

ARTS1032

HY'@hYfUhifY'@UVcfUhcfm'Kf]h]b['6YmcbX'@]a]hg

HYfa 'Hk c''##'&\$%-

7ci fgY'Cj Yfj]Yk

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B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
<y`yb'; fch\<="" td=""><td>\"[fch\4 i bgk "YXi "Ui</td><td>Hi YgXUm'%%1 %&</td><td>K YVghYf"&&\$</td><td>Ž*%'&'-',)</td></y`yb';>	\"[fch\4 i bgk "YXi "Ui	Hi YgXUm'%%1 %&	K YVghYf"&&\$	Ž*%'&'-',)
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D\cbY.'f\$&Ł'-',)'(,)*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##gUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'``fPYbg]b[tcb'WJa di g\cong \cong \cong \cong mUbX'5fti/'8Yg][b'7Ua di gYg\cong \cong \con$

7ci fgY 8YW] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

H\]g`]bffcXi WcfmWci fgY`W.U``Yb[Yg`mci 'lc'h\]b_'UVci hk f]h]b[`Ug`Yl dYf]a YbhU`žk cfX!Vi]`X]b[ž UXj Ybhi fci gž'W.U``Yb[]b[žfUX]\W]žUbX`Yj Yb`XUb[Yfci gž'UbX'lc'k f]hY`Ug'h\ci [\']h\Wb`W.Ub[Y'h\Y'k Um h\]b[g`UfY"'=hhYUW.Yg'h\Uh`]hYfUhi fY`\Ug`VYYb'h\Y'Yb[]bY!fcca 'cZa UbmcZ\]ghcfmg'a cghXY\\]g]j Y \\Ub\[Yg'hc'gc\]U\Z\W' hi fU'ž]XYc'c[]\W]\Z'UbX`VY\Uj]ci fU`bcfa g"'8]j]XYX`]bhc'Zci f'df]a Ufma cXi `Yg'—I hcd]Ub'GdYW`Uh]cbž'H\Y'I f[Yb\mcZBck z'6YmcbX': cfa 'UbX': cfa `YggbYggz'F\Yhcf]\\g'cZH\Y'=b\i a Ub—']hYI dcgYg'mci 'hc'gca Y'cZh\Y'a cghYI \\Jh]b[`UbX'i b\\cbjYbh]cbU`k f]h]b[']b`'9b[`]g\Z'UbX'Yei]dg'mci 'k]h\gca Y'cZh\Y'g_]``g'bY\\ggUfmhc'fYg]ghh\Y'W`hi fU``\\cbzfa]h]Yg'h\UhXYUXYb'h\Y'a]bX"'HfYUh]b[``]hYfUhi fY Ug'dfc\W'ggz']bhYfj Ybh]cbz'UbX`YI dYf]a Ybhz'h\Y'\\ci fgY'Ug_g'mci 'hc'gYY'k f]h]b[`Ug'U``]ZY`cb['YXi \Wh]cb']b\ck '! hc'i gY'GUa i Y``6Y\\Y\ff\g'\d\fugY'! hc'ZJ)`\Y\hYf''

7cifgY@YUfb]b[CihWcaYg

%" h\]b_`UbX`k f]hY`UVci h``]hYfUhi fY`]b`]hg`gcW]U``WcbhYI hg &" h\]b_`UbX`k f]hY`UVci h``]hYfUhi fY`Ug`Ub`cb[c]b[`dfcWYgg ' " fYWc[b]gY`UbX`X]gW gg`h\Y``cb[`hfUX]h]cb`cZYI dYf]a YbhUh]cb`]b`9b[`]g\``]hYfUhi fY` (" dUfh]W]dUhY`]b`h\Y`YI dYf]a YbhU``dfc^YWhcZ`]hYfUfmdfcXi Wh]cb`

HYUW]b['GhfUhY[]Yg'

H\]g:Wci fgY:k]``VY:HJi [\hiVm%I:%"):\f:`YWh fY:`UbX:%I:%"):\f:h hcf]U``dYf:kYY_'UbX:k]``XYd`cm@A G"

5ggYgga Ybh

D`YUgY`VY`Uk UfY`h\Uh5@`UggYgga YbhHUg_g`UfY`fYei]fYX`Wci fgY`Wca dcbYbhg`f\i fX`Y`Wca dcbYbhg\!' Mci A I GH`UhhYa dh`YUW\`hUg_']b`cfXYf`hc`dUgg`h\Y`Wci fgY'': U]`i fY`hc`Wca d`YhY`UbmcZh\YgY`hUg_g`k]``fYgi `h]b U`ZU]`i fY`cZh\Y`Ybh]fY`Wci fgYžYj Yb`]Zmci f`fYgi `hg`]b`h\Y`ch\Yf`hk c`hUg_g`a][\h'UXX`i d`hc`U`bi a Yf]WU` dUgg]b[`[fUXY"

#i]g'U`gc'Ub'YI dYWUIJcb'cZh\Y'Wci fgY'h\Uhmci 'UlhYbX'U``'WUggYgž'YWh fYg'UbX'h hcf]U`g''

K Y'dfYZYf'h\Y'i gY'cZh\Y'7\]WU[c'A Ubi U``cZGhmYž'%+h\`YX]h]cbž'Zcf'U```ghmY'UbX'fYZYfYbW]b['a UhhYfg"

\hd.#kkk"WX]WU[ca Ubi U`cZghm`Y"cf[#cc`gSW]hUh]cb[i]XY#N]hUh]cb![i]XY!%"\ha``

5ggYgga Ybh'HUg_g'

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@hYfUfm91 dYf]a Ybh	')	\$) #\$+#&\$% `%%\$\$`DA	% <u>*</u> &. <u>Z</u>
H\Y`K f]h]b[`@UV`	')	\$-#\$, #&\$%-`%%\$\$`DA	&ž(
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5ggYgga Ybh'8YhJ]`g

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GhUfh XUhY.

