUmGUhi fXUm]b

K YY_'&'Zcf'dYfZd	cfa UbWYg"
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FYgci fWyg

DfYgWf]VYX F Ygci fWYg

 $GYY^*A\ ccX^*Y^*UbX^*C\ b^*]bY^*F\ YUX]b[\ ^*F\ Ygci\ fWYg^*Zcf^*h^Y^*Zc^*^ck\]b[\ ^*UbX^*ch^Yf^*fYUX]b[\ g^*UbX^*Ui\ X]c!j\]gi\ U^*a\ UhYf]U^*g.$

 $K = "M'9"] bcf": i W g z f P: f g J] g] h hc U G a U D U A G ca Y E i Ygh c b g hc 5 g 5 D U m z F Z W S f W z ' (z & f \& $ (! - ")) (! - ")) } c f C S g S D U m z F Z W S f W z C S G U C S G C$

K _ '&. 'h/W

FYWca a YbXYX FYgci fWyg

GYY'A ccX`Y'UbX'Cb`]bY'FYUX]b['@]gh'

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dhJb['tc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]hJbž'd'YUgY
hY'Yd\cbY'9| hYfbU'Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'#c'i'#c'i'
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\ 'Hi fb]hJb'mci 'a Um'Udd'm\Zcf'Ub'Yl hYbg]cb\ZVi hmci
a i ghfYhU]b'mci f'hJ\W_Yhbi a VYf'\Zfca '9| hYfbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj]XYbW''hc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WUh]cb'''#c''' Zmci 'Ya U]''9| hYfbU''Gi ddcfhmci 'k]'`

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/