7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

7cbj Ybcfg

B Ua Y		

7ci fgY 8YHJ] g

7 fYX]hDc]bhg (

Gi a a UfmcZh\Y 7ci fgY

Gi VYWn5fYU. ? ge[UBWdXad_S`UW!? ge[U7`eW_T'W

#Jih]g'dfuWjWi`Wci fgYžmci k]``XYj Y`cd'mci f'UXj UbWX`]bglfi a YbHJ`žj cWI`cf'Wca dcg]ljcb'g_]``g'k]h\cbY! hc!cbY'gi ddcfhZca 'U'gdYWjU']ghhi hcfž'UbX'[U]b'Yl dYf]YbW'']b'Udd`mjb['h\cgY'g_]``g'h\fci [\'dUfh]WjdUh]cb]b'k cf_g\cdgža UghYfWUggYg'UbX'YbgYa V'Yg"''H\YgY'Wca dcbYbhg'k]```Wca V]bY'hc'gi ddcfhmci f'gdYWjU']gh ghfYb[h\g'UbX']bhYfYghgž'UbX'hc'VfcUXYb'UbX'WcbhYl hi U`]gY'mci f'dYfgcbU``ghi X]Yg"''A]`YghcbYg'Zcf'h\Y Wci fgY'UfY'hYWXb]WU``Zci bXUh]cbgž'Zcf'Vch\'dYfZcfa Yfg'UbX'Wca dcgYfgž'UbX'\YU'h\mdfUWj\W'z'Ug'h\Y'VUg]g cZcdh]a U`'dYfZcfa UbW''UbX'gi ghU]bUV'Y'a i g]Wa U_]b["'''

7 ci fgY @YUfb]b[Ci hWca Yg

- % 6i]`X`UbX`Udd`miZci bXUh]cbU``hYWXb]\WU``g_]``g`]b`h\Y`XYj Y`cda YbhcZUfh]gh]\WYI dfYgg]cb
- &" 5dd`mih\Y`df]bW]d`Yg`cZ\YU`h\midfcZ/gg]cbU``dfUWf]WY`hc`j cWU``UbX`]bglfi a YbhU``hYW\b]ei Y`UbX dYfZcfa UbW
- '" → XYbhjZmfY`Uhjcbg\]dg'Uacb[']ggiYg']bZcfa]b['dYfZcfaUbWY'dfUWhjWY
- ("5dd`mdf]bWdYgcZhYUakcf_hcaig]WYbgYaVY

HYUWX]b['GhfUhY[]Yq'

5ggYgga Ybh

 $: i fh Yf XYHJ g dYfHJb b [ihc Ugg Ygga Ybhik] ``VY dcgh YX cb A cc X Y Ub X X g W gg Y X] b WUgg] b K YY _ Wc Z HYfa "&"$

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
9bgYa V`Y`Wcblf]Vi ljcbž dYfZcfa UbWYgždUfhW.YW_]b[lYgh	&\$I	Bch5dd`]WUV`Y	(
A UghYfWUgg'Wf]h]ei Yg'	%) ।	k YY_`%\$	1
DYfZcfa UbWY#dfYgYbHJHjcb	')	K YY_g ⁻ , !%\$	%ž&
DfUWjWU``YI Ua]bUhjcb'	' \$1	Bch5dd`]WUV`Y	%ž&

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% '9bgYa V`Y'Wcblf]Vi h]cbždYfZcfa UbWgždUfhWkYW_]b['hYgh

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g. '8]fYWiZYXVUW_Xi f]b['fY\YUfgU`g'UbX'k cf_g\cdg'UbX'U'Zcfa U``fYdcfh'Uhi\Y'WbWi g]cb'cZh\Y gYgg]cb"'Ghi XYbhgfWbhf]Vi h]cb'hc'YbgYa V`Y'dYfZcfa UbW'g'UbX'k cf_g\cdg'k]```VY'UggYggYX'h\fci [\'U dUfh\W\YW_]b['YI Ua]bUh]cb"9bgYa V`Y'dUfh\W\YW_]b['k]```VY'i bXYfhU_Yb']b'H%z'H&'UbX'H' "'=bX]j]Xi U``mcf']b ga U```[fci dgz'ghi XYbhg'k]``'dYfZcfa 'k cf_g'cf'YI Wlfdhg'cZk cf_gz'ghi X]YX'UbX'dYfZcfa YX'Xi f]b['h\Y hf]a YghYf'''K\YfY'Uddfcdf]UhYzYI Wlfdhg'k]```VY'gY'YWhYX'Vmh\Y'dUfh\W\YW_]b['YI Ua]bYf''

