

# 7ci fgY'Cj Yfj ]Yk

#### Ghuzz 7 cbhuWh8 Yhu]`g

'7cbj Ybcfg'

BUa Y	9a U]`	5j U]`UV]`]hm	@cWJhjcb	D∖cbY
?]a '6i fk Y``	_"Vi fk Y``4 i bgk "YXi "Ui	Vm'Uddc]bha Ybh	KYVghYf*%%\$	

#### GWXcc`7cbHJWh =bZcfa Uhlcb

Fcca " %&ž~Yj Y~" FcVYfhK YVghYf~6i ]X]b[

D\cbY.'f\$&Ł'-',)'(,)\*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. \hdg.##gUa "Ufhg"i bgk "YXi "Ui

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'h\Y'HfUX]tjcbU``7igtcX]Ubgž h\Y'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fti/'8Yg][b'7UadigYgŁUbX't\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`cWJhYX''$ 

# 7ci fgY 8YHJ] g

7 fYX]hDc]bhg\*\*

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. ? ge[U

6YWca ]b[ 'U'DYfZcfa Yf']g'ZcW gYX'cb'h\Y'bUhi fY'UbX'UWei ]g]h]cb'cZa i g]\W''dYfZcfa Ub\W'g\_]``''\#iXYYdYbg UbX'WcbhYl hi U']gYg'mci f'gdY\\JU']gh\_bck `YX[ Y'k ]h\ fYZYfYb\W''hc'mci f'ck bž'fY`Uh]j Y`m'UXj Ub\W'Xž'ghU[ Y'cZ XYj Y`cda Ybh'Ug'U'a i g]\\JUb\"'H\Y'cj YfU```cV'Y\\fi]g'hc \Y`d'mci 'hc'i bXYfghUbX'h\Y'XYj Y`cda Ybh'cZmci f'ck b Yl dYfn]gY'cV'Y\\Jj Y`mž'gc'h\Uh'mci '\Wb'\\f\]h]\W'`mYj U'i UhY'h\Y'ghi X]c'hfUX]h]cb']b'h\Y'`][ \hcZVfcUXYf UddfcU\W\Yg'hc'a i g]\WYXi \Wb]cb'UbX'hc'ch\Yf'\_]bXg'cZ\_bck `YX[ Y''

### 7ci fgY @YUfb]b[ 'Ci hWca Yg

- %" i bXYfghUbXžXYa cbghfUhY`UbX#cf`X]gW gg`h\Y`bUhi fY`cZa i g]\W\`g\_]```UbX`]hg`U\\ei ]g]h]cbžk ]h\ fYZYfYbW\`hc`h\Y`Yd]ghYa c`c[ mcZdfU\W]\W\/
- $\& "fYWc[b]gY`UbX`Wf]h]WU``mUggYgg`U'j Uf]YhmcZUddfcUWXYg`hc`h\Y`hYUWX]b[`UbX``YUfb]b[`cZ dYfZcfa UbWY`g_]``gž'h\fci [\`fYZYWf]cb`cb`h\Y]f`ck b`YI dYf]YbWY`]b`]bglfi a YbHU``UbX'j cWU```Yggcbg/$
- '"]XYbfjZmUbX`WfjfjWU``mYj U`i UfY`dfcZYgg]cbU``UbX`fYgYUfW\`\_bck `YX[Y`ZcWigYX`cb`]bgffi a YbfU``/ j cWU```YUfb]b[`UbX`fYUW\]b[

#### HYUWX]b[ GhfUhY[]Yg

@YWi fYg`k]``YI d`cfY`a i g]WJUbg\]d'Ug`U`Wca d`YI 'g\_]``ž'UfYUg`cZghi XmUbX`h\Y`Yd]ghYa c`c[ mcZdfUWj\W'/fUX]h]cbg`cZa i g]WJ``hYUW.]b[ 'UbX``YUfb]b[ ž']bWi X]b[ 'UddfYbhJWfg\]d'UbX`h\Y`WcbgYfj Uhc]fY`a cXY`/Wca a i b]WJh]cb'UbX`a cXY``]b[ /UbX`ghi XYbhUddfcUW.Yg`hc`]bX]j ]Xi U``dfUWJ\W''``5 'X]ghJbWf]cb'k]```VY`XfUk b` VYhk YYb`dfcZYgg]cbU``UbX`fYgYUfW.`\_bck `YX[ Yž'fY`UhYX`hc`a i g]WJ``g\_]```XYj Y`cda Ybhž'UbX`ei U`]hUh]j Y` fYgYUfW.`a Yh\cXg`k]```VY`]bhfcXi WfX"

