

7 ci fgY 8 YHJ]`g 7 fYX]hDc]bhg ** Gi a a UfmcZh\Y 7 ci fgY

Gi VYWn5fYU. ? ge[U

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
9h\bc[fUd\]WfYgYUfW\ dfcYW#dfYgYbHJhcb	(\$1	&%#\$, #&\$% `\$(`.\$\$`DA	%ž&Ž
8]gW gg]cb 'V'c['	' \$ı	Bch5dd`]WUV`Y	%ž&Ž
@]gh/b]b['HYgh'	' \$1	Bch5dd`]WUV`Y	%ž&Ž

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% 9h\bc[fUd\]WfYgYUfWX dfc YWHdfYgYbHJh]cb

GHJfh:XUhY.

@Yb[h\. '9h\bc[fUd\]WfYgYUfW\ fYdcfh '&\$\$\$ k cfXg"

8YHJ]`g. '±0'[fci dg·cZ(!) 'ghi XYbhg·ghi XYbhg·a i ghiWca d`YhYihk c'Wca dcbYbhg·-'U'fYgYUfW.'fYdcfhik cfh\ &\$i 'fa]b" &\$\$\$'k cfXgŁ'UbX'Ub'cfU``dfYgYbhUh]cb'fl%) 'a]bgŁ'k cfh\ '&\$i ''Ghi XYbhg·k]``VY'dfcj]XYX'k]h\ k f]hhYb'ZYYXVUW_cb']bY'UZhYf'gi Va]gg]cb'cZh\Y'fYgYUfW.'fYdcfh'

5XX]h]cbU`XYhU]`g.

<ck 'hc 'gi Va]h'h\ Y 'Yh\ bc[fUd\]WfYgYUfW\ 'dfc YWh'fYdcfh3</pre>

5 hith Y 'VY[]bb]b['cZh\Y'thYfa ž'mci f'Wci fgY 'UXa]b]ghfUhcf'k]```gYbX'mci 'Ub']bj]hUh]cb'hc'h\Y'l BGK 'K cfX A i g]W9h\bc[fUd\]WF YgYUfW. 'Dfc YWhK YVg]hY"'H\Y'Ya U]``k]```WcbhU]b'U'i gYf'bUa Y 'UbX'U'dUggk cfX" : c``ck 'f\Y'`\mdYf]b_'[]j Yb']b'h\Y'Ya U]``hc'gYhi d'mci f'UWWci bh'

C bWY 'mci '\Uj Y 'gYhi d 'mci f'UWWci bhmci 'k]``'Uff]j Y 'UhU'XUg\VcUfX"'H\YfY'mci 'k]``'VY'UV'Y 'hc 'WfYUhY 'mci f 'dcght"' i i a Um'VY 'U' [ccX']XYU'hc 'Vcc_a Uf_'h\Y'UXa]b 'dU[Y 'fXUg\VcUfXŁ'cZh\]g 'g]hY 'Ug'mci 'k]``'dfcVUV'm fYhi fb 'hc']hU'bi a VYf'cZh]a Yg"' i mci 'ZbX'mci 'Wbb 'bchfYhi fb 'hc'h\Y'XUg\VcUfX'UZhYf'mci '\Uj Y 'gYhi d 'mci f UWWci bhmci 'k]``'VY'UV'Y 'hc'ZbX'h\]g 'dU[Y 'Vmi g]b['h\Y'Zc``ck]b['I F @ '\hhdg.##\`c[g"i bgk "YXi "Ui #k cfXa i g]Wdfc YWg#k d!UXa]b#

9UW\"ha Y`mci 'j]g]hd`YUgY`gUj Y`mci f`k cf_`Ug`U`XfUZh"H\Y`Wci fgY`UXa]b]ghfUhcf`k]```VY`UV`Y`hc`gYY`mci f k cf_`]b`XfUZhZcfa 'Vi hbc`cbY`Y`gY`k]```VY`UV`Y`hc"`K \Yb`mci 'UfY`gi fY`mci f`dcgh`]g`fYUXmž'd`YUgY`W]_'h\Y 'di V`]g\'`Vi hhcb"

Gi Va]gg]cb bchYg.fl&; fci d gY ZgY YWljcb hc VY ZbU]nYX Vmghi XYbhg Vm>i bY '\%'' f&E'5 \cdot']ghic ZfYgYUfW [fci dg 'UbX h\Y]f'a Ya VYfg\]d k]``VY dcghYX cb 'A ccX'Y Vm>i bY '\%' "'fl E'F YgYUfW 'hcd]Wg 'UbX 'WcbhUWlg k]h\ fYgYUfW\ 'gi V YWlg ZbU]nYX Vm>i bY '\&) "'f(E'6f]YZfYdcfhcb 'fYgYUfW\ 'dfc[fYgg Xi f]b['hi hcf]U 'g 'cb '>i `m ' 'UbX '("'fl) E'Gi Va]gg]cb 'cb '>i `m\%' fl\%)) da E'UbX 'X]gW gg]cb 'cZ(\$\$!k cfX 'fYgYUfW\ 'UVghfUWlg 'Uhih hcf]U 'g cb '>i `m\%' / '\%+"'fl*E'K f]hhYb 'dfc YWlifYdcfhgi Va]gg]cbg '5i ['\%(\tilde{z}' (.\$\$da "'fl+E'Dfc YWlicfU'' dfYgYbhUh]cbg 5i ['\%\%!(da E'')