5ggYgga Ybh& A UghYfWUgg Wf]h]ei Yg

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.') \$\$ k cfXg

8 Yhu]`g. '7 f]h]ei Yg`k]```VY`gi Va]hhYX`UWfcgg`H%žH&`UbX`H' "``H\YgY`k]```VY`fY`UhYX`hc`a UghYfWUggYg dfYgYbhYX`Vmj]g]h]b['Yl dYfhg`]b`U`fUb[Y`cZa i g]WU``ghm`Yg`UbX`UWfj]h]Yg"``7 f]h]ei Y`ei Ygh]cbg`k]```VY g\W\Zc`XYX`hc`Yb\%i fU[Y`ghi XYbhg`hc`]XYbh]Zm]ggi Yg`Zcf`X]g\W gg]cbž`UbX`hc`XfUk ``]b_g`Ua cb['dYfZcfa Ub\W Y`Ya Ybhg`UbX`g\W\c`Uf`m`]hYfUhi fY"```

5XX]h]cbU`XYhU]`g.

Ghi XYbhgʻa i ghiUhhYbXʻUʻa]b]a i a ʻcZ%a UghYfWUggʻ]bʻHYfa ʻ&ʻUbXʻgi Va]hi%a UghYfWUggʻWf]hjei Yʻ]bʻK YY_ %\$ʻcZHYfa ʻ&"

Gi Va]gg]cb bchYg.j]U A ccX`Y dcfhU`

Hi fb]h]b\$]b[:.

5qqYqqa Ybh' DYfZcfa UbWY#dfYqYbHJhcb

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\."%\$ a]bi hYg

8 YHU]`g. DYfZcfa UbWY#dfYgYbHUh]cbg`k]```VY`WcbXi WhYX`Xi f]b[`k cf_g\cd`gYgg]cbg`]b`H%`UbX H&"``±ghfi a YbHu`]ghg`UbX`j cWU`]ghg`k]```dYfZcfa `cbY`Wca d`YhY`]hYa `]b`YUWX`hYfa žk\]WX`a Um`VY`U a cj Ya YbhcZU```Uf[Yf`k cf_/Wca dcgYfgžg]a]`Uf`mžk]```dfYgYbhh\Y`dYfZcfa UbWY`cZcbY`]hYa `cfa cj Ya Ybh''`: YYXVUW_.``k f]hhYb`fYdcfhg

5XX]h]cbU`XYhU]`g.

 $\label{eq:hamiltonian} $$HY'' = \mathbb{E}^{a} \left[h'' \cdot UggYggYX' \cdot dY'fZ' \cdot fa \quad UbW''g' \cdot dfYgYbh'X' \right] b'' + Y'fa \quad \&'WUb' \cdot bch' \cdot bW' \quad XY''h \cdot Y'' \cdot fYdYfhc \cdot fY' \cdot dfYgYbh'X' \cdot fY'' \cdot fY'' \cdot dfYgYbh'X' \cdot fY'' \cdot fY'' \cdot dfYgYbh'X' \cdot fY'' \cdot fY''' \cdot fY'' \cdot fY''$

