

# 7ci fgY'Cj Yfj ]Yk

### Ghuzz 7 cbhuWh8 Yhu]`g

'7cbj Ybcfg'

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D∖cbY
9f]b 6fUbb][ Ub	Y"VfUbb][ Ub4 i bgk "YXi "Ui	Q	F "‱ 'K YVghYf 6i]`X]b[	-',)+,)(

#### GWkcc~7cbhJWh~abZcfa Uh]cb

Fcca " &ž Yj Y`" FcVYfhK YVghYf 6i ]X]b[

D\cbY. f\$&L'-',) '(,) \*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##gUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJadi gtž; UX][U'`fP]tmUbX'5fth' '8Yg][b'7Uadi gYgt'UbX'thY'B[i bbUkU' dYcd'Y'fbi gtfU']Ub'8YZYbW': cfW'5WJXYam']b'7UbVYffUt'cZthY'`UbXg'k\YfY'YUWX'WJadi g'cZl BGK ']g`cWJtYX''$ 

# 7ci fgY 8YHJ] g

7 fYX]hDc]bhg\*\*

Gi a a UfmcZh\Y 7ci fgY

 $\label{eq:Giv_wish} \text{GiV_WishYU.`FZWSfdWS`VBWdAd_S`UWEfgV[WeH\]gWcifgY.WcifgY.Wbb'U`gc`VY`ghiX]YX`]b'h\Y`Zc``ck]b[`gdYW]U`]gUh]cb. 5 dWSf[hWI d[f[`YHT]gWcifgY.Wbb'U`gc`VY`ghiX]YX`]b'h\Y`Zc``ck]b[`gdYW]U`]gUh]cb. 5 dWSf[hWI d[f[`YHT]gWcifgY.Wbb'U`gc`VY`ghiX]YX`]b'h\Y`Zc``ck]b[`gdYW]U`]gUh]cb. 5 dWSf[hWI d[f[`YHT]gWcifgY.Wbb'U`gc`VY`ghiX]YX`]b'h\Y`Zc``ck]b[`gdYW]U`]gUh]cb. 5 dWSf[hWI d[f[`YHT]gWcifgY.Wbb'U`gc`VY`ghiX]YX`]b'h\Y`Zc``ck]b[`gdYW]U`]gUh]cb. 5 dWSf[hWI d[f[`YHT]gWcifgY.Wbb'U`gc``ck]b'h\Y`Zc``ck]b[`gdYW]U`]gUh]cb. 5 dWSf[hWI d[f[`YHT]gWcifgY.Wbb'U`gc``ck]b'h\Y`Zc``ck]b'h$ 

 $F \ Yj \ ]Yk \ ]b[ \ 'h\ Y'5 \ fhg \ Yl \ d`cfYg \ U'j \ Uf]Y \ fmcZk \ f]h]b[ \ 'a \ cXYg \ ]b \ 'ufhg \ 'ci \ fb \ U`]ga \ z']b \ Vi \ X]b[ \ ']b \ Yk \ gz' \ dfYj \ ]Yk \ g" \ 'H\ fi \ ]h \ 'k \ f]h]b[ \ 'a \ cXYg \ 'mci \ 'k \ ]`` \ 'VYWca \ Y' \ ZJa \ ]`]Uf' k \ ]h \ 'k \ f]h]b[ \ dfUW]W \ g' \ UWcgg' a \ Ubm \ Zcfa \ g']b \ Vi \ X]b[ \ 'h\ YUh'Y' \ UbX' \ dYf \ Zcfa \ UbW \ z'XUbW' \ z'a \ i \ g]Wzj \ ]gi \ U``Ufhg \ UbX' \ Z]a \ "`H\ ]g$ 

# 5ggYgga Ybh

K CF? GI 6A ≠H98.

5```WJhUhJcbg'a i gh']bWi XY'dU[Y'fYZYfYbWYg"'Y['fbi h\cf'gi fbUa Y'MYUfz'dU[Y'bc"Ł'fGa ]h\ &\$\$\$z') Ł

=b!hYl hfYZYfYbWb[ cf\*ZcchbchYg\*UfY\*UVWYdhUV`Y\*

: i ``V]V`]c[fUd\mUhh\Y`YbX`cZmcif`fYgYUfW\dUdYfg`]g`fYei]fYX"

 $I\ bfY`]UV`Y`k\ YVdU[\ Y`fYZYfYbWfg`UfY`bch`UVWYdfUV`Yž`Y[\ "`K\ ]\_]dYX]U"`$ 

: cbhg]nY''&'d'YUgY'!'a mYmYg'UfY'[Yhf]b['C@8"

