



ARTS2036

A cXYfb]ga . HYI hUbX GWfYYb

HYfa 'Hk c''##'&\$%-

7ci fgY'Cj Yfj]Yk

Ghuzz 7 cbhuWh8 Yhu]`g

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
>c\b`5lff]X[Y	9	Hi YgXUm'‰!%'cf'Vm Uddc]bha Ybh	KYVghYf*&&,	-',)((,(

GWkcc~7cbhJWh~abZcfa Uh]cb

Fcca " &ž Yj Y`" FcVYfhK YVghYf 6i]X]b[

D\cbY. f\$&L'-',) '(,) *

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##gUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]tjcbU``7igtcX]Ubgžh\Y'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5ft\'v'8Yg][b'7UadigYgŁ'UbX't\Y'B[ibbUkU`dYcd'Y'fbigtfU']Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`cWJhYX''$

7ci fgY 8YW] g

7 fYX]hDc]bhg * *

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. 7`Y1eZ

 $\label{eq:linear_continuity} $$H\g^Wb^U\gc^VY^ghX]YX^]b^hY^Zc^ck\b[-gdYV]U^]gUh]cb.^8[^{\underline{\ }}\ EfgV[We]$

H\]g'Wci fgY']blfcXi Wrg'mci 'hc'h\Y'k Uj Y'cZYI dYf]a YbhUhjcb']b'h\Y'Uffg'h\Uhk Y'\Uj Y'Wca Y'rc'W\'``
_ aVWd [e_ "'8i f]b['h\Y']fghZcffmcf'gc'mYUfg'cZh\Y'lk Ybh]Yh\ WYbhi fmž'hfUX]h]cbU``UfhZcfa g'i bXYfk YbhUb
i bdfYWYXYbhYX'dfcWrgg'cZfYj c'i h]cb'UbX']bbcj Uhjcbž'Ug'Ufh]ghg'UbX'k f]hYfg'gci [\hfUX]WJ``mbYk 'k Umg'hc
YI dfYgg'h\Y'YI dYf]YbW'cZa cXYfb]m''K Y'k]``Wcbg]XYf'h\Y'k cf_g'cZU'fUb[Y'cZ]bX]j]Xi U``9i fcdYUb'UbX
Bcfh\'5a Yf]WUb'a cXYfb]ghk f]hYfg'UbX'Z]'a !a U_Yfgž'Ug'k Y```Ug'YI Ua]b]b['gca Y'cZh\Y'a Ubm'Vc``YWh]j Y
a cj Ya Ybhgž'[fci di gW 'Yg'UbX'']ga g'''h\UhgdfUb['i d'Xi f]b['h\]g'dYf]cX'''5 'hci WkghcbY'cZci f']bei]fmk]``
VY'h\Y'j YI YX'fY`Uhjcbg\]d'VYhk YYb'''a cXYfb]m'''Ug'U'\]ghcf]WU`'WcbX]h]cb'UbX'''a cXYfb]ga "''Ug'U'W''h IfU`
a cj Ya Ybh''K \UhXcYg']ha YUb'hc'VY'a cXYfb3'K \Uh\\UddYbg'hc'Ufhk \Yb''hfUX]h]cbU`'VY`]YZg'UbX'k Umg'cZ
']ZY'YbhYf'U'dYf]cX'cZdYfa UbYbhWf]g]g'-'U'dYf]cXž'h\Uh]gžk \Yb'''U``'h\Uh]g'gc']X'a Y`hg']bhc'U]f''3'5bX'XcYg
a cXYfb]ghW''hi fY'hYUW\'i g'k Umg'hc'ZYY`'Uh\ca Y']b'a cXYfb]mzcf'XcYg']hfYa]bX'i g'cZci f'U`]YbUh]cb3'

7ci fgY @YUfb]b[Ci hWca Yg

%" F YUX a cXYfb]gh`]hYfUhi fY 'UbX 'Z]`a 'Wf]hJWU``m

&" FY`UhY`a cXYfb]gh`]hYfUh fY`UbX`Z]`a 'hc'h\Y]f`W\`h fU`'WcbhYl h

' "8fUk WcbbYWgcbg VYrk YYb _Yma cXYfb]ghrYl rg UbX Zj`a g`

("AU_Y'U'dYfgi Ug]j Y'Uf[i a Ybh

)": cfa i `UhY`UbX`XYZYbX`Ub`]bXYdYbXYbhdc]bhcZj]Yk `cb`U`gi V^YWžk \]`Y`hU_]b[`]bhc`UWVci bhh\Y j]Yk g`UbX`Uf[i a Ybhg`cZch\Yfg

*" 9a d`cmfYgYUfW\.'g_]``gž']bWi X]b['Z]bX]b['Uddfcdf]UhY 'gYWcbXUfma UhYf]U``UbX'i g]b[']h'Wf]h]WU``m"

HYUWX]b[GhfUhY[]Yg

HYUWX]b[ghfUhY[]Yg UbX fUh]cbU`Y.

