

MUSC4706

Aig]W9bgYaV`Y

HYfa 'C bY''##'&\$&%

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWJhjcb	D\cbY
>c\b'BUd]Yf	- 0	5		\$&'-',) *-)''cf -\$*)')-\$&

GWkcc~7cbhJWh~abZcfa Uh]cb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca ' %&ž@Yj Y`' žFcVYfhK YVghYf6i]X]b[fl %(Ł

D\cbY. f\$&L'-',) '(,) *

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. 'k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7UadigYgŁ'UbX'thY'B[ibbUkU`dYcd'Y'fbigtfU]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZthY'`UbXg'k\YfY'YUW\cong WJadig'cZIBGK']g`cWJtYX''$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7c]igfgY

H\]g`]g`U`dfUW]WU``a i g]WWci fgY`h\UhcZZYfg`mci `Ub`cddcfh b]mhc`dUfh]W]dUhY`]b`hk c`gY`ZgY`YWYX YbgYa V`Yg"H\Y`A i g]WDfc[fUa `cZZYfg`U`k]XY`W\c]W`cZga U```UbX``Uf[Y`YbgYa V`Yg`h\Uh]bWi XY`WUgg]WU`ž\UnnžY`YWfc!UWci gh]WYbgYa V`Yg`UbX`k cf`X`a i g]WYMci `a UmgYY_'Uddfcj U``hc`Zcfa `mci f`ck b`YbgYa V`Yi bXYf`h\Y`gi dYfj]g]cb`cZU`a Ya VYf`cZh\Y`a i g]WghUZZ/Uddfcj U``]g`ghf]WhmVUgYX`cb`a Yf]hUbX`ghUZZUJi U]`UV]`]mm`A i g]W9bgYa V`Y`UXj UbWrg`mci f`dYfZcfa UbWrg_]``gžh\Y`gh XmcZj Uf]ci g`YbgYa V`Y`ghmYg`UbX dfUWf]WYgžUbX`h\Y`fY`Yj UbhiYbgYa V`Y``]hYfUhi fY"`A cfY`VfcUX`mžh\Y`Wci fgY`]g`]bhYbXYX`hc`XYj Y`cd`mci f`YI dYf]YbWr`cZa i g]Wa`U_]b[`Ug.`U'_ ge[US^SUfZhc`YI hYbX`mci f`a i g]WU``_bck`YX[Y`UbX`i bXYfghUbX]b[/U`eaU[S^SUf]

5ggYgga Ybh

, \$I `UHY'bXUbW'`]g'h\Y`a]b]a i a `fYei]fYa Ybh'ZcfYUW\YbgYa V'Y''
? YYd']b`a]bX'h\Uh`fl\Zca Y`YbgYa V`Yg`fYei]fY`Ui X]h]cbg'df]cf'hc'k YY_'\%''f\ZcYy YfU``fYei]fY`mci 'hc`\Uj Y
Ub`]bghfi a Ybh'h\Uhmci 'WUb'd`Um''fl \L'Gca Y`\Uj Y``]a]hUh]cbg']b'bi a VYf`cZdYcd`Y`k \c'\WUb'VY
UVW\dh\X''\8 YhU]`YX`]b\Zcfa Uh]cb`UVci hYUW\YbgYa V`Y`]g``]gh\X`\YfY.
\hdg.##gUa "Ufhg"i bgk "YXi "Ui #ghi XYbhg#Ygci f\W'g#a i g]\Wghi XYbhg#a i g]\WYbgYa V`Yg#`C b`mUddfcj YX
YbgYa V`Yg'`c]bYX`]b'h\Y'W ffYbh'h\fa '\WUb'VY'i gYX'\Zcf'\wci fgY'UggYgga Ybh''F Y`Yj Ubh']b\Zcfa Uh]cb'\UVci h
YUW\YbgYa V`Y`]g'i gi U``mi dXUh\YX'Vm'C!K YY_'UbX'ghi XYbhg'g\ci `X'\W\Y\W_'\Zcf'\Ubmi dXUh\Yg'\Uh'h\]g'h]a Yž
UbX`]Z\Z fh\Yf`]b\Zcfa Uh]cb']g'fYei]fYX\Zd`YUgY'\Wcbh\U\h'h\Y`G\W\cc``c\Z5fhg'\UbX'A YX]U'C\Z]\W'.
\hdg.##gUa "Ufhg"i bgk "YXi "Ui #\wcbh\U\h'i g'Ghi XYbhg'gY`Y\h']b['h\Y]f'hk c'YbgYa V`Yg'\Zcf'h\]g'\wci fgY`Xc'gc'cb