5ggYgga Ybh& 8]gW gg]cb V`c[

GhUfh XUhY.

Gi Va]gg]cb'bchYg.6'c['%Xi Y'>i bY'% 'fl\%))da Ł/6'c['&'Xi Y'>i bY'&) 'fl\%))da Ł/6'c['' 'Xi Y'>i 'm'& fl\%))da Ł

5ggYgga Ybh' @]ghYb]b[HYgh

GHJfhXUhY. Bch5dd`]WUV`Y

 $8YhU]`g. `H\]g`UggYgga Ybh'hUg_`Wcbg]ghg`cZhk c`hYghg`cZYei U``k Y][\h]b[" c[U``) $$`k cUXi Y`>i `m&$

5 hm/b XUb VW F Yei]fYa Ybhg

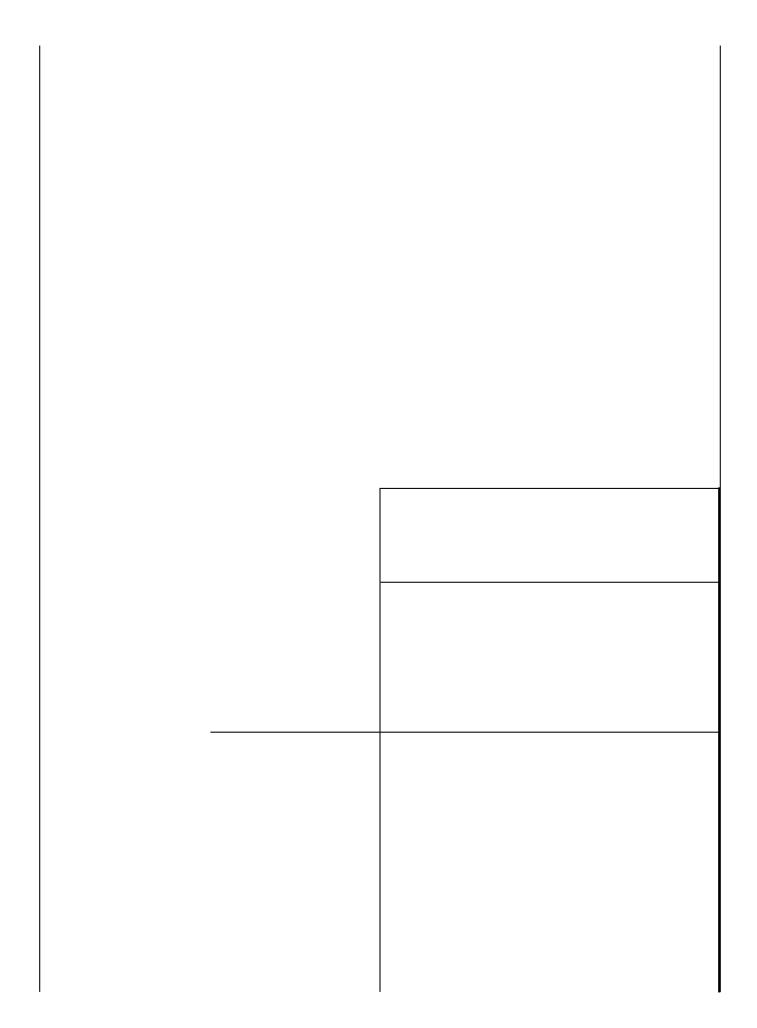
 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] } \\$

