

7ci fgY Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`] m	@cWJh]cb	D\cbY
6fmcb] ⁻ HfYn]gY	V"ffYn]gY4 i bgk "YXi "Ui	D`YUgY'Ya U]`'Z]fgh	K YVghYf	-',)()%
		Zcf'Ub'Uddc]bha Ybh	6i]`X]b[[.] %%+	

Hi hcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWJhjcb	D\cbY
A Uf_'A]h/XY``	a "a]h/XXY``4 i bgk "YXi "Ui		€ A mYfg	- ' ,) '(, * '
DfcXi Wjcb'A UbU[Yf			H\YUlfY	

GWXcc`7cbHJWhi±bZcfa Uhjcb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@Yj Y`" žFcVYfhK YVghYf'6i]`X]b[ff %(Ł

D\cbY. f\$&£'-',) '(,)*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhq"i bqk "YXi "Ui #qUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5\Lock\`YX[Y'h\Y'HfUX]tjcbU``7i\gtcX]Ubg\cong h\Y'6YXY[U'`fP\Ybg]b[tcb'WJa\di\gt\cong \cong m5\Lock\`YX][U'`fP\Ybg]b[tcb'WJa\di\gt\cong \cong m5\Lock\`Y'B[i\bbUk\U'\dYcd'Y'fbi\gth\'Y'B[i\bbUk\U'\dYcd'Y'fbi\gth\'Y'\UW\\`WJa\di\g'\cong \cong \cong m5\Lock\Y'\UbXg'k\YfY'YUW\\`WJa\di\g'\cong \cong \cong\cong \cong \cong \cong \cong \cong \cong \cong \cong \cong \cong$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

Gi VYWh5fYU. FZWSfdWS` V BWdAd_S`UWEfgV[We

5fY mci '`cc_]b['Zcf'U'WlfYYf'Ug'U'dYfZcfa Yf3'8 c'mci 'k Ubhhc'i gY'dYfZcfa UbW'hc 'Yl dfYgg'mci f'ck b]XYUg3'H\]g'Wci fgY']g'Zcf'U``'ghi XYbhg']bhYfYghYX']b'k cf_]b['k]h\'dfUWlJW'hc 'XYj Y`cd'U'dYfZcfa UbW'k cf_ Zcf'di V']WdfYgYbhUh]cb"'DYfZcfa UbW'hU_Yg'd'UW']b'h\Y'\YfY'UbX'bck "'H]a Y''cdYbg'i d'UbX'VcX]Ygž]a U[Yg'UbX'k cfXg'fYg\UdY'k\Uhh\Y'Z hi fY'Wb'cc_'`]_Y'''K\]'Y'XfUk]b['cb'ZfUa Yk cf_g'Zfca 'hfUX]h]cbg'cZ ']j Y'UfhUbX'dYfZcfa UbW'ZYUW\ k YY_`mk cf_g\cd'k]``dfcj]XY'mci 'k]h\'hYW\b]ei Yg'Zcf'WfYUh]b['mci f'ck b dYfZcfa UbW''''Mci 'k]``Yl d`cfY'U'k]XY'fUb[Y'cZUff]gh]WdfUWJ\Wgz'WfYUh]j Y'dfcW'ggYg'UbX'Wf]h]WU``WcbhYl hg Zcf'a U_]b['mci f'k cf_"'Mci 'k]```k cf_'h\fci [\ci hh\Y'Wci fgY'cb'dfUW]W'zk\Yh\Yf'mci 'UfY'a U_]b[dYfZcfa UbW'''mci fgY`Zcf'XYj Y'cd]b['mci f'XfUa Uhi f[]\W'`ZYYXVUW_'g_]``g'cb'h\Y'dfcW'gg"