@YWh fYgž'h hcf]U'g'UbX'UggYgga Ybh]b'h\]g'Wti fgY'g\ci `X'[]j Y'mci 'h\Y'g_]``g'bYW'ggUfmhc'dYfZcfa]bhY``][Ybhž']bXYdYbXYbhfYUX]b[g'cZa cXYfb]ghhYl hg'UbX'hc'Wta a i b]\WlhY'h\cgY'fYUX]b[g']b'Wt[YbhgdYYW\'UbX'dfcgY''

@YWnifYg.

!`]blfcXi WY`UbX`XYZ]bY`h\Y`Wti fgYfg`WtfY`WtbWYdlgž`]bWi X]b[``YgdYW]U``m''"a cXYfb]ga ""``UbX`'"a cXYfb]lm'" !`XYa cbglfUhY`h\Y`dfUWf]WYg`cZWf]h]WU``fYUX]b[``h\Uhmci `bYYX`hc`Udd`m`]b`UggYgga YbhHUg_g !`UbU`mqY`hYI hq`UbX`g]hi UhY`h\Ya `]b`h\Y]f`gcW]U``UbX`Wi`hi fU``WtbhYI hg

Hi hcf]U`g.

K YY_`mhi hcf]U`g`YbWci fU[Y`mci `hc`fYUX`Wcbh]bi ci g`mh\fci [\ci hh\Y`gYa YghYf`UbX`dfcj]XY`U`Zcfi a `hc XYj Y`cd`mci f`ck b`fYUX]b[g`]b`X]U`c[i Y`k]h\rh\cgY`cZmci f`]bghfi Whcf`UbX`dYYfg"

5ggYgga Ybh

5ggYgga YbhHUg_g

5 https://bxubwww.Fyei]fya Ybhg

Hi hcf]U``UhhYbXUbWY`]g'a UbXUhcfm'

@YWi fY`UHYbXUbW']g`glfcb[m`YbWci fU[YXžVi h`bcha UbXUhcfm''<ck Yj Yfž`]Zmci `Xc`bch`UHYbX`H\Y `YWi fYg`]b`dYfgcbžmci `a i gh_YYd`i d`k]h\`h\Y`fYWcfX]b[g`cb`]bY"

7 ci fgY GWX YXi ` i

S u dat epo p u tŁ	n •n Tupure.						

FYgci fW/g

DfYgWf]VYX F Ygci fWYg

GYY'A ccX'Y

FYWca a YbXYX FYgci fWYg

GYY'A ccX'Y

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

=j U`i Y`ghi XYbhZYXVUW_`\][\`mUbX`i gY`]hhc`fYZ]bY`a mWci fgYg`Yj YfmmYUf"`H\UhgU]Xžh\]g`Wci fgY`\Ug`bch fi b`VYZcfY`]b`]hg`dfYgYbhZcfa "`Ghi XYbhg`]b`h\Y`dUgh\Uj Y`Wca a YbhYX`h\Uhh\Y`Wci fgY`a cj Yg`ei]\W_`mUbX\Uj Y`YI dfYggYX`U`k]g\`hc`gdYbX`a cfY`h\Ub`cbY`k YY_`cb`WYfhU]b`hYI hgžk\]W\`=i gYX`hc`hfmhc UWWca a cXUhYžVi hh\Uhfg`bc``cb[Yf`dcgg]V`Y`]b`h\Y`hf]a YghYf`Zcfa Uh'`Cb`h\Y`ch\Yf`\UbXžk Y`bck `\Uj Y a cfY`h]a Y`ZcfX]gW`gg]cb`YUW\`k YY_žk\]W\`=\cdY`k]``VY`Ub`]a dfcj Ya Ybh'`

=`cc_7zfk UfX'hc^\YUf]b['mcif7zYYXVUW_'h\fci[\^A m9I dYf]YbWY^UbX'ch\Yf^W\UbbY`g^UWfcgg'h\Y`gYgg]cb"

Gi Va |gg|cb cZ5ggYgga YbhHUg_g

Gi Va |qq|cb cZ5qqYqqa YbhHUq_q

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''#mci
UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'YI hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW'a Ybhgt'hc
]bWi XY'Ug'Yj]XYbWY'hc'gi ddcfhmci f'YI hYbg]cb'Udd']WUh]cb'''#mci 'Ya U]''9I hYfbU''Gi ddcfhmci 'k]''
Ui hca Uh]WJ''mfYWY']j Y'U'h]W_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'hc'gdYW]ZWJ''mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\ci fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUh]j Y'gi Va]gg]cb'XYhJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY