

7ci fgY^{*}Cj Yfj]Yk

GhJZZ 7 cbhJWh 8 YhJ] `g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWUh]cb	D\cbY
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D\cbY.'f\$&Ł'-',)'(,)*

9a U]`. gUa 4 i bgk "YXi "Ui

7ci fgY 8YHJ] g

7 fYX]hDc]bhg * *

Gi a a UfmcZh\Y 7ci fgY

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I dcb'Ybfc`a Ybhzmci 'a i ghgi Va]h'U'W ff]W `i a 'j]h'UY']bWi X]b['g_]``g'UbX'YI dYf]YbW'z'UbX'U'ghUhYa YbhcZ dYfgcbU```YUfb]b['UbX'WUfYYf'[cU`g"'Mci 'k]``k cf_ 'cb'd`UWYa Ybhk]h\ 'Ub']bXi ghfmcf[Ub]gUh]cb'zcf'-\$!%\$\$\ci fgža UbU[YX'Ufci bX'h\Y'cf[Ub]gUh]cbfg'fYei]fYa Ybhg'UbX'h\Y'ch\Yf'Wci fgY'fYei]fYa Ybhg'cZmci fXY[fYY"

7 ci fgY @YUfb]b[Ci hWca Yg

- %" FYWc[b]nY'h\Y'j Uf]ci g'ZJWYhg'cZh\Y'dYfZcfa]b['Ufhg']bXi ghfmUbX'\ck 'hYUW\]b['UbX'`YUfb]b[ž WfYUh]j Y'XYj Y`cda Ybhž'dfcXi Wh]cb'UbX'UXa]b]ghfUh]cb'UfY']bhYf!WcbbYWYX"
- &" 5dd`mU'j Uf]YhmcZWf]hJWU```YbgYg`hc`h\Y`fc`Y`cZd\mg]WU``dYfZcfa UbWY`Ug`U`hmdY`cZ_bck`YX[Y`]b`h\Y VfcUXYf`UYgh\YhJWZW``hi fU`z`gcWJU``UbX`YWcbca]WWcbhYl hg`h\Uha U_Y`i d`h\Y`dYfZcfa]b[``Ufhg]bXi qhfm'
- ' "8Ya cbglfUhY fYgYUfWX g_]``g`UbX`Wf]h]WU``h\]b_]b[`]b`[Uh\Yf]b[ž`WUgg]Zm]b[`UbX`Wcaaib]WUh]b[]bZcfaUh]cb`UVcihh\Y`dYfZcfa]b[`Uflg"
- ("5dd`mik f]h]b['g_]``g'hc'h\Y'hfUbg`Uh]cb'UbX'Wca a i b]WUh]cb'cZdYfZcfa UbWY'dfUWh]WYg'UbX _bck `YX[Yg"
-) " 8 Ya cbglfUhY'd\mg]WU``Uk UfYbYgg`UbX`_bck `YX[Y'h\fci [\'dUfh]W]dUh]cb']bž'cf'cVgYfj Uh]cb'cZz WfYUh]j Y'dfcWYgg`UbX'dYfZcfa UbWY"

HYUW]b['GhfUhY[]Yg'

- 7 YbhfU``WcbWfdhg`Zfca 'h\Y'a U'cf`fBUbWf`Ghi X]Ygž'H\YUhfY`UbX`DYfZcfa UbWf`Ghi X]Ygž'A i g]WE'k]``
 VY`WUff]YX`]bhc`h\]g`gi V`YWfik\YfY`ghi XYbhg`k]```VY`Z]bY!hi b]b[`g_]``g`]b`]bXYdYbXYbhfYgYUfW\\
 i h]`]g]b[`_bck `YX[Yg`UWWfi YX"
- H\YgY`WcbW'dhg`k]```VY`fY!UddfcUWXYX`h\fci[\`U`Wcbg]XYfUh]cb`cZ\ck`'h\Ym'ZbWfjcb`]b`VfcUXYfUYgh\Yh]WzW`hifU`z`gcW]U``UbX`YWcbca]WWcbhYlhg`cZh\Y`dYfZcfa]b[`Ufhg`]bXighfm'

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5ggYgga Ybh

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g\\\YXi `Y'U'[YbYfU`]bZcfa Uh]cb'gYgg]cb'Ufci bX'K YY_') "'=Zmci f']bh\fbg\]d'd\f]cX'YI \\YXg'h\Y'f\dcfh
XYUX`]bYZd`YUgY'VY']b'hci \\X'k]h\ 'a Y"

K CF? GI 6A ≠H98.

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I bfY`]UV`Y`k YVdU[Y`fYZYfYbWrg`UfY`bchUVWYdrUV`Yž'Y["'K]_]dYX]U"`

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8YW]`q.