7ci fgY @YUfb]b[Ci hWca Yg

- %" gmbh\Yg]gY`Wca d`YI`a UhYf]U`g`Zfca `fYUX]b[gž`WUgg`YIYfW]gYgž`dYYfg`UbX`gY`ZIXYjY`cda Ybh`]bhc`U Wc\Yg]jY`)`a]bi hY`gY`ZIXYj]gYX`dYfZcfa UbWY`Zcf`di V`]WdfYgYbhUh]cb
- &" i bXYfgHJbX`UbX`UffjWi`UhYfVch\`]b`k f]h]b[`UbX`]b`dYfZcfa UbWYŁ'h\Y`bUhi fY`cZXfUa Uhi f[]\WJ```Ub[i U[Y`k\]W\]g`]b`h\Y`dfcWYgg`cZVY]b[`XYj Y`cdYX`Zcf`di V`]Wg\ck]b[
- ' "YZZYWNji Y`mWf]njei Y`n\Y`k cf_`cZdYYfg`Ug`U`k UmcZcZZYf]b[`XfUa Uhi f[]WU``gi ddcfh
- ("i bXYfgHJbX'h\Y'bYWfgg]hmcZWcbh]bi YX'UhhYa dhgžWca dcg]h]cbU``ghfUhY[]Yg'UbX'fi]j YfidYfZcfa UbWf dfUWf]Wf']b'h\Y'di fgi]hcZXYj Y`cd]b['U'dYfZcfa UbWf'`Ub[i U[Y'cZcbYfg'ck b
-) " fYZYWni dcb 'UbX 'Ufn]W `UhY 'UVci hih\Y fY`Uh]cbg\]d VYhk YYb 'dfcWYgg 'UbX 'h\YcfYh]WU`ž\]ghcf]WU`cf WcbhYl hi U``]XYUg 'UbX'h\Y'dYfZcfa UbWY'hYl h]hgY`Z

UbX`\]ghcf]WJ``dYfZcfa UbWY`dfUWf]WYg`[i]XY`mcif`h\]b_]b[`UbX`\Y`d`hc`dcg]h]cb`mcif`ck b`WfYUh]jY`aYh\cXg"H\Y`YaVcX]YX`UbX`YI dYf]Ybh]U```YUfb]b[`h\Uhmci i bXYfhU_Y`k]```YbUV`Y`mci hc`WfUZhWfYUh]jY`cihWcaYgh\UhacjY`cf`UkU_Yb`UiX]YbWYg`VmfYZYWf]b[`U`Wf]h]WU``dcg]h]cb"`Mci k]````YUfb`hc`[]jY`UbX`fYWY]jYXfUaUhif[]WU``ZYYXVUW_`]b`U`giddcfh]jY`UbX`Wc``UVcfUhjY`UvcfUhcfmgYh]b["

Gi Va]gg]cb bchYg.:]bU`dYfZcfa UbW k cf_dfYgYbhYX'j]U``]j YghfYUa

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh' . DfY!ZYWjj Y 5bU`mg]g

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. &&\$\$!&) \$\$ k cfXg

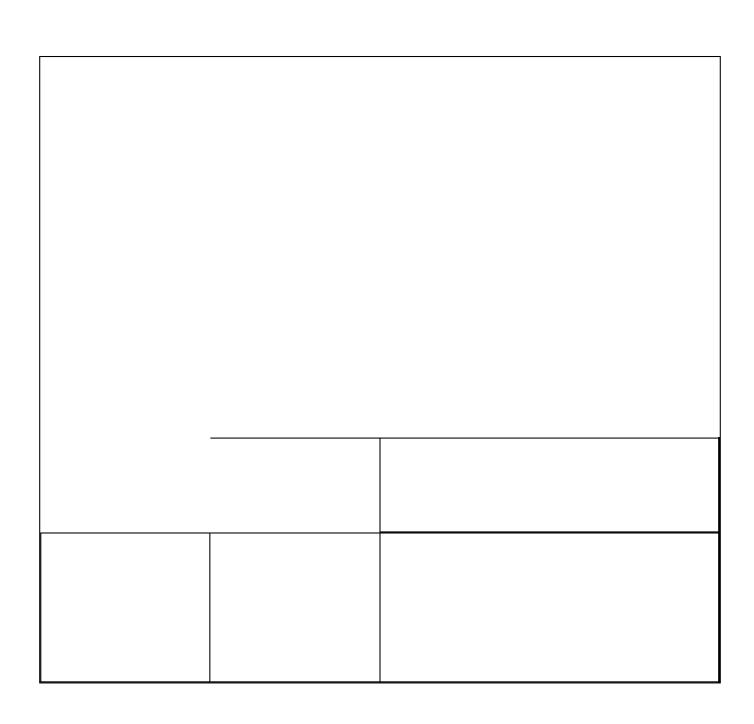
8YHJ]`g.