+b`k cf\_g\cdgž'ghi XYbhg`k]```dfYgYbhWf]h]ei Yg`cZfYgYUfW\`ZcW`gYX`cb`a i g]\W\``dYfZcfa Ub\W`g\_]```UbX`]hg` U\\e'i ]g]h]cbž'XYj Y`cd]b[ 'U'gYbgY'cZ\\ebhYl h\Zcf'h\Y]f`dYfgcbU``Yl dYf]Yb\W`UbX`gdY\\]U`]ghhfUX]h]cbg`k ]h\]b` h\Y`VfcUXYf`a i q]\\U``W`hi fY"

# 5ggYgga Ybh

5XX]hjcbU``]bZcfa Uhjcb`k]``VY`dfcj]XYX`cb`a ccX`Y"

#### 5ggYgga Ybh'HUg\_g'

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7 f]h]ei Y	&) ı	Bch5dd`]WUV`Y	4
9ggUmi	')	\$&#\$, #&\$% `%%) - `DA</td><td>%₹</td></tr></tbody></table>	

#### 5ggYgga Ybh'8YhJ]`g

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GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\."%\\$\$\$\Ž\%\\\$\$\$

8 YHU]`g. `Ghfi Wi fYX`V`c[g`fWU`&ž\$\$\$`k cfXg`hchU`Łk]```Ug\_`ghi XYbhg`hc`fUŁ`fYZYWhicb`h\Y``]b\_g`VYhk YYb`h\Y fYgYUfW\ YbWei bhYfYX`]b``YWi fYg`UbX`h\Y]f`ck b`ghi X]Ygž'UbX`h\Y]f`dYfgcbU``YI dYf]YbWY`]b`UW&i ]f]b[gdYW]U`]ghdYfZcfa UbWY`g\_]``g/UbX`fVŁ`Wc``YWWžg\UfY`UbX`Wf]h]WU``mYj U`i UhY`k YV!VUgYX`dfcZYgg]cbU`a UhYf]U`g`fY`UhXXhc`h\Y`gi VYWh': YYXVUW\_`k]``VY`dfcj ]XYX`Y`YWhfcb]WU``mžj ]U`Wca a Ybhg`cb`h\Y`V`c[g"`

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' Hi & fb \end{tabular} $$ h' \end{tabular}$ 

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GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'%)\$\$'kcfXg

8 YhJ]`g. 'H\]g`k]```VY`U`ghfi Whi fYX`Wf]h]ei Y`fWU`%) \$\$`k cfXgŁcZUb`Ya d]f]WU``fYgYUfW.`fYdcfhž`fY`UhYX`hc`h\Y ghi XYbhfgi gdYWJU`]ghUfYU`cZYI dYfh]gY"'5 'fYWca a YbXYX``]ghik]```VY`[]j Ybž?fca 'fYZYfYYX'ci fbU`g/ghi XYbhg k]```\Uj Y`h\Y`cddcfhi b]hmhc`]XYbhJZmdUdYfg`a cfY`WcgY`mfY`UhYX`hc`h\Y]f`ck b`]bhYfYghgžgc``cb[`Ug`h\YmUfY Uddfcj YX`Vmh\Y`hi hcf"'H\Y`YI YfW]gY`k]```VY`UggYggYX`UhU`fY`Uh]j Y`mYUf`mghU[Y`cZh\Y`Wci fgY`VYWUi gY`]h k]```\Y`d`k]h\`dfYdUfUh]cb`cZh\Y`YggUm': YYXVUW\_k]`y``VY`dfcj]XYX`]b`h\Y`Zcfa 'cZU'k f]hhYb`fYdcfh''