## 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
K cf_g\cd^5Wfj ]hJYg^	&\$I	% #\$+#&\$% `%%' , `5A	%ž&
Dcfh7c`]c'cZFYj]Ykg'	(\$1	\$%\$,	%ž&
9ggUmi	(\$1	&) #\$+#&\$% `\$) .\$\$`DA	%ž&Ž

# 5ggYgga Ybh8YhJ]`g

5ggYgga Ybh% K cf\_g\cd 5Whjj ]hjYg

GhUfh'XUhY. "&\$#\$\*#&\$%- "%%", "5A

@Yb[h\.'b#J

 $8YHJ]^*g.^*\&!(^!k\ cf_g\ cd^!UWIj)\ JHYg^!k\ J^*'VY^!XYg][\ bUHYX^!Ug^!UggYgga\ YbH^!Ug_g^!UbX^!k\ J^*'VY^!a\ cXY^*YX^!]b^!WUggVyZcfY^!UggYgga\ YbH^!5ggYggYX^!Vmfi\ VfJW^!$ 

#### 5XX]h]cbU`XYhJ]`g.

Mci 'k ]``'VY'UggYggYX'VUgYX'cb'h\Y'Zc``ck ]b[ 'Wf]hYf]U"

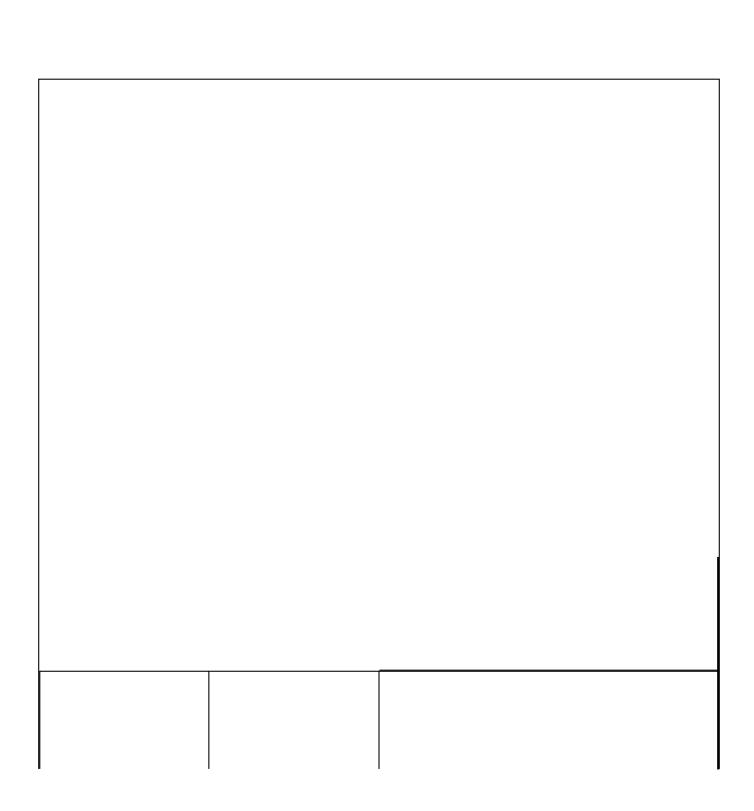
Wf]hYf]U	BYYXg k cf_	gUh]gZUWhcf m	[ccX	JYfm[ccX	YI WY``Ybh
i bXYfghUbX]b[ cZ	•		•		
h\Y`a cXYgž					
WX UfUWY f'UbX					
Y`Ya Ybhq'cZi Uf]ci o	q				

fYj ]Yk ]b[ gmYg UV] ]mhc ]XYbh]m h\Y j Uf]ci g Y Ya Ybhg UbX ghfUhY[ ]Yg cZ j Uf]ci g fYj ]Yk ]b[					
ghmYg					
gY`ZfYZYW¶cb'UbX		•		•	
UggYgga Ybh					
YI dfYgg]cb					
cf][]bU`]lmi]b 'h\Y	•	•	•		
ZFUa]b['cZ					
fYgdcbgYg'hc'h\Y					
hUg_g"					
109_9					

.