5ggYgga Ybh (DfUWh) WU Yl Ua]bUh]cb

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ]`g.: cf'dYfZcfa UbW' gh XYbhgžYl Ua]bUh]cbg'k]``VY'i bXYfHJ_Yb']b'H' "'Gh XYbhg'k]``dfYgYbhU &\$!a]bi hY'dfc[fUa a Y'XYj]gYX']b'bY[ch]Uh]cb'k]h\'h\Y]f'gdYVJU`]ghh hcfgžhc'gi]hh\Y]f'dYfgcbU``]bhYfYghg UbX'hfUYWcf]Yg"''Gh XYbhg']XYbh]Zh]b['Ug'Unn'gdYVJU`]ghg'k]``dfYgYbhhYWb]WU`k cf_Zfca 'h\Y'Unn gm`UVi g'XYj]gYX'Zcf'h\]g'Wci fgY"'H\Y'Yl Ua]bUh]cb'k]``U`gc']bWi XY'U'g\cfng][\hfYUX]b['HYghZcf'U` gh XYbhg"'H\]g'k]``VY'h\Y'Z]bU`UggYgga YbhHUg_Zcf'dYfZcfa Yfg": cf'Wca dcg]h]cb'gh XYbhgz'dcfhZc`]cg'k]``VY'gi Va]hhYX']b'H' "'DcfhZc`]cg'a Um]bWi XY'Ubmbi a VYf'cZk cf_gžVi hh\Y'hchU`Xi fUh]cb'cZk cf_g'g\ci `X'VY'%) 'a]bi hYg"''GWcfYg'g\ci `X'VY'gi Va]hhYXz'U`cb['k]h\'fYWcfX]b[g'cZU``k cf_g"''H\]g'k]``VY'h\Y'Z]bU`UggYgga YbhHUg_Zcf'Wca dcg]h]cb'dcfhZc`]c"

5 https://bxubwww.Fyei]fYaYbhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj]Yk '`YWhi fY`fYWcfX]b[g"

7ci fgY GWX YXi `Y

J]Yk 'WUgg'hja YHUV'Y

H]a YtUV`Y

8UhY	Hmd Y	7 cbhYbh
K YY_'%'' '>i bY'!'+'>i bY	@YWi fY	±blfcXi Whcb'hc'DUi``A UW!``YWhfYf'Zcf'HYfa'&" GhiXYbhg']blfcXi WY'h\YagY'jYg'hc'DUi`"'HYfa'd`Ub X]gW ggYX'UbX'UggYggaYbhXYhU]`g'UbX'XUhYg'UfY YId`U]bYX"
K YY_`&.`%\$`>i bY`!`%(>i bY	@YWn fY	HYUW.Yf``YX`YbgYa V`Y`]a dfcj]gUlfjcb"'Df]bWjd`Yg`cZ]a dfcj]gUlfjcb'UbX`YbgYa V`Y`k cf_'k cf_g\cddYX`]b WUgg"
K YY_'' .'%+'>i bY'!'&% >i bY	@YWn fY	6Y[]bikcf_icbiH5G?i%":HYUWXYfik]``\YUf gi[[Ygh]cbgiZcfifYdYfhc]fYzighm`YicZ]adfcj]gUh]cb UbXiUgg][biXicg":GhiXYbhgik]``kcf_g\cdiWxc]Wig k]h\ih\Y]fiXicidUfhbYfiUbXihYUWXYfihciVcibWi]XYUg Zcfih\YidfYgYbhUh]cb"
K YY_'(.'&('>i bY'!'&, >i bY	@YWn fY	HUg_`%DYfZcfa UbWY`UbX`ZYYXVUW_#k cf_g\cd VY[]bg"
K YY_').'%>i `m!') '>i `m	@YWhifY	HUg'%DYfZcfa UbWY`UbX`ZYYXVUW_#k cf_g\cd VY[]bg"`GffUhY[]Yg`Zcf`H5G?`'&`X]gWYggYX"
K YY_'*.', '>i `m'!'‱'>i `m	< ca Yk cf_	FYUX]b['k YY_"
K YY_'+.'%) '>i `m'! '%- '>i `m	@YWn fY	DfUWNWTWXb]ei YgʻUbXʻdfYdUf]b[ʻZcfʻZ]bU``H5G?ʻ&Z]bU``dYfZcfa UbWY"DfYgYbhUh]cbʻg_]``gž'dYfZcfa UbWYUbI]Yhmž'gdYU_]b[ʻhcʻUbʻUi X]YbWYž'UbXʻa U_]b[ʻYmYWcbhUWfik]```VY`X]gWiggYX`UbX`k UmgʻhcʻdfYdUfY`k]``VY`k cf_g\cddYX"
K YY_', .'&&'>i `m'!'&*'>i `m	5ggYgga Ybh	Gh XYbh]b!WUgg dYfZcfa UbW UggYgga Ybhg VY[]b"
K YY_''&-'>i`m!'& 5i[igh	5ggYgga Ybh	Gh XYbh']b!WUggʻdYfZcfa UbWYʻUggYgga Ybhg Wcbhjbi Y"
K YY_`%\$.`) '5i [i gh!'- 5i [i gh	5ggYgga Ybh	Gh XYbh']b!WUgg'dYfZcfa UbW''UggYgga Ybhg Wca d'YhY'UbX'WUgg'XYVf]YZ'

