

7ci fgY'Cj Yfj]Yk

GHJZ:7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUhjcb	D∖cbY
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GWkcc~7cbhJWh~abZcfa Uh]cb

GVXcc``cZhY`5fhg`UbX`AYX]U

Fcca ' %&ž@YjY`' žFcVYfhK YVghYf6i]`X]b[fl %(Ł

D\cbY. f\$&£'-',) '(,) *

9a U]` gUa 4 i bgk "YXi "Ui

K YVq]hY. k k k "Ufhq"i bak "YXi "Ui #qUa

 $\label{eq:hamiltonian} $$HY'GWKcc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]hjcbU``7ighcX]Ubgžh\Y'6YXY[U'`fPYbg]b[hcb'WJadigŁž'; UX][U'`fP]hmUbX'5fh\/'8Yg][b'7UadigYgŁ'UbX'h\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cf\W'5\WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`c\WJhYX''$

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Yg 5ggYggYX
@hYfUhi fY`fYj]Yk`	' \$ı	&*#\$*#&\$&\$`\$*.\$\$`DA	%ž&
A YX]U'Dfc'YWfD`Ub') \$1	' \$#\$+#&\$&\$`%&.\$\$`DA	&Ž
A YX]U'Dfc'YWhD'Ub DfYgYbhUh]cb'	&\$ı	Hi hcf]U`g`K YY_`%\$	' ž(

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% @]hYfUhi fY fYj]Yk

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'%) \$\$'k cfXg

8 YHJ]`g.

%) $\$ k cfXg"'H\]g']g'h\Y'Z]fghUggYgga Ybh'hUg_"': YYXVUW_k]``VY'dfcj]XYX']b'k f]hhYb'Zcfa ž'h\fci [\'U ei U']hUh]j Y'fi Vf]WUbX'bi a Yf]WU`[fUXY"

5XX]h]cbU`XYhU]`g.

 $D^*YUgY^*i \ gY^* < Ufj \ UfX^*F \ YZYfYbW]b[\ ^*GfmY^*Zcf^*U^*^*UggYgga \ Ybfg''^5 \ ^*ei \]W_*[i \]XY^*hc^*h\ Y^*i \ gY^*cZf\]g^*gfmY^*WUb^*VY \ Zci \ bX^*\ YfY.$

\hdg.#kkk"X]h']Y#aYX]U#]VfUfm#XcWaYbhg#8+Hi&\$@]VfUfm &\$<UfjUfXi&\$Ei]\w_i&\$;i]XY"dXZ

5 'gi a a UfmicZh\Y'UggYgga YbhUbX'i gYZ\`fYgci fW'g'Zcf'h\]g'Wci fgY'WUb'U`gc'VY'Zci bX`]b'h\Y'5ggYgga YbhXcW a Ybhicb'A ccX`Y"

Hi fb]h]b gYh]b[. `H\]g Ugg][ba Ybh]g gi Va]hhYX h\fci [\`Hi fb]h]b UbX gh XYbhg Xc bchgYY Hi fb]h]b q]a]`Uf]hmfYdcfhg"

5ggYgga Ybh'& 'A YX]U'Dfc YWh'D`Ub

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. &\$\$\$ k cfXg

8YHJ]`g.

K cf_]b[']b'[fci dg'cZ(!) 'YUW\ ']bX]j]Xi U'`ghi XYbhik]``Wcbhf]Vi hY'U'&\$\$\$!k cfX`gYWf]cb'hc'U', ž\$\$\$!%ž\$\$\$ k cfX`fXYdYbX]b['cb'[fci d'g]nYŁ'A YX]U'DfcYWhD`Ub"'Mci 'k]``VY'a Uf_YX'cb'mci f']bX]j]Xi U``&ž\$\$\$'k cfX Wcbhf]Vi hJcb'': YYXVUW_'k]``'VY'dfcj]XYX']b'k f]hHYb'Zcfa ž'h\fci [\ 'U'ei U`]hUhj Y'fi Vf]WUbX'bi a Yf]WU`'[fUXY''

5XX]h]cbU`'XYhU]`g.