8 YHJ]`g. '%) \$\$!k cfX YggUnž ZYYXVUW_dfcj]XYX ij]U Hi fb]h]b".

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhdb gYYhd for $[\ \ \]h]YXh\fci [\ \ \]h]b UbXgh XYbhg WUb gYYhd fb]h]b g]a] Uf]hm fYdcfhg" \end{tabular}$

5ggYgga Ybh& H\Y K f]h]b[@UV

GHJfh:XUhY.

@Yb[h\.'&\$\$\$'kcfXg'dYf'ghiXYbh

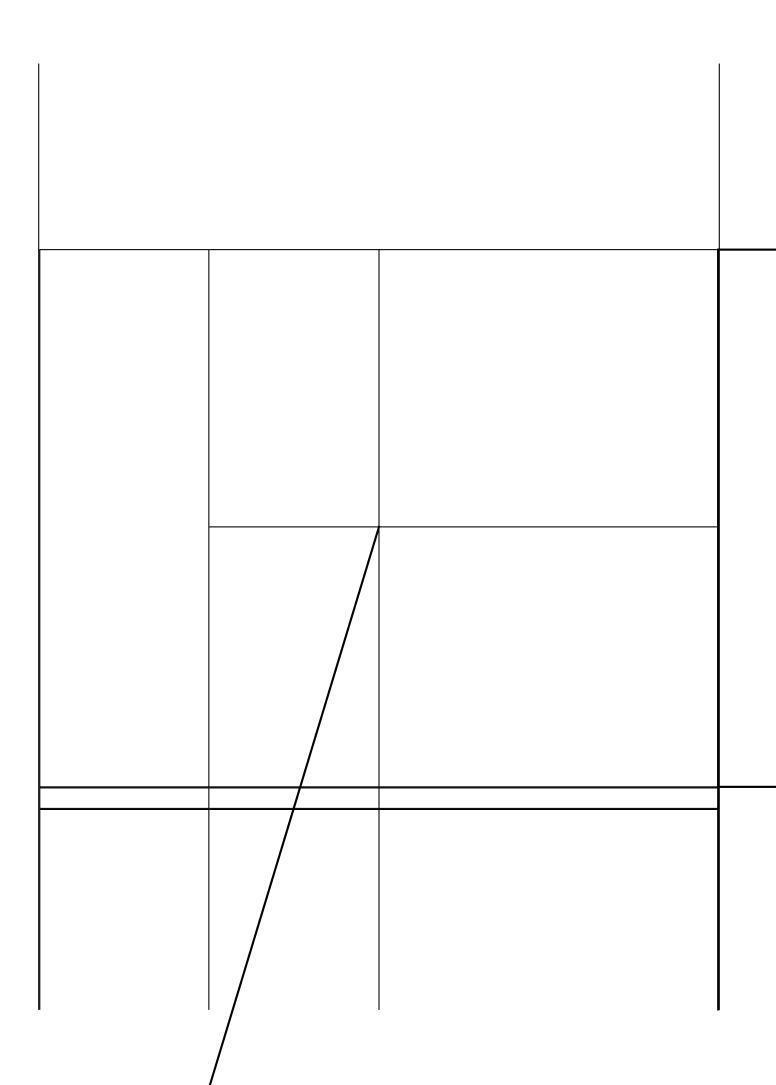
8 YHJ] g.

5 HMY BUXUU VIVI F YEA YEYA Y bhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] }$

700 fg**fg**Y GWX YXi **%**Y

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_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'\lefi\Wb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UVW]XYbhU`mWcdm]b['Zfca `U`gci f\W' k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]Uf]ga]bhch\YZc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUlfYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfglUbX`mci f`f][\hg`UbX`fYgdcbg]V]`]h]Yg`Ug`U`gh XYbh'Uhi BGK
- VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hzil BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5WWdhUV`Y'I gY'cZ I BGK '= H'F Ygci fWfg'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wcaaib]hm
- `cWUHY 'gYfj]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@G9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.#\gi \YY\\f\i \\XY\g''\\f\f\f\i \bgk\\\YX\i \"\J\\Y\\\f\\\]aY

5 WUXYa]W = b Zcfa Uh cb.

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- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU'th 'UbX'GUZYlmidc']W[Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUljcb`]b`l\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]mUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

.

a U[Y⁻7fYX]h

 $$$ \frac{d^cUX''k \,]_a \, YX]U''cf[\# \,]_dYX]U\#\% a \, a \, cbg\#J\#JY\#C \, VYf\Ui \, gYbS!S: \, Ugca \, YhYfS!S8 \, YfSgWX ObYSGWXY]b \, S!SH\YS5 bWYbhSc ZS8 \, UmgSi \, \& \, K \,]^]Ua \, S6 \, `U \, Yi \, \& - S$\%S]Yg''d[$$$

7F = 7CG

7F = 7CG Dfc|]XYf 7cXY. \$\$\$-,;