a Uf_	<b>Vf</b> ] <b>H</b> Yf]U
a Uf_ <8	



		5 'W.YW_`]ghcZdfYgWf]dh]cbg'Zcf'[ccX'cd]b]cb'kf]h]b[žk]h. 'gca Y'Yl Uad'Yg"'5dd`m]b['h.Y'dfYgWf]dh]cbg'Zcf[ccX'cd]b]cb'kf]h]b["'Kf]hY'U'g\cfhf%)\$'kcfXtcd]b]cb'd]YWYg'cb'Ub'Ufhg']ggiY"'Y[žk\m`cW_cih'ukg'UfY'WfUd#[fYUh"
	GYa ]bUf	6fYU_`]bhc`[ fci dg`cZlk c#\\fYY`UbX`Wca dUfY`YUW\ ch\Yffg`k cf_"`8]gW gg`Ug`U`WUgg"`
K YY_`) . '%'>i `m! ') '>i `m	@YWhifY	@YWifYf. 9f]b 6fUbb][Ub
		7 f]h]w]ga 'UbX'a U&f'UYgh\Yh]wg'g\]Zrg/7 UgY'Gh Xm A ]w\UY`: f]YXžF cVYfhA cff]g'UbX'A ]b]a U`]ga "'H\Y Ufh]ghUg'gY`Z Wca a YbhUrcf"'K \UhUfY'h\Y'a U&f W`h fU`'UbX'UYgh\Yh]Wg\]Zrg'cWW ff]b[']b'mci f'Z]Y`X3 K \Uh]g'h\Y'fc`Y'cZh\Y''Wca a YbhUrcf''cf'Wf]h]W]b'h\]g ]bghUbWY3'
	GYa ]bUf	G\Uf]b[ kf]h]b[ UbXkcf_g\cdd]b[ ]bdU]fg" 8]ghf]Vih]cbcZYgUmeiYgh]cbg"
K YY_'*.', '>i `m!'%&'>i `m	@YWhi fY	; i Ygh@YWn fYf`f%") \fgL` <ck `\ug`w\ub[yxcjyf`h]a="" `ubx`h\yb"<br="" `wf]h]w]ga="" y"`5ygh\yh]w^x[ya="" ybhž'bck="">f9f]b`%") \fgŁ`FYj ]g]cb`cZdfYgWf]dh]j Yg`Zcf`[ccX k f]h]b[`UbX`hf]U``Zcf`k cf_g\cd`UggYgga Ybh' </ck>
	GYa ]bUf	K cf_]b[ ']b'dU]fg'cb'h\Y'gUa Y'YI Ua d'Yž'k f]hY'U'WfUd fYj ]Yk 'UbX'U'[ ccX'fYj ]Yk 'cZU'WcbhYa dcfUfmk cf_ cZUfh']b'mci f'a YX]U'fa i g]WžXUbWYž'h\YUhfYž'Ufhz'h\Uh WUb'VY'UVWY'ggYX'cb']bY"'H\Yb'k f]hY'U'Wf]h]ei Y'cZ YUW\'ch\Yffg"''Gi Va ]h'mci f'fYj ]Yk g'UbX'mci f Wf]h]ei Y'cZmci f'dUfhbYf'g'k f]h]b[ 'Zcf'UggYgga Ybh'Uh h\Y'YbX'cZh\Y'WUgg"'
K YY_'+.'%) '>i `m'! '% '>i `n	n@YWhifY	; i Ygh@YWi fYff%") \fgL F Yj ]Yk ]b[ ``]hYfUfmhYl hg. Zfca K cc`Zhc`K ccX`UbX`VYmcbX"`f9f]b`%") \fL F YUX]b[ `UbU`mg]g.`K \UhUfY`h\Y`WU]a g`cZh\Y`d]YW\ cZk f]h]b[ ž`]a d`]W]hmicf`Yl d`]W]hm3`K \UhghfUhY[ ]Yg UbX`hYW\b]ei Yg`XcYg`]hi gY`hc`Xc`h\]g3`K cf_g\cd Zcf`bYl hk YY_'g`UggYgga Ybh'
	GYa ]bUf	K cf_g\cd`5ggYgga Ybh&.`FYUX]b[`UbU`mg]g.`K \Uh UfY`h\Y`WU]a g`cZh\Y`d]YWY`cZk f]h]b[ž]a d`]MJhmcf YI d`]MJhm3`K \UhghfUhY[]Yg`UbX`hYW\b]ei Yg`XcYg`]h i gY`hc`Xc`h\]g3
K YY_', .'&&'>i `m'!'&*'>i `n	n @Y Whi fY	D\]`cgcd\]Yg`cZUYgh\YhJWg/7UgY`GhiXm`; ]``Yg 8Y`Yi nY`cb`: fUbW]g`6UWcb"'Cb'h\Y`WUHJ`c[iY YggUm'K f]hjb[`HUg`GHJfh]b[`k]h\`U`fYj c`i h]cbUfmž WUbcb]WJ``k cf_`cZUfh'Zfca `mci f`X]gW]d`]bYž'UbX k cf_]b[`]b`mci f`X]gW]d`]bY`[ fci dgžh\]b_'h\fci [\`h\Y Z bXUa YbHJ`g`cZh\UhX]gW]d`]bY`UbX`\ck 'h\Y YI Ua d`Y`\UbX`Yg#Y]a U[]bYg`h\cgY`Z bXUa YbHJ`g"