3 &" " " Z " " i dWbadffZSfW YSYWe i [fZ S Ud[f[US^[eegWXadfZWZaefadYS` [eSf[a` S` V SVVdWeeWe S bdaTW_ fZWk ` WWW fa dWeWSdJZ S` V dWeba` V fažFZWdWbadf VaWe ` af ZShWfa TWW f[dWk fW] fTSeWW /YdSbZeł V[SYdS_ e WfUfz

GHUfhXUhY. Bch5dd`]WUV`Y @Yb[h\. b#J 8YW]`q. 3 &" " " Ž&' " " i dWbadffZSfWYSYWei [fZ S Ud[f[US^[eegWXadfZWZaefadYS`[eSf[a` S`V SVVdNeeWe S bdaTW_ fZWk`WWW fa dMeWSdUZS`V dMeba`V fažFZWdMbadfVaWe`afZShWfa TWW ffdMk fW/fTSeWW /YdSbZełV[SYdS e WfUfž 5 Wfcgg h\Y hk c df]cf hUg_g h\YgY ``YUfb]b['ci h\ka Yg k YfY 'UXXfYggYX" =fiY gd`]hih\Ya ']bhc hk c a Uf_g hc gi]h h\Y`k\Umh\Y`UggYgga\Ybhg`UfY`gYhi\d`cb`5GHF5`fUbX`k\]W\`=WUbffiV\\Ub[Y\!\\Y`(\\$i\\Zcf`h\]g`hUg_`k]``VY UggYggYX'U[U]bgh ŒŁ FYWc[b]nY h\Y j Uf]ci g ZJWYhg cZh\Y dYfZcfa]b['Ufhg]bXi ghfmUbX \ck 'hYUW\]b['UbX ``YUfb]b[ž WYUhji Y'XYj Y`cda Ybhž'dfcXi Whjcb'UbX'UXa]b]ghfUhjcb'UfY']bhYf!WcbbYWhYX" @L'5dd`mU'j Uf]YmrcZWf]h]WD```YbgYg'hc'h\Y'fc`Y'cZd\mg]WD``dYfZcfa UbWY'Ug'U'mdY'cZ_bck `YX[Y']b'h\Y VfcUXYf'UYgh\Yh]WzWi`hi fU`zgcW]U``UbX`YWcbca]WWcbhYl hg'h\Uha U_Y'i d'h\Y'dYfZcfa]b['Ufhg']bXi ghfm' CRL 8 Ya cbglfUlY fYgYUfW g_]`g]b [Uh\Yf]b[ž m 'dY

a Uf_	Wf] W f]U
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	Y`UVcfUh]cb`VYmcbX`h\cgY`WcffYWh]cbg`fYgi`h]b[`]b`U`WYUf`UbX
	dYfgi Ug]j Y'7J "'9l WY``YbhYl dfYgg]cb'UbX'cf][]bU`]hm]b'h\Y'ZfUa]b['cZ
	k.cf_'UbX'YXi WUh]cb'\][\`][\hg"
8	5b'Udd`]WUhjcb'cZWcffYWhjcbg'hc'h\Y'7J'UbX'U'fYZjbjb['UbX'Y`UVcfUhjcb
	VYmcbX`h\cgY`WcffYWh]cbg`fYgi`h]b[`]b`U`WYUf`UbX`dYfgi Ug]jY`7J"`7`YUf
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7	5b'Udd`]WUhjcb'cZWcffYWhjcbg'hc'h\Y'7J'UbX'U'fYZjbjb['UbX'Y`UVcfUhjcb
	VYmcbX`h\cgY`WcffYWf]cbg"`7`YUf`YI dfYgg]cb`UbX`gca Y`cf][]bU`]hm]b`h\Y
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	Bc'Udd`]WUhjcb'cZWcffYWhjcbg'hc'h\Y'7J"'IbWYUf'YIdfYgg]cb'UbX
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 $\label{thm:condition} \begin{tabular}{ll} Hi fb]h]b"gYH]b"[."H\]g"Ugg][ba Ybh]g"gi Va]hHYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"Xc"bchigYY"Hi fb]h]b g]a]`Uf]hmfYdcfhg" \\\end{tabular}$

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DfYgWf]VYX F Ygci fWYg

GYY'cb`]bY'fYUX]b['`]ghUhcb'AccX`Y"

FYWca a YbXYX FYgci fWyg

GYY'cb`]bY'fYUX]b['``]ghcb'AccX`Y"

7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y`cda Ybh

 $H\]g'WcifgY'k\]``VY'Yj\ U`i\ UhYX'h\ fci\ [\ \ 'U'A\ m9I\ dYf]YbW''gi\ fj\ Ym'Uhh\ Y'YbX'cZh\ Y'WcifgY''$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

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HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-',) "''' %cf'Ya U]''h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui ""Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ub'Y! hYbg]cb\mathred{Image: Wa Jhmci h\Yf'f'Y'j UbhXcW a Ybhg\mathred{Image: Wa Uj''9! hYfbU''Gi ddcfhmci 'k]'`

JbWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hYbg]cb'Udd']\wUh]cb"'\mathred{Image: Wa Uj''9! hYfbU''Gi ddcfhmci 'k]'`

Ui hca Uh]\wU'`mfYW']j Y'U'h]\w_Yhbi a VYf\mathred{Image: hY'Yd\cbY'mci 'k]\tag{Y}\wu''mug_'\mathred{Image: Wa'\mathred{Image: hY}\wu''mug_'\mathred{Image: hY}\wu''mug_'\mathred{Image: hY}\wu''mug_'\mathred{Image: hY'Yd\cbY''mci 'k]\tag{Y}\wu''mug_'\mathred{Image: hY'\mathred{Image: hY'\mathred{I

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUhjj Y'gi Va]gg]cb'XYHJ]`g''

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5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

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- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWhg'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'Uh'l BGK "'\\hd.##gi V'YWfi |XYg"`]VfUfm'i bgk "YXi "Ui #\`]qY#JVci hY`]qY

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