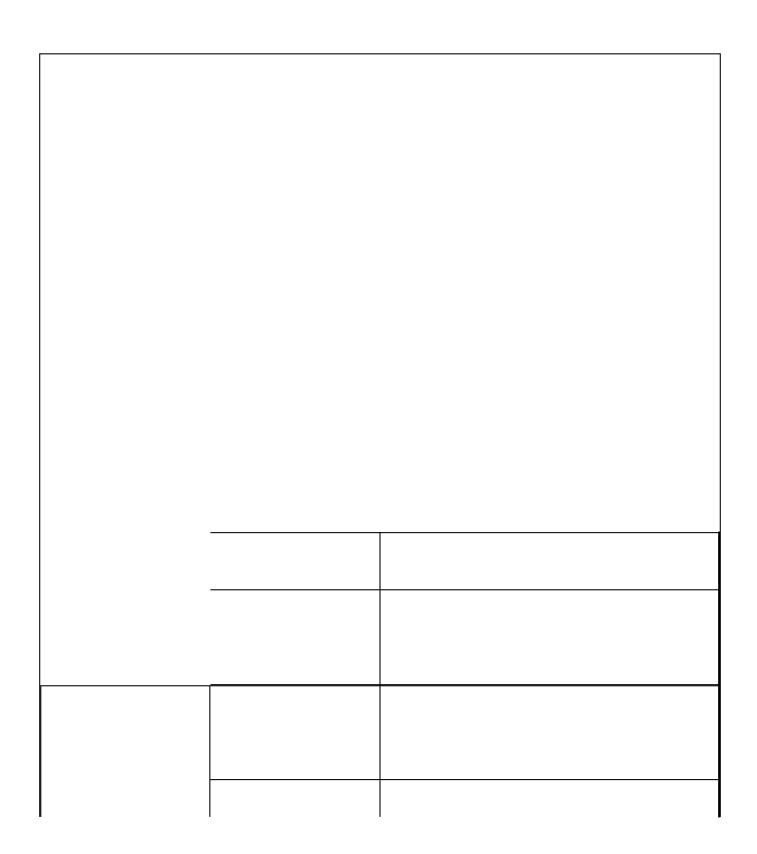
Hi fb]h]b gYh]b[. `H\]g Ugg][ ba Ybh]g gi Va ]hhYX h\fci [\`Hi fb]h]b UbX gh XYbhg Xc bchgYY Hi fb]h]b g]a ]`Uf]hmfYdcfhg"

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GhUfhXUhY. Bch5dd`]WUV`Y

**@Yb[h**. &ž\$\$\$ k cfXg

**8 YHJ]`g**. `H\Y`hcd]WZcf`h\]g`YggUm`fWU`&ž\$\$\$`k cfXgŁ`k]```VY`fY`UhYX`hc`h\Y`ghi XYbhfig`gdYWJU`]ghiUfYU`cZ YI dYfh]gYž`UbX`k]```XfUk `cb`U`Wf]h]WU``UWWci bhcZU`fUb[Y`cZfYgYUfW\`ghi X]Yg"`H\]g`]g`h\Y`Z]bU``UggYgga Ybh



		V"``Gh XYbh`YX`gYa]bUf`f) fcid`5Ł
		; fci d'9.''6`c['f&Ł'Xi Y'%%) - ': F =85M') '>i `m
		; fci d'5.''7f]h]ei Y'Xi Y'‱) - ': F -85M') '>i `m
K YY_'*.', '>i `m!''&&'>i `m	@YWh fY	U"``Ghi XYbhiUddfcUVXYg`hc`dfUWhJWY
		V"``Gh XYbh`YX`gYa]bUf`fl fcid`6Ł
	GYa ]bUf	U"``FYZYW¶b[`cb`UddfcUVXYg`hc`dfUW¶WY
		V"``Gh XYbh`YX`gYa]bUf`f} fcid`7 Ł
		; fci dg 6 / '7.''7 f]h]ei Y'Xi Y'%) - ': F = 8 5 M'&'>i `m
K YY_'+.'%) '>i `m'! '%- '>i `m	eYWh fY	U"``DfcV`Ya UhJWghi X]c`fY`UhJcbg\]dg
		V"``Gh XYbh`YX`gYa]bUf`fl fcid`8Ł
	Hi hcf]U`	U"``9ggUmhdg7 'gi [[Ygh]cbg
		V"``Gh XYbh`YX`gYa]bUf`fVm; fci d`9Ł
		; fci dg'8 '/ '9.''7 f]h]ei Y'Xi Y'%%) - ': F =8 5 M'%- '>i `m
K_YY_', .'&&'>i `m!'&* '>i `m	@YWhifY	±bX]j ]Xi U``hi hcf]U`gž'UgʻgW\YXi `YX
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5i[igh	Hi hcf]U`	U"``5ZZYWh]b`UXjUbWYX`ghX]c``Yggcbg
		V"``8fcd!]b`YggUmgi ddcfh
		5```ghi XYbhg.``9ggUmXi Y`;‰) - ': F -85M1&'5i [i gh

# FYgci fWfg

## DfYgWf]VYX F Ygci fWYg

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## FYWca a YbXYX FYgci fWYg

 $\label{eq:hammer} H\Y'fYWca\ a\ YbXYX'fYUX]b[\ 'UddYUfg']b``Y[\ Ubhcž'cb'a\ ccX`Y''$ 

.

### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\Y'Wc'i fgY'Yj U'i Uhjcb'k]``'VY'WcbXi WhYX'h\fci [\'h\Y'fA mYl dYf]YbWYfigi fj Ym'

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Hi fb]h]b Gi Va ]gg]cb

#mci YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dr]b[ 'rc'gi Va ]hmci f'Ugg][ ba Ybht\fci [ \'Hi fb]r]bž'd'YUgY
hY'Yd\cbY'9I hYfbU'Gi ddcfhcb'-', ) '''' %cf'Ya U]''h\Ya 'cb'YI hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''#c'ici
UfY'i bUV'Y 'rc'gi Va ]hmci f'Ugg][ ba YbhXi Y'rc'U'ZJi 'hk ]h\'Hi fb]r]b'mci 'a Um'Udd'mZcf'Ub'YI hYbg]cbž'Vi hmci
a i ghfYhJ]b'mci f'r]WYhbi a VYf'Zfca '9I hYfbU''Gi ddcfhfU'cb[ 'k ]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybfgt'rc
]bWi XY'Ug'Yj ]XYbWY 'rc'gi ddcfhmci f'YI hYbg]cb'Udd']WUh]cb'''#mci 'Ya U]''9I hYfbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ''mfYWY]j Y'U'r]W\_Yhbi a VYfž'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]'''bYYX'rc'gdYW]ZJWJ''mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhYg'cb'h\Y]f'gmghYa 'gfUh' g'cb'Hk]hYf"

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]``'VY'gHJhYX'cb'mci f'\\ci fgY'g'A ccX`Y'g]hY'k ]h\ U`hYfbUh]j Y'gi Va ]gg]cb'XYhJ]`g''

cfi]bZcfa Uh]cbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbh'i bgk "YXi "Ui #\ck! qi Va ]hUgq][ ba Ybha ccXiY

#### 5 WUXYa ]W<cbYghmUbX D`U[ ]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"'=hWJb'hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhU`mWcdm]b[ 'Zfca `U`gci fWY'k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidgdu[]uf]ga ]bhch\YZc``ck]b[ WuhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`'hYl h'cf']XYU'k]h\ci h'UW\_bck `YX[]b['h\Y gci fW''cf'i g]b['ei chUh]c<u>b'a Uf\_g</u>"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

♣Uddfcdf]UhY 'dUfUd\fUg

\Ub[ ]b[ 'U'ZYk 'k cfXg'UbX'd\fUgYg'k \] Y 'a cghmfYhU]b]b[ 'h\Y'cf][ ]bU`

ghfi Wh fY 'UbX#cf'dfc[ fYg

Z]XYUg'cZh\Y'cf][ ]bU`žUbX ']bZcfa Uh]cb'k ]h\ci hUW\_bck `YX[ Ya Ybh''H\]g'U`gc

Udd`]Yg']b'dfYgYbhUh]cbg

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Uf

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±bUddfcdf]UhY`W]hUh]cb.`7
gcifWYg'k\]WX`\UjY'
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8 i d`]\WJh]cb`fl`gY`Zd`U[ ]U' k cf\_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\\ VYYb`dfYdUfYX`cf`gi Va ] cf`Ub. ga Ybhcf'\\cir\\tip\ BGK `cf'Ubch\Yf'i b]j Yig.

7 cffYWhfYZYfYbWb[ dfUWhWyg.

- i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]]hJYg'Ug'U'ghi XYbh'Uhi BGK VY'Uk UfY'cZd'U[]Uf]ga žWcdmf][\hz'I BGK 'Ghi XYbh'7cXY'cZ7cbXi Wh'UbX'5VWWdhJV'Y'I gY'cZ I BGK "=7 H"F Ygci fWYg"Dc`]Wm