	GYa ]bUf	FYj]Yk'cZkf]h]b['Zfca'kcf_g\cd']b'`YWfifY"	
K YY_''&-'>i`m!'& 5i[igh	@YWn fY	C b 'h\Y']bhYfj ]Yk #dfYj ]Yk "'K f]h]b[ 'HUg'7\ccgY'l i dWta ]b[ 'dYfZcfa UbW#YY'YUgYž'UbX'k cf_]b[ ']b dU]fgži bXYfhU_Y'fYgYUfW\'hc'gi ddcfhUb ]bhYfj ]Yk #dfYj ]Yk 'd]YWY'cb'h\Y'Ufh]ghUbX'h\Y W\cgYb'k cf_"	
	GYa ]bUf	DfYgYbh]bhYfj]Yk#dfYj]Yk fYgYUfW\]b'dc]bhZcfa hch\Y'WUgg"'7cifgY'YjU'iUh]cb"	
K YY_'%\$.') '5i [i gh!'- 5i [i gh	@YWn fY	@YWn fYf. '9f]b '6fUbb][ Ub 8 YVUhY 'cb 'h\Y 'fc 'Y 'cZh\Y 'Wf]h]W]b 'h\Y 'X][ ]hU``U[ Y 	
	GYa ]bUf	Dcgh8YgWf]dhj Y`FYj]Ykg	
K YY_'%%''%&'5i [i gh!'%* 5i [i gh	@YWh fY	@YWn fYf. '9f]b '6fUbb][ Ub	
		7 f]h]W]ga 'Ubx'A U'cf'5Ygh\Yh]WG\]Zhg	
	GYa ]bUf	Mci f`X]gWjd`]bY`UbX`h\Y`fc`Y`cZ'h\Y`Wf]hjW	

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

Image: Ybwci bhYf'U'dfcV'Ya k \Yb'UhYa dhJb[ 'hc'gi Va ]hmci f'Ugg][ ba Ybht\fci [ \ 'Hi fb]hJbž'd'YUgY

HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U] 'h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'\\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUfŁ" Image:

UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]hJb'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'hJW\_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fY'Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hYbg]cb'Udd']WUh]cb" Image: Ya U]''9! hYfbU''Gi ddcfhmci 'k ]''

Ui hca UhJWJ''mfYW'] Y'U'hJW\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]'''bYYX'hc'gdYWJZWJ''mUg\_'Zcf'cbY"

Hi fb]hJb'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk ]hYf"

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mj ]U'Y]h\Yf'Hi fb]hjb'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]``'VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\ U`hYfbUhj Y'gi Va ]gg]cb'XYHJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va ]hUgq][ ba Ybha ccXiY

### 5 WUXYa | W < cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'\lefi\WJb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UVW]XYbhU`m\Wcdm]b[ 'Zfca `U`gci f\W' k ]h\ci h\U\W\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]Uf]ga ]bhch\YZc``ck]b[ WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a ]`Uf'k cfXg'hc'h\Y'cf][]bU`h\I hcf']XYU'k ]h\ci hUW\_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf\_g"'H\]g'U'gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfc YWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k ]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]Y a cghmfYhJ]b]b['h\Y'cf][]bU`gffi WhifY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`ž'UbX']bZcfa UhJcb'k]h\ci hUW\_bck `YX[Ya Ybh''H\]g'U`gcUdd`]Yg']b'dfYgYbhUhJcbg'k\YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc

### 5 WUXYa ]W = b Zcfa Uh cb ·

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb[ 'hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va ]gg]cbg`[i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua ]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a ]gUXj Ybh fY/
- gh XYbhYei ]mUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa ]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

.

⇒a U[Y'7fYX]h

| 'F YU'H]a Y 5 fhg

7F = 7CG

7F = 7CG Dfcj ]XYf 7cXY. \$\$\$-,;