h\Y`i bXYfqhUbX]b['h\Uhih\Ym\Uj Y`fYUX`UbX`i bXYfqhccX`h\]q`]bZcfa Uh]cb"

5ggYgga Ybh& 9bgYa V`Y`6 7cbff]Vi h]cb UbX DUfH7\YW_]b[

 $\textbf{GhUfhXUhY}. \ \texttt{Bch5dd`]} \textbf{WUV`Y}$

8YW]`q.

Ghi XYbhgfidUfhjWjdUhjcb: UbX: WcbhfjVi hjcb: hc: YbgYa V:Y:fY\YUfgU`g:k]``VY: UggYggYX: Zcfa Uhjj Y`mž: Vmh\Y YbgYa V:Y:X]fYWhcf: fY[i `Ufma cb]hcf]b['ghi XYbhg:]b: fY\YUfgU`g: hc: Ybgi fY:h\Uhi`YUfb]b[']g: hU_]b[d`UWY": Gi a a Uhjj Y: UggYgga Ybh]g: UXa]b]ghYfYX: h\fci [\'U'dUfh\W\Y\W_]b['h\gh\Uhi\Y'bX'cZh\Y'gYgg]cb"

: YYXVUW_. '8]fYWhUbX`cb[c]b[`ZYYXVUW_Xif]b[`fY\YUfgU`g``UbX`UZhYf`U`aU]b`dYfZcfaUbW\"

5XX]h]cbU`XYhU]`g.

H\Y`dUfft\W\Y_]b[`fryfi gi U``mfU_Yg`d`U\W`]b`fh\Y`k\YY_'UZfryf`fh\Y`Z]bU``fY\YUfgU`#di V`]\WdYfZcfa Ub\W\fK\YY_'%\$`cf`ff\\#\\Y`Z]bU``fY\\YUfgU`#di V`]\WdYfZcfa Ub\W\fK\YY_'\%\$`cf`ff\\#\\Y`Z]bU``fY\\YUfgYX\X'\K\\YfY`dcgg]\V`Yž_YYd`U`\\cdot\CZU`` a i g]\WdUffg#g\\cf\fY\YUfgYX`Xi f]b[`fh\Y`fr\fa`'UbX`Vf]b[`fh\Ya`fc`fh\Y`dUfft\W\Y_]b[`fr\gh'\Gfi XYbfg`g\ci `X\X]g\W\gg`fh\Y`YI dY\\Uff\cf\f\Y\fr\ybgYa V`Y`k]fh\`fh\Y`YbgYa V`Y`X]fY\\cf\f\: cfa Uffj Y`UggYgga Ybfia UmfU_Y]bfc`U\\\chi\f\\fr\ybfg\Zcf\fY\\YUfgU`\"

5ggYgga Ybh' . A U]b DYfZcfa UbW

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ] g.