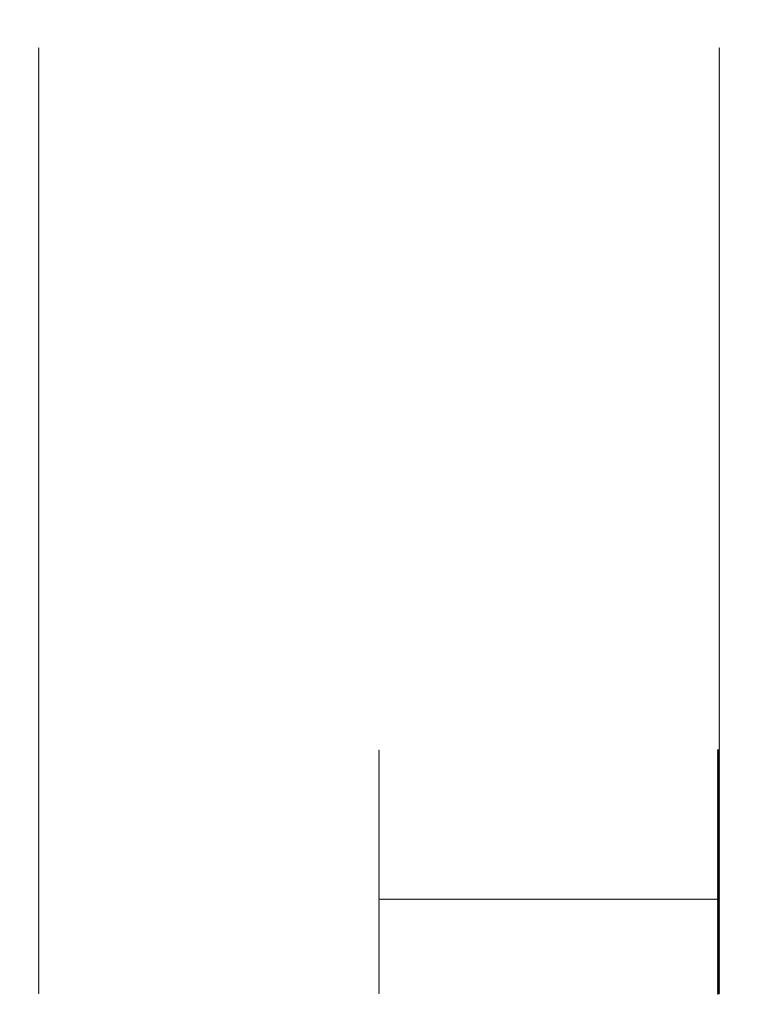
7ci fgY'GWXYXi `Y

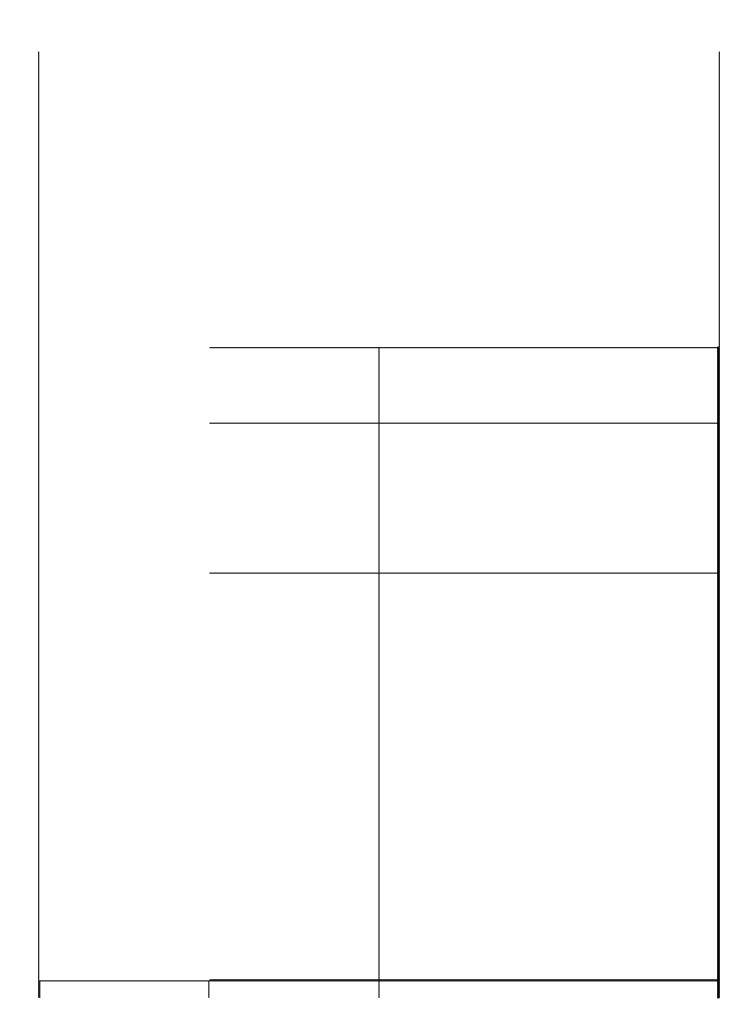
J]Yk 'WUgg'hja YhUV'Y

H]a YtUV`Y

8 UhY	HmdY	7 cbhYbh
K YY_'%'' '>i bY'!'+'>i bY	@YWn fY	@%" =blfcXi Wfjcb. 'gh Xmjb[/ ``]ghYb]b['hc'k cf'X a i g]W'a i g]W/ 'gcW]Ym" A Yh\cXg" A U]b'a i g]W/ `WcbW/dhg" C h\Yf WcbW/dhg. 'W 'hi fU`fY`Uhjj]ga ža i `hjW 'hi fU`]ga ž Wcga cdc`]hUb]ga ž]bhYfW 'hi fU`]ga žgcWjU`Z bWfjcbž gY`Z\ccX"
K YY_'&.'%\$'>i bY'!'%(>i bY	@YWi fY	Hi hcf]U``fYUX]b[g'k]``VY`Ubbci bWX`]b`h\Y``YWi fYg" @&" H\Y`6`UW_`5hUbhJW%—'HfUX]hJcbU``A i g]WZfca ; \UbU\/ 'GYbY[U` %" +blfcXi WJcb'hc'K Ygh5Zf]WUb







K YY_'*.', '>i `m!'%&'>i `m	5ggYgga Ybh	@]ghYb]b["HYgh%fXi f]b["gWXYXi "YX""YWh fY"h]a YŁ		
		Bc'@YWifY		
K_YY_"+."%)	@YVVi fY	<i>@</i> *"'H\Y'6`UW_'5hUbŊWf7iVUŁ		
		H\Y'7iVUb'ea`'Wcad`YI'fea`ł_S_Tałf[_TSŁ		
	Hi hcf]U`	Gi Va]gg]cb^fbi `m%* ž'%%)) da Ł'UbX'X]gW gg]cb^cZ (\$\$!k cfX^fYgYUfW\"UVglfUWg"		
		5bU`mg]g`cZXSVa Zcfa "`		
K YY_', .'&&'>i `m!'&*'>i `m	@YWhifY	@+"`H\Y`6`UW_'5hUbhJWf6fUn]`Ł		
		5S`Va_TWVeS_TS * fdab[US1[S		
	Hi hcf]U`			

FYgci fWYg

DfYgWf]VYX`F Ygci fWYg`

 $\label{eq:hamiltonian} $$H\YfY^{-}]g^{-}U^{-}F\ Ygci\ fWYg^{-}gYWY]cb^{-}]b^{-}A\ ccX^{-}Y^{-}h^{-}Uh^{-}W^{-}cb^{-}U]bg^{-}h^{-}Y^{-}Z^{-}C^{-}ck\]b[\ .$

 $\underline{5"`F\ Ygci\ fVWg`Zcf'a\ UbXUhcfmWci\ fgY``]ghYb]b[\ `UbX`fYUX]b[\ .}$

%"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b['rc'gi Va]hmci f'Ugg][ba Ybht\fci [\'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU''Gi ddcfhcb'-',) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft\"'=\maintention
UfY'i bUV'Y 'rc'gi Va]hmci f'Ugg][ba YbhXi Y'rc'U'ZJi 'hk]h\'Hi fb]r]b'mci 'a Um'Udd'm\Zcf'Ub'YI hYbg]cb\ZVi hmci