FYgci fW/g

DfYgWf]VYX F Ygci fWYg

%''9j Yfmt\]b[']g'7cbbYWYX.'H\Y'Dck Yf'cZA i g]WVm8Ub]Y``6UfYbVc]a

&" ``A chicbž'9a chicbž'UbX'@cj Y. `H\Y'B Uhi fY'cZ5fhighiWDYfZcfa UbWY'VmiH\ca Ug'7 Ufgcb'A Uf

' "'@YhhYfg'hc'U'Mcib['DcYhVmFU]bYf'F]`_Y

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FYWca a YbXYX FYgci fWfg

Bch'Uj U]`UV`Y

7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

@f]YZmci h]bY`\ck 'ghi XYbhZYYXVUW_flVch\ 'Zcfa U``UbX`]bZcfa U`E'cb'h\Y`Wci fgY`k]```VY`[Uh\YfYXž\ck ']h k]``VY`UbU`mgYX`UbX`\ck ']hk]```VY`UbWYX`i dcb'hc']a dfcj Y`h\Y`ghi XYbh`YUfb]b['YI dYf]YbW''': cf'YI Ua d`Yž mci 'a][\hX]gW gg`k \Uhk Ug`]XYbhZ]YX`]b'dUghZYYXVUW_UbX`\ck 'h\]g'Wci fgY`k Ug'WXUb[YX'hc'UXXfYgg'h\Y]ggi Y"Q

Gi Va |gg|cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''#c'ici
UfY'i bUV'Y 'rc'gi Va]hmci f'Ugg][ba YbhXi Y'rc'U'ZJi 'hk]h\'Hi fb]r]b'mci 'a Um'Udd'mZcf'Ub'YI hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'r]WYhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybfgt'rc
]bWi XY'Ug'Yj]XYbWY 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']WUh]cb'''#mci 'Ya U]''9I hYfbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ''mfYWY]j Y'U'r]W_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'rc'gdYW]ZJWJ''mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\ci fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUh]j Y'gi Va]gg]cb'XYhJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"'=hWJb'hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhU`mWcdm]b['Zfca `U`gci fWY'k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidgdu[]uf]ga]bhch\YZc``ck]b[WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`'hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW''cf'i g]b['ei chUh]c<u>b'a Uf_g</u>"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

♣Uddfcdf]UhY 'dUfUd\fUg

\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \] Y 'a cghmfYhU]b]b['h\Y'cf][]bU`

ghfi Wh fY 'UbX#cf'dfc[fYg

Z]XYUg'cZh\Y'cf][]bU`žUbX ']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh''H\]g'U`gc

Udd`]Yg']b'dfYgYbhUh]cbg

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Uf

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\text{X} k 'k \c`Yžk]h\ci h'Uddfcdf]UhY 'fYZYfYb\M}

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g|cb'U`gc h\Ya 7 c``i g]cb. 'k cf_]b['k]h\ ' ridUgg]b['cZih]bWi XYg'dfcj]X]b['mci f' h\Y`XiY`XUhYž`cf`Zcf`h\Y`difd Ubch\Yfgh XYbh\ d`U[]Uf]q]b[UhUbmha Yž 'Ubch\Yf'dYfqcb'hc a Ub UWUXYa |WhUq_žghYU`i UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXY hc Wca d'YhY Ubch\Yf dYfgi dUma YbhZcfWca d`Yhb[/a]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7
gcifWYg'k\]WX`\UjY'
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YX"

8 i d`]\WJh]cb`fl`gY`Zd`U[]U' k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\\ VYYb`dfYdUfYX`cf`gi Va] cf`Ub. ga Ybhcf'\\cir\\tip\ BGK `cf'Ubch\Yf'i b]j Yig.

7 cffYWhfYZYfYbWb[dfUWhWyg.

- i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]]hJYg'Ug'U'ghi XYbh'Uhi BGK VY'Uk UfY'cZd'U[]Uf]ga žWcdmf][\hz| BGK 'Ghi XYbh'7cXY'cZ7cbXi Wh'UbX'5VWWdhJV'Y'I gY'cZ I BGK "=7 H"F Ygci fWYg"Dc`]Wm