



# **ARTS2036**

A cXYfb]ga . HYI hUbX GWfYYb

HYfa "Hk c"#"&\$&\$

# 7ci fgY<sup>-</sup>Cj Yfj ]Yk

#### GHJZZ7cbHJWh8YHJ]`q

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUhjcb	D\cbY
DfcZYggcf <sup>-</sup> 9`]nUVYh\	Y"a Wa U\cb4 i bgk "YXi "Ui		K YVghYf'&&'	- ' , ) %/* (
A WA U\cb	_		_	

#### GWkcc`7cbhJWhi ≠bZcfa Uh]cb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca " %&ž@Yj Y`" žFcVYffiK YVgfYf'6i ]`X]b[ 'f) %( Ł

D\cbY. 'f\$&Ł'-',) '(,) \*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $H\Y'GVX cc``cZh\Y'5fhg`UbX`A YX]U'k ci `X``] \underline{Y'hc'F YgdYWM} ``m'5 \underline{W}bck `YX[Y'h\Y'HfUX]h]cbU``7 i ghcX]Ubgž h\Y'6YXY[U``fP Ybg]b[hcb'WJa di głž; UX][U``fP ]hm'UbX'5fh/ '8Yg][b'7 Ua di gYgŁ'UbX'h\Y'B[i bbUk U`$ 

7 ci fgY 8 YHJ]`g 7 fYX]hDc]bhg \* Gi a a UfmcZh\Y 7 ci fgY

## 5ggYgga Ybh

5 fUb[Y cZgi ddcfha UhYf]U g k ] ``VY Uj U] `UV Y cb A ccX Y

#### 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'NJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
9ggUmi	))।	%%#\$, #&\$&\$`\$, .\$\$`DA	%ž&Ž ž( ž) ž*
FYUX]b['; Ua Y'	<b>%\$</b> I	Bch5dd`]WUV`Y	%Z
F YgYUfW\. '91 YfW]gY '	')।	&&#\$*#&\$&\$`\$, .\$\$`DA</td><td>%ž&ž*</td></tr></tbody></table>	

### 5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh'% '9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'&\$\$\$'k cfXg

8YHJ]`g.

&\$\$\$ k cfXg"

H\]g^]g^h\Y^Z|bU^`UggYgga YbhHUg\_"^Gi Va ]gg]cb^j ]U^@A G"

.

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$ 

5ggYgga Ybh& FYUX]b[; Ua Y

GhUfhXUhY. Bch5dd`]WUV`Y

8 YW] g.

Gi Va ]gg]cb'j ]U'@A G"'H\]g']g'h\Y'Z|fghUggYgga YbhHUg\_"

5XX]h]cbU`XYhJ]`g.

H\]g [ Ua Y fi bg Wcbh]bi ci g mh\fci [ \ci hh\Y Wci fgY

5ggYgga Ybh' . F YgYUfW '9I YfW]gY

GhUfhXUhY. Bch5dd`]WUV`Y

**@Yb[h**\."\&\$\$"kcfXg

8YHJ]`g.

\$ k cfXg"'H\]g']g'h\Y'h\]fX'UggYgga Ybh'lUg\_"'Gi Va ]gg]cb'j ]U'@A G"

# 5 hm/b XUb VW F Yei ]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] }$ 

# 7ci fgY GWX YXi `Y

## J]Yk 'WUgg'hja YhUV'Y

# H]a YtUV`Y

8 UhY	HmdY	7 cbhYbh
K YY_'\$.'&) 'A Um!'&- A Um		
K YY_'% '%'>i bY '!') '>i bY	@YWi fY	@YWNifY`%`±blfcXiWNjcb
		@YWnfY'&.'G\cfhiZcfa'dfcgY.'g\cfhighcfmifbYY AccX'Y'Zcf'h\Y'hYIhgE''H\Y'8YUX''Zfca'>UaYg >cmWYžFZW6gT^EWbZ?Uh\Yf]bY'AUbgZ]Y'Xž'>YUb F\mgž'>cgYd\'7cbfUXž':"'GWchh:]m[YfU'Xž'81bU 6UfbYgž'
	GYa ]bUf	±btfcXi WfjcbgʻUbXʻG\cfhiZcfa 'dfcgY.'g\cfhighcfmffgYY A ccX`YʻZcfʻth\YʻthYI hgŁʻth\Yʻ8YUX'Zfca '>Ua Yg >cmWYžFZW6gT↑ Wdež′? Uth\Yf]bYʻA UbgZJY`Xž'>YUb F\mgž'>cgYd\`7cbfUXž': "'GWcthi: ]m[YfU`Xž'8↑bU 6UfbYgž
K YY_'&.', '>i bY'!'%& >i bY	@YWhifY	