a i ghfYhJ]b'mci f'r]\W_Yhbi a VYf'\Zfca '9I hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybfg\L'rc
]bWi XY'Ug'Yj]XYbW' 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']\WJh]cb'''=\maintention
UfY'' bUYY''mfYWY']j Y'U'r]\W_Yhbi a VYf\Zfvi h]Zmci 'hY'Yd\cbY'mci 'k]'''bYYX'rc'gdYW]\ZfwJ''mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]``'VY'gHJhYX'cb'mci f'\\ci fgY'g'A ccX`Y'g]hY'k]h\ U`hYfbUh]j Y'gi Va]gg]cb'XYhJ]`g''

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! qi Va]hUgq][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±i\WbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiW\YUh]b[ihciU\W]XYbhU`m\Wcdm]b[iZfca iUigci f\Wik]h\ci hiU\w_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b[WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[. '7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]Y 'a cghmfYhU]b]b['h\Y'cf][]bU`ghi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU`žUbX']bZcfa Uh]cb'k]h\ci hUW_bck`YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k\YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'UbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk'k\c`Yžk]h\ci h'Uddfcdf]UhY'fYZYfYbW]b['"

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhiUbmh]a Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_ž'ghYU']b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[`gcifWrg`k\]W\`\UjY`bchVYYb`fYUXžk]h\cihUW_bck`YX[]b[`h\Y`´gYWcbXUfm´gcifWrZfca`k\]W\`_bck`YX[Y`cZh\Ya`\Ug`VYYb`cVhU]bYX"

8 i d`]\WUh]cb`ffgY`Zd`U[]\Uf]ga "Ł gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWnfYZYfYbWlb['dfUWnWyg.

• DUfUd\fug]b[z'gi a a Uf]q]b[z'YggUmk f]hb['UbX'ha Y'a UbU[Ya Ybh