 $Gh XYbhg `UfY `fYei]fYX `hc `dUfh] W dufY `]b `h\Y `a U]b `dYfZcfa UbW g `cZh\Y]f `W\cgYb `YbgYa V`Ygžk \] W `k]```VY UggYggYX `Vmih\Y `YbgYa V`Y `X]fY W cf''$

: YYXVUW_. []j Yb]b k f]h]b ['UZhYf'h\Y'a U]b 'dYfZcfa UbWY" 'H\]g]g 'h\Y'Z]bU` 'UggYgga Ybh'hUg_"

5XX]h]cbU`XYhJ]`g.

Bcfa U``mYUW\`YbgYa V`Y`A U]b`DYfZcfa UbW\`]g`U`[fci d`UW\]j]lmiUbX`Wcblf]Vi hYg`%\$i 'lc'h\Y`Wci fgY`a Uf_f&\$i ']b`lchU`Ł"

.

5ggYgga Ybh(. 7ca dUfUhjj Y Yj U`i Uhjcb

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. %\\$\$\$!\%\\$\$ k cfXg

8 YW] g.

 $\label{eq:hug_fye} $$H^g'U''^2$$$

 $: YYXVUW\underline{\hspace{-0.05cm}}. \ 'k \ f] \hspace{-0.1cm} \text{h} \hspace{-0.1cm} 'b \ 'fYgdcbgY \ 'dfcj \] \hspace{-0.1cm} XYX''$

5 X X] h] c b U` ` X Y h U] ` g.

 $K \ f] \hbox{\it th} Y b `Y j \ U`i \ U \hbox{\it th}] \hbox{\it cbg} `U f Y `g i \ Va \] \hbox{\it th} Y X `j \] U `\hbox{\it ti} \ f b] \hbox{\it ti} b `U b X `] \hbox{\it bWi} \ XY `\hbox{\it th} Y `Zc ``ck \] b [\ . \ . \] b [\ . \] .$

%"

5 m/Yb X Ub WY F Yei	1fYa	Ybha

Gh	iXYbhgʻUfYʻYl	dXMAX.pc.	·UddfcUW	h\]g`Wti	fgY	'Ug'dfcZY	ˈgg]cbU`g	".5 m /bX	:UbWY	g\ci	$Y^{Y^{Y}}X^{Y}$	Wcbg]gh	Ybh'UbX
di	bWi U`"												

 $A\ cghYbgYa\ V`Yg`k\]```\ Uj\ Y`U'di\ V`]WdYfZcfa\ UbW'h\ Uh']g`bch]b`h\Y`fY[\ i\ `Uf`fY\YUfgU``h]a\ Y''`Gh\ XYbhg`A\ I\ GH\ UhhYbX`h\]g`dYfZcfa\ UbW''':\ U]`i\ fY`hc`Xc`gc`bchcb`ma\ YUbg`h\Uhmci `k\]``bchfYWY]j\ Y`h\Y`a\ Uf_`Zcf`h\Uh$

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

5g'dfcj]XYX'VmYbgYa V`Y'X]fYWcfg"

FYWca a YbXYX FYgci fWYg

7 ci fgY '9j U`i Uh]cb 'UbX '8Yj Y`cda Ybh

 $8]fYWhUbX`cb[c]b[`ZYYXVUW_Xif]b[`cb`]bY`gYgg]cbg`UbX`UZYf`Z]bU``UggYgga~Ybhgi~Va~]gg]cb''$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]hJbž'd'YUgY

HY'Yd\cbY'9| hYfbU'`Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUfŁ"'=Imci
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]hJb'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU'`Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WUhJcb"'=Imci 'Ya U]''9! hYfbU'`Gi ddcfhmci 'k]'`
Ui hca UhJWJ'`mfYWY]j Y'U'hJW_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'``bYYX'hc'gdYWJZJWJ'`mUg_'Zcf'cbY''
Hi fb]hJb'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf"

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

5 WUXYa]W + DZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb fY`UhJb[hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va]gg]cbg '[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK < YU`h\ UbX GUZYmdc`]W[Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a]gUXj Ybh fY/
- gh XYbhYei]hmUbX'X]gUV]`]hm/

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