

# **ARTS2050**

5 WUXYa ]WK f]h]b[ Zcfh\Y < i a Ub]h]Yg

HYfa "Hk c" ## "&\$&\$

# 7ci fgY<sup>·</sup>Cj Yfj ]Yk

# GhJZZ 7 cbhJWh 8 YhJ] `g

# 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
5ggcWJUHY DfcZYggcf				

## 7ci fgY 8YHJ] g

#### 7 fYX]hDc]bhg\*\*

#### Gi a a UfmcZh\Y'7ci fgY

H\]g`Wci fgY`XYj Y`cdg`ghi XYbhg``WJdUWJhmhc`k f]hY`UWJXYa ]WYggUmg`UWVcfX]b[ 'hc`h\Y`Wcbj Ybh]cbg`cZh\Y <i a Ub]h]Yg"`\\defidcj ]XYg`h\Y`\_bck `YX[Y`UbX`j cWJVi `UfmVmk\]W\`ghi XYbhg`WJb`Wf]h]WU``mUbU`mgY`h\Y`k cf\_cZch\Yfg`Ug`k Y```Ug`h\Y]f`ck b"`H\Y`Wci fgY`ZcW`gYg`gi ghU]bYX`UhhYbh]cb`cb`j Uf]ci g`a cXYg`cZUWJXYa ]W k f]h]b[ 'h\UhiUfY`i gYX`]b`YggUmk f]h]b[ 'UbX`g\ck g`\ck 'h\YgY`a UmVY`XYd`cmYX`hc`dfYgYbhU`ghfcb[ Uf[i a Ybh''H\Y`Wci fgY`]bWi XYg`UbU`mg]g`cZU`fUb[Y`cZYI Ya d`Ufg`Ug`k Y```Ug`YggUmg`Zca 'UWJXYa ]W ci fbU`g`fYgYUfW\YX`Vmghi XYbhg`UbX`fY`UhYX`hc`h\Y]f`X]gWJd`]bUfm]bhYfYghg"`H\]g`ghi Xmk ]``VY UWWca dUb]YX`Vmk f]h]b[ 'YI YfWJgYgž'k f]h]b[ 'k cf\_g\cdg`UbX`XYhJ]`YX`ZYYXVUW\_`Zca 'Vch\`dYYfg`UbX`ghUZ'

### 7ci fgY @YUfb]b[ 'Ci hWca Yg

- %" %" 8 Ya cbghfuhY ZJW] hmik jh\ [fua a Uf'UbX'gYbhYbWY Wcbghfi Whjcbž'UggcWjuhYX'k jh\ Z bXUa YbhU`Yl dYWfuhjcbg'cZUWUXYa jWk fjhjb [h]b'h\Y'\i a Ub]hYq"
- &" &" 5dd`mUgdYWg`cZh\Y`df]a Ufma cXYg`cZUWUXYa ]Wk f]h]b[ `]b`h\Y`\i a Ub]h]Ygž`]bWi X]b[ YI dcg]h]cbžXYgWf]dh]cbžbUffUh]j Y`UbX`Uf[ i a YbhUh]cb"
- '"'"Ghfi WhifYžYX]hžUbXdfYgYbhUd]YWYcZkf]h]b['UWWcfX]b['hc'WcbjYbh]cbg'cZUWUXYa]Wkf]h]b[']bhXY`\iaUb]h]Yg"

#### HYUWX]b[ 'GhfUhY[]Yg'

#### HYUW ]b[ 'GlfUh'[ ]Yg

 $\label{thm:condition} $$HY''HY'UWX]b['a Yh'cXg''i gYX'UbX'h'Y'UggYgga Ybh'Wca dcbYbhg'\Uj Y'VYYb'XYj Y`cdYX'hc''fYZYWh'U d\]cgcd\mcZ'YUfb]b['UbX'hY'UWX]b['k\]WX$ 

- ZcghYfg Ub Yb[ U[ ]b[ 'UbX']bWi g]j Y "YUfb]b[ 'Yl dYf]YbWYž
- dfca ch/g dYYf YUfb]b[ 'h\fci [\'dYYf fYj]Yk 'UbX'UggYgga Ybh
- gYY\_g'fc'[]j Y'Yj Yfmghi XYbh'U'ghU\_Y']bž'UbX'Ub'cddcfhi b]hmhc'Wcbhf]Vi hY'hcž'h\Y'Wci fgY'Ug'U'Zcfi a Zcf''YUfb]b["

