7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
>cX] ⁻ 6fcc_g		K YXbYgXUmi%4% UbX'ch\Yf'h]a Yg'Vm Uddc]bha Ybh	V	·)) * ¹)

GWkcc~7cbhJWh~abZcfa Uh]cb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca ' %&ž@Yj Y`' žFcVYfhK YVghYf6i]X]b[fl %(Ł

D\cbY. f\$&L'-',) '(,) *

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. 'k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7UadigYgŁ'UbX'thY'B[ibbUkU`dYcd'Y'fbigtfU]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZthY'`UbXg'k\YfY'YUW\cong WJadig'cZIBGK']g`cWJtYX''$

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gii aaa autolfora oo Zirhoy fayo i fgy

H\]g`Wci fgY`lfUWYg`h\Y`Ya Yf[YbWY`UbX`g][b]Z]WUbWY`cZ5i glfU`]Ub`W]bYa U`W``hi fY'''Mci `k]```ghi Xmlh\Y XYj Y`cda YbhcZh\Y`5i glfU`]Ub`Z]`a `]bXi glfmž`]bWi X]b[`UbU`mg]g`cZh\Y`YWcbca]WžgcW]U``UbX`dc`]h]WU`
ZUWcfg`UbX`h\Y`a ml\g`k \]W\`\Uj Y`g\UdYX`h\Y`]bXi glfm''Mci `k]```U`gc`ghi Xmlh\Y`fc`Y`cZh\Y`5i glfU`]Ub
gWfYYb`]b`g\Ud]b[`h\Y`YI dYf]YbWY`cZa cXYfb`5i glfU`]U`UbX`]ng`d`UWY`]b`h\Y`k cf`X"`H\Y`Wci fgY`U`gc
YI d`cfYg`U`fUb[Y`cZa U]bghfYUa `UbX`bcb!a U]bghfYUa `Zcfa g`cZUi X]c!j]gi U``dfcXi Whcb`]b`5i glfU`]U"`H\Y
hcd]WgžZ]`a g`UbX`Wf]h]WU``hYI hg`\Uj Y`U```VYYb`gY`YWYX`hc`YbWci fU[Y`h\Y`ghi XmcZU`k]XY`fUb[Y`cZ
5i glfU`]U'g`bUh]cbU``W]bYa U"``

7 ci fgY @YUfb]b[Ci hWca Yg

- %" ⇒XYbhjZmgca Y`cZh\Y`\]ghcf]\W`žYWcbca]\Wžgc\\jU``UbX`W`hi fU``\WcbhYl hg`cZ5i ghfU`]Ub`\\jbYa U &" 9j U`i UhY`h\Y`d`U\W`cZ5i ghfU`]Ub`\\jbYa U`]b`fY`Uhjcb`hc`< c``mk ccXžfY[]cbU`žUbX`ch\Yf`bUhjcbU`\\WbYa Uq
- '"→XYbhJZmÜbX'Wf]hJWU``miUggYgg'h\Y'X]j Yfg]hmcZ5i ghfU`]u'g'bUhJcbU`'WJbYa U'UbX'dfYgYbhmci f'ZJbX]b[g]b'Wc\YfYbhzk Y```ghfi Whi fYXz'cfU`'UbX'k f]hhYb'Zcfa"

HYUW]b['GhfUhY[]Yg'

H\Y`Wci fgY`k]``VY`HJi [\hi g]b[`U`Wca V]bUh]cb`cZ`YWfi fYgžgWfYYb]b[gž`WJgg`X]gW gg]cbž[fci d`fYgYUfW. UbX`dfYgYbHJn]cbž]bX]j]Xi U``fYgYUfW.`UbX`k f]h]b["``H\Y`hYUW.]b[`UddfcUW.`U``ck g`gh XYbhg`hc`YI dUbXgž`gWfYYb]b[g

5ggYgga Ybh

 $8YHJ]^*YX^*]bZcfaUH]cb^*cb^*Vcfh^*UggYggaYbh^*HJg_g^*k]^*^*VY^*UjU]^*UV^*Y^*cb^*AccX^*Y''^5ggYggaYbh^*HJg_g^*k]^*^*U^*gcVY^*X]gW^*ggYX^*]b^*WUgg^*hc^*Ugg]gh^dfYdUfUH]cb^*UbX^*h_YfY^*k]^*^*VY^*cddcfh^*b]H^Yg^*hc^*dfYgYbh^*UgdYW^*g^*cZmcifkcf_f^*h_Y^*WUgg^*Zcf^*ZYYXVUW_"$