Í	I	% &+Łž6i bi Y`G` 5Z[W 3`VS/ag fZ]a '% &-Ł
	GYa ]bUf	A Uf]bYtt]'g:8gfgd[ef?S`[W\efa+A]bU @cmlg8W_[ [ef?S`[W\efa
		: f]m`@Ub[ž? Wfcaba'[e f1]`a '% &+tž6i bi Y`'G` 5Z[W 3`VS'ag f1]`a '% &-t
K YY_'*.'*'>i `m'!'%\$'>i `m	FYUX]b[	F958∃; K99?
K_YY_"+."%		7]hmgWUdYg
		@YWnifY`%%~z¨H"`G"`9`]ch~z¨FZWISefW>S`V
		>WUfgdW#\$H"'G"'9`]chzFZWI_SefW>S`V_UbX cdYb]b['WXUdhYf'cZ>Ua_Yg'>cmWY'g'G1keeWe
	GYa ]bUf	
		H"'G"'9`]chëFZWI SefW>S`VUbX'cdYb]b['WXUdhYf'cZ>Ua Yg'>cm\W'g'G1keeWe
K YY_', .'&\$'>i `m!'&( '>i `m	n@YWhifY	DUf]g': fUbW'
		@YWnffY'%'; Yfhfi XY'GhY]bžFZW3gfaT[aYdSbZk aX 31[UW4 Fa] 'Se
		@YWnffY`%(`; YffrfiXY`GhY]bžFZW3gfaT[aYcSbZkaX 3^[UW4Fa]^Se
	GYa ]bUf	
		DUf]g`: fUbW
		; Yfffi XY GhY]bžFZW3gfaT[aYcSbZk aX3′[UW4 Fa] 'Se
K_YY_''&+'>i `m'!'' %'>i `m	n@YWhifY	@UhY A cXYfb]ga
		@YWfi fYg'%) 'UbX'%* '>i `m'GUa i Y`'6YW_Yhhz'=dSbb e >Sef F SbWfd `Umz'%), Ł
	GYa ]bUf	GUa i Y`6YW_Yhhë=cSbb e >SefFSbWfd`Umž%), Ł
K YY_'%\$.'' '5i [ i gh! '+	@YWh fY	Dfcj ]bWJU`A cXYfb]ga g
5i [ i gh		@YWn fy"%+"9"YUbcf"8Uf_žFZWDaSV fa 5aa'S_ [ž
		@YWNifY`%, `9fb`AU``YmžgY`YWNYX`kcf_g`FcVmb <mxyžgy`ywnyx`kcf_g`fld`cuxyx`hc`accx`ył< td=""></mxyžgy`ywnyx`kcf_g`fld`cuxyx`hc`accx`ył<>
I		ı

GYa ]bUf	9 YUbcf 8 Uf_ž F ZWD a SV fa 5 a a ′S_ [ž
	'9fb'AU``YmžgY`YWNYX'kcf_g'FcVmb' <mxyžgy`ywnyx kcf_g</mxyžgy`ywnyx 

## FYgci fWyg

### DfYgWf]VYX F Ygci fWYg

 $H\Y'A\ ccX`Y'g]hY'k\ ]```]bWi\ XY'U'V]V`]c[\ fUd\mz'dXZg'cZgca\ Y'g\cfh'hYI\ hgž'UbX'fYgci\ fWfg'j\ ]U @Y[\ Ubhc']bWi\ X]b[\ ``]b\_g'hc'Z]`a\ g$ 

#### FYWca a YbXYX FYgci fWfg

F Ygci fWg'k ]``'VY'dfcj ]XYX'j U]'h\Y'A ccX`Y'g]hY'UbX'@Y[ Ubhc'

### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\Y`Wcbj Ybcf`k Y`Wca Yg`ZYYXVUW\_`h\fci [\ci hh\Y`Wci fgY`UbX`k]```UWfj Y`mgYY\_`gh XYbhgfij ]Yk g`cb`h\Y Wci fgY`h\fci [\ci hh\Y`h\fa "`A m9I dYf]YbWf`k]```YbUV`Y`gh XYbhg`hc`Wca a Ybh]b`h\Y`Wci fgY`UbX`h\Y h\UW\]b[`cj YfU``"

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

# 5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/