#### F Uhcbu'Y 'cZ5qqYqqYX'HUq\_q

- ```H\Y`DfUWJ\WU``YIYfWJgYg`UfY`U]aYX`UhUW\]Yj]b[``YUfb]b[`ciHWcaYg`%UbX''`UbX`XYjY`cd]b[`9b[`]g\ [fUXiUhY`Uhf]ViHYg`%~(ž\*"
- ````h\Y`G\cfhYf`k f]h]b[g`hUg\_`]g`U]a YX`Uh`UW\]Yj ]b[``YUfb]b[`ci hWca Yg`%'` 'UbX`XYj Y`cd]b[`9b[`]g\ [fUXi UhY`Uhf]Vi hYg`%ž' ž'\*"

````h\Y`9| hYbXYX`YggUm]g`U]a YX`UhUW\]Yj ]b[ ``YUfb]b[ `ci hWca Yg`%' ž`UbX`XYj Y`cd]b[ `9b[ `]g\ `[ fUXi UhY Uhf]Vi hYg`%ž' ž`(ž') `UbX`\*

## 5ggYgga Ybh

5FHG&\$) \$1.5WUXYa ]WK f]h]b[ 'Zcf'h\Y'<i a Ub]h]Yg

A cfY`]bZcfa Uh]cb`cb`UggYgga YbhiHUg\_g`Zcf`th\]g`Wci fgY`]g`Uj U]`UV`Y`cb`th\Y`A ccX`Y`g]hY"

#### 5ggYgga YbhHUg\_g

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
G\cfnk f]rhYb^d]YVWg^	) \$1	\$'#\$+#&\$&\$`%%)-`DA	%ž&
9ggUmi	(\$1	\$-#\$,#&\$&\$`%%)-`DA	% <u>ž&amp;</u> Ž
±b!7`UggʻHYghʻ	%\$I	%+#\$+#&\$&\$`%%) - `DA	%

#### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% G\cfhk f]hYb d]YWg

GhUfhiXUhY. \$\%\\$\*#&\$&\$ \%. \$\$ \5A

**@Yb[h**. '%\$\$!%) \$\$ k cfXg

8YHJ]`g.

Gh XYbhg k ]``dfcXi W'h fYY k f]hh/b d]YWg Ug dUfhcZh Y `YUfb]b[ 'UWhj ]h]Yg ]b h Y Z]fghZj Y k YY\_g cZh/fa " cfa Uhj Y ZYXVUW\_k ]``VY dfcj ]XYX Ug U dUfhcZh YgY `YUfb]b[ 'UWhj ]h]Yg "H\Y h fYY k f]hh/b d]YWg a i gh h Yb gi Va ]hh/X Zcf gi a a Uhj Y UggYgga YbhUhh Y YbX cZk YY\_Zj Y" C fU fY WcfXYX ZYYXVUW\_ž U bi a Yf]WU [fUXY UbX U fi Vf]Wk ]``VY dfcj ]XYX k c k YY\_g UZh/f gi Va ]gg]cb"

 $\label{eq:higher} \mbox{Hi fb]h]b'gYh]b[.'H\]g'gg][ba Ybh]g'gi Va ]hhYX'h\fci [\'Hi fb]h]b'UbX'gh XYbhg'Xc'bchgYY'Hi fb]h]b g]a ]'Uf]hmfYdcfhg''$ 