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'tUg_	K Y][\h	

FYgci fWyg

DfYgWf]VYX F Ygci fWfg

H\Y`Wci fgY`A ccX`Y`]g`mci f`_YmfYgci fWY`Zcf`h\]g`Wci fgY"`H\Y`Wci fgY`A ccX`Y`]g`k\YfY`mci `k]```Z]bX`Ub]blfcXi Wfjcb`hc`YUW\`k\YY_fg`hcd]Wz`h\Y``]b_g`hc`ci f`cb`]bY`WUggYg`UbX`gWfYYb]b[gž`UbX`U``]b_`hc`h\Y`Wci fgY @Y[Ubhc`fk]h\`mci f`k\YY_`mfYUX]b[g`UbX`YI lfU`gWfYYb]b[gŁ"`C b`h\Y`Wci fgY`A ccX`Y`mci `k]```Z]bX``]b_g`hc _Ym5i glfU`]Ub`Z]`a `UfW\]j Ygž`Z bX]b[`VcX]Yg`UbX`dfcZYgg]cbU``UggcW]Uh]cbg`Ug`k\Y```Ug``]b_g`hc`j]XYc dfYgYbhUh]cbg`Vm!! `UbX`]bhYfj]Yk\g`!!`k]h\`5i glfU`]Ub`Z]`a a U_Yfg"`D`YUgY`gYhUg]XY`gca Y`h]a Y`hc`YI d`cfY UbX`ZJa]`]Uf]gY`mci fgY`Zk]h\`h\Y`Wci fgY`A ccX`Y`VYZcfY`h\Y`Wci fgY`gHUfng"

FYWca a YbXYX FYgci fW/g

FYWca a YbXYX fYgci fW/g UfY Uj U] UV Y h\fci [\@Y [Ubhc"

7 ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

@]_Y'U```ci f'Wci fgYgz'h\]g'Wci fgY']g'fY[i `Uf'mfYj]gYX'UbX'XYj Y`cdYX'gc'h\Uh]h\Wb'Yb[U[Y'k]h\ W ffYbh UbX'Ya Yf[]b['XYVUhYgz'h\Ycf]Yg'UbX']XYUg'UbX'gc'h\Uh]h\Wb'Ya VfUW'bYk 'fYgci f\W'g''Gh XYbhZYXVU\W_Wb'VY'U'j U'i UV'Y'fYgci f\W'jb'\Wci fgY'fYj]g]cb'UbX'XYj Y`cda YbhUbX'Ug'U'dUff]\W]dUbh]b'h\]g'\Wci fgY'mci f ZYXVU\W_cb'\Wci fgY'gfi \Wi fY'UbX'\Wci fgY'a UhYf]U'g'k]``VY'gci [\h\Uhj Uf]ci g'dc]bhg']b'h\Y'\Wci fgY'' 5`cb[g]XY'h\Y'a cfY'i gi U''fYj]g]cbg'UbX'XYj Y`cda YbhcZ\\bhYbhfi dXUh]b['fYUX]b[gz''fYZ\Yg\]b['' hcd]\\g'Yh\\Zh\]g'\\ci fgY'\Ug'U'gc'\YYb'fYXYg][bYX'gc'h\Uh]h\\Ub'k cf_\Zf'cb']bY'XY']j Yfm]b'ci f'\W ffYbh 7 C J =8!\% 'UX\\ghY\X'k cfX'''\\ci k]``gYY'\Zfca 'k YY_'\%h\Uh]b'fUbg]h]cb]b['h\Y'\\ci fgY'hc'cb']bXi ghfm\UbX']g mYUf'=\Uj Y'U'gc']b'fcXi \W'X'gca Y'a cfY'''\UbXg!cb'''fYgYUf\\X']b'nc'h\Y'5i glfU']Ub'g\\YYb']bXi ghfm\UbX']hg \]ghcfm'

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgHUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'ghi XYbh'UhI BGK
 VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hz'I BGK 'Ghi XYbh'7cXY'cZ7cbXi Wh'UbX'5VWWdhUV`Y'I gY'cZ

5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/