 $D^YUgY'i gY'< Ufj UfX'F YZYfYbW]b['GhmY'Zcf'U```UggYgga Ybhg''`5'ei]W_[i]XY'hc'h\Y'i gY'cZh\]g'ghmY'WUb'VY Zci bX'\YfY.$

\hdg.#kkkk"X]h']\#a YX]U#]VfUfm#XcWa Ybhg#8 +Hi &\$@]VfUfm &\$<Ufj UfXi &\$Ei]\W_i &\$; i]XY"dXZ

5 'gi a a UfmcZh\Y'UggYgga Ybh'UbX'i gYZ\`fYgci fWYg'Zcf'h\]g'Wci fgY'WUb'U'gc'VY'Zci bX']b'h\Y'5ggYgga Ybh XcW a Ybh'cb'A ccX'Y"

Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh']g'gi Va]hhYX'h\fci [\'Hi fb]hjb'UbX'gh XYbhg'Xc'bchgYY'Hi fb]hjb q]a]`Uf]hmfYdcfhg"

5 ggYgga Ybh' . 'A YX]U'Dfc 'YWh'D 'Ub 'DfYgYbhUh]cb

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\., !\% a]bi hY [fci d'dfYgYbhUh]cb

8YHJ]`g.

K cf_]b[']b'[fci dg'cZ(!) ž'gh XYbhg'[]j Y'Ub', 'hc'%\$'a]bi hY'dfYgYbhUh]cb'cb'h\Y]f'A YX]U'Dfc'YWhd`Ub"'H\Y a Uf_'k]``'VY'U'[fci d'a Uf_": YYXVUW_'k]``'VY'dfcj]XYX']b'k f]hhYb'Zcfa 'UbX'bi a Yf]WJ`'[fUXY"'H\]g']g'h\Y ZbU`'UggYgga Ybh'hUg_'Zcf'UhhYbXUbW'di fdcgYg"

Gi Va]gg]cb'bchYg.5``]b_'hc'h\Y'j]XYc'dfYgYbhUh]cb'a i ghVY'gi Va]hhYX'cb'A ccX`Y'VmHi YgXUm(h\5i [i gh-'\DA

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5 https://bxubwww.Fyei]fya Ybhg

 $\pm 0.\% h hcf]U^*g^*JbWi X]b[^*gY^*Za UbU[^*YX^k YY_m'[^*fci d^k cf_'UbX^*[^*fci d^W]b]Wg^*zmci ^k]^*^UWf] Y^mYb[^U[^*Yk]h WcfY^Wci ^fgY^WcbfYbMh Uhk]^*^Yd^mci ^hc^UffU]b^7 @C^*&UbX^* "$

7ci fgY GWX YXi `Y

J]Yk 'WUgg'ha YhUV'Y

H]a YHJV`Y

8 UHY	HmdY	7 cbh/bh
K YY_"\$."&) "A Um"! "&-		D@95G9'G99':1@@'G7<981@9'C:'H<9
A Um		7 C I F G 9 'C B 'A C C 8 @ 9

FYgci fWfg

DfYgWf]VYX F Ygci fWYg

D`YUgY`j]Yk 'h\Y`5ggYgga YbhXcW a Ybhcb`A ccX`Y"

FYWca a YbXYX FYgci fWYg

7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y`cda Ybh

±bZcfa U``ZYYXVUW_`cb'h\Y`WcifgY`k]```VY`gci[\h'h\fci[\`Ub'cb`]bY`eiYgh]cbbU]fY`Xif]b['h\Y`WcifgY`cZh\Yh'fa"

: cf⁻Z|bU⁻·Wc⁺i fgY⁻Yj U⁻i Uh|cbž⁻ghi XYbhg⁻k]⁻-⁻fYWV]j Y⁻U⁻]b_hc⁻hkY⁻A m9I dYf]YbWV⁻gi fj Ymhck UfXg⁻hkY⁻YbX⁻cZ hkY⁻hYfa "

Gi Va]gg]cb cZ5ggYgga YbhHUg_g Hi fb]h]b Gi Va]gg]cb

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$



5 WUXYa]W = b Zcfa Uh cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg 'Zcf'Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYi]Yk 'cZa Uf_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc`]\\\Yg/
- YI Ua]bUh]cb dfcWYXi fYg/
- qdYWJU``Wcbq]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybh fY/
- gh XYbhYei]miUbX'X]qUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbz'gYY

\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#

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5 i ghb '4 Ui ghbh\YdUd] `cb" D\chc[fUd\YX cb 'GYd '&\$% "

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7F = 7CG Dfc|]XYf 7cXY. \$\$\$-,;

5W_bck `YX[Ya YbhcZ7ci bhfm

 $K_{Y'UW\underline{b}ck} YX[Y'hY'6YXY[U''dYcd'Y'k \c'UfY'hY'ffUX]h]cbU''W ghcX]Ubg'cZhY''UbXg'cb'k \]WX'I_BGK_{Ybg]b[hcb'WJa_di_q']q''cWJhYX'''$