5ggYgga Ybh& 9ggUm

GhUfh'XUhY. '\$\%\\* \* #&\$&\$ '\\&. \$\$ '5 A

@Yb[h\.'\\\$\$

8YHJ]`g.

8 YHJ]`g.`K cfX``Yb[h\.`%\$\$"'H\]g`]g`h\Y`Zbu``UggYgga YbhHJg\_"`: YYXVUW\_`k]``VY`dfcj]XYX`]b`h\Y`Zcfa `cZ cfU``ZYYXVUW\_`UbX`U`a Uf\_]b[ `fi Vf]W

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$ 

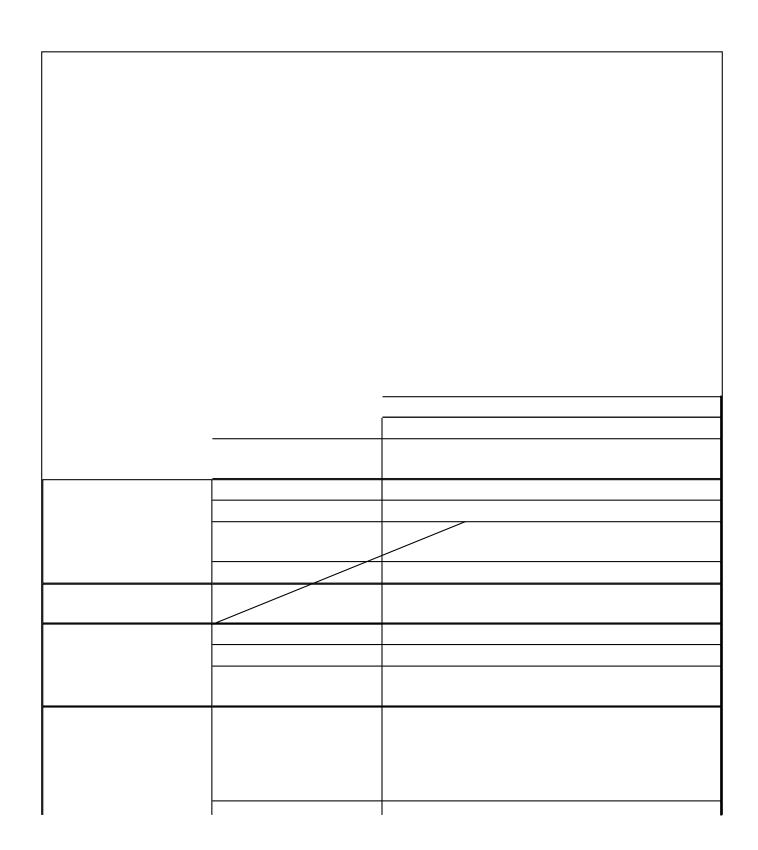
5ggYgga Ybh'' . '\dg'HYgh

GHJfh'XUhY. '&-#\$\*#&\$&\$'%&.\$\$'5A

8YHJ]`g.

 $C\ b`]bY'[fUa\ a\ Uf'hYgh''C\ b'hcd]Wg'Wcj\ YfYX']b'h\Y'Wci\ fgY'''A\ Uf\_YX'hYgh'fYh'\ fbYX'hc'gh'\ XYbhg'k\ ]h\ 'bi\ a\ Yf]WU` [fUXY''$ 

 $\label{eq:higher_problem} \mbox{Hi fb]h]b$^{-}gYh]b[.^{-}H\]g$^{-}g$^{-}bchU$^{-}Hi fb]h]b$^{-}Ugg][$ba$^{-}Ybh$^{-}h]b$^{-}Ugg][$ba$^{-}h]b$^{-}Ugg][$ba$^{-}h]b$^{-}Hi fb]h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]b$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}h]h$^{-}$ 



# FYgci fWfg

#### DfYgWf]VYX F Ygci fWYg

 $\label{eq:hammer_hammer} $$H\YfY^{g'bc'gYh'hYI} h'Vcc_{Zcf'h} ]g'Wci fgY$$ 

### FYWca a YbXYX FYgci fWYg

5bmch\YfigYZ\`fYgcifWYg'k]``VYid`cUXYX'hc'h\Y'WcifgY'AccX`Y'g]hY'''

### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Gh XYbhZYXVUW\_cb'h]g'Wci fgY'k]``'VY'[Uh\YfYX'i g]b['i b]j Yfg]hm'Ui hca UhYX'dfcW'Xi fYgž'Ug'kY``'Ugh\fci [\']bZcfa U`ž'Ubcbma ci g'ZYXVUW\_Zcfa g'': YYXVUW\_k]``'VY'i gYX'hc'UbU'mgY'UbX'UggYgg'h\Y'Wci fgYž

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi V'YWfi |XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY

# 5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['lc.