FiVf]WzWcaaYblUfmUbX^[fUXY^!XiY^YbX^kYY__^)"

5XX]h]cbU`XYhU]`g.

D`YUgY`gYY`AccX`Y`Zcf`UXX]hJcbU``]bZcfaUhJcb`fY[UfX]b[`h\]g`Ugg][baYbh'

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"UbX"gh"XYbhg"Xc"bchgYY"Hi fb]h]b"UbX"gh"Xhybg"Xc"bchgYY"Hi fb]h]b"UbX"gh"Xhybg"Xc"bchgYY"Hi fb]hg"Xhybg$



FYgci fWyg

DfYgWf]VYX F Ygci fWYg

F YUX]b[g'Uj U]`UV`Y']b'@Y[Ubhc"

FYWca a YbXYX FYgci fWfg

D`YUgY`gYY`fhY`WcifgY`AccX`Y`Zcf`Ub`YIfYbg]jY``]ghcZfYUX]b[g`UbX`X][]fU``fYgcifWYg''

7 ci fgY 9j U'i Uh]cb UbX 8 Yj Y'cda Ybh

H\]g`Wci fgY`]g`fci h]bY`m`]a dfcj YX`cb`h\Y`VUg]g`cZ]bZcfa U``ZYYXVUW_[]j Yb`Vmgh XYbhg`hc`hYUW\Yfg`Ug`k Y``Ug`h\fci [\`Zcfa U``a YW\Ub]ga g`gi W\`Ug`A m9I dYf]YbW`]b`h\Y`Z]bU``k YY_g`cZh\Y`Wci fgY''`

A m9I dYf]YbWY`ZYYXVUW_`]g`j]hU``hc`gi ghU]b]b['h\Y``]ZY`cZ'h\]g`Wci fgY"`K Y`YbXYUj ci f`hc`a U_Y`h\Y dfYW]ci g`gdUWY`cZh\]g`dYfZcfa UbWY``UVcfUhcfmUg`]bbcj Uhjj Yž`Zi]X`UbX`f][cfci g`Ug`dcgg]V`Yž`gc`d`YUgY dfcj]XY`mci f`ZYYXVUW_`k\Yb`kY`fYei Ygh`]h'

±0`&\$&\$`k Y`UfY`ZUW]b['i bdfYWYXYbhYX`W]fW a ghUbWYg`h\Uhk]```fUX]WU``mfYg\UdY`h\Y`XY`]j Yfma cXY`Vi h U`gc`dYfZcfa Uh]j Y`ci hWca Yg`UggcW]UhYX`k]h\`h\]g`Wci fgY"`H\]g`]g`U`[fYUhcddcfhi b]mh'`Ghi XYbhg`k]```VY`Uh h\Y`ZcfYZfcbhcZYI dYf]a YbhUh]cb`Ufci bX`h\Y`bYI i g`cZdYfZcfa]b[`h\Y`X][]hU``mf]j Yf'i

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

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Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) "''' %cf'Ya U] hya 'cb'Yl hy'fbu'hy'ngi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Yl hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9| hy'fbu''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Yl hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9| hy'fbu''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYf'Z'Vi h]Zmci 'hy'Yd\cby'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cby''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hYf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghcZmci f'gh X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/