

MUSC2603

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HYfa "Hk c" ## "&\$&\$

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H\]g`WcfY`Wci fgY`YI hYbXg`h\Y`ghi XYbhg`_bck `YX[Y`cZh\Y`a UhYf]U`gž`ghfi Wi fYgž`UbX`dfcW*ggYg`cZhcbU` a i g]Whc `Ub`UXj UbW*X``Yj Y``j]U`h\Y`ghi XmUbX`Udd`]WUhjcb`cZ\Ufa cb]WUbX`a Y`cX]WdfUWfJW*g`]b`Vch\UXj UbW*X`Wci bhYfdc]bhž`ZcW`g]b[`dUfhmcb`h\Y`ghi XmcZh\Y`a i g]WcZ>"G"`6UW\ž`Ug`k Y```Ug`YUf`m&\$h\Wbhi fmK YghYfb`Ufha i g]WWca dcg]hjcbž`UbX`]bWi X]b[`Ub`YI Ua]bUhjcb`cZh\Y`]a d`]WUhjcbg`Zcf`gi W\a i g]WU``dfUWfJW*g`cb`ch\Yf`ghm`Yg`gi W\`Ug`dcdi `Uf`a i g]WUbX`\Unn"`Ghi XYbhg`k]```XYj Y`cd`hY\b]WU`_bck `YX[Y`UbX`g_]```]b`h\Y`UbU`mg]g`UbX`Udd`]WUhjcb`cZUddfcdf]UhY`a Y`cX]WUbX`\Ufa cb]WY`UVcfUhjcbž Wci bhYfdc]bhUbX`WcblfUdi bhU``hY\b]ei Yžj c]WY!`YUX]b[ž`UbX`a i g]WU``Zcfa "`9I YfW]gYg`]b`UXj UbW*X Wca dcg]hjcbU``WfUZhdfcj]XY`h\Y`ghi XYbhk]h\`h\Y`cddcfh b]hmhc`XYa cbghfUhY`h\Y]f`]b]hJhJhj Y`UbX`\WfYUhjj]hm

5ggYgga Ybh

5```UggYgga Ybh']hYa g`UfY`hc`VY`gi Va]hhYX'j]U`A ccX`Y"

 $\pm i]g'l BGK 'Dc'] Whith Uh'U'''ghi XYbhg'a i gh'UhhYa dh'5@'UggYgga Ybhg'] hYa g'k]h\]b'h\]g'Wc'i fgY''' \\$

 $\label{thm:condition} \text{Ghi XYbhg'a i gh'UWX]Yj Y'U'fDUggfia Uf_'Zcf'h\Y'< Ufa cbm'Wca dcbYbh'hc'VY'Uk UfXYX'U'fDUggfia Uf_'Zcf'h\Y'Wci fgY'cj YfU'`''' }$

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A i g]W@JhYfUh fY	%) ၊	Bch5dd`]WUV`Y	(<u>ž</u>)
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8 YW] g.

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K f]hhyb ZYYXVUW_dfcj]XYX Vm`YWn fYf"

5XX]h]cbU`XYhU]`g.

 $H\Y'<Ufa\ cbmDcfhZc`]c`Wca\ dcbYbh]g`VUgYX`cb'h\fYY`UggYgga\ Ybh]hYa\ g'h\UhUfY`UggcW]UhYX'k\]h\'h\Y WcbhYbhcZVch\'h\Y'<Ufa\ cbm@YWi\fY'UbX'h\Y'<Ufa\ cbmGh'X]c.$

%" < Ufa cbmHYghf%) ı $\$ '\Y`X`Uhh\Y` < Ufa cbmGh X]c`h]a Y`Uh'%\$" \$Ua !\%\"\$\$da `cb`Hi YgXUm &' `>i bY`fK YY__(\Łžh\]g`hYgh]g`k cfh\`%) ı `cZh\Y`hch\`a Uf_\Zcf`h\Y`\Wci fgY"

&"`A]bcf`5gg][baYbhf%)ı Ł`hc`VY`giVa]hYX`j]U`AccX`Y`Vm("\$\$da`: f]XUm'%+`>i`mfKYY_`+Łž'h\]gUggYggaYbh]g`kcfh\`%)ı `cZh\Y`hchU``aUf_`Zcf`h\Y`WcifgY"

 $\label{eq:continuity} D`YUgY`BchY. `gh`XYbhg`bYYX`hc`UhtU]b`U`ffDUggfia~Uf_`Zcf`h\Y`< Ufa~cbmWca~dcbYbh'hc`VY`Uk~UfXYX`Ub~cj~YfU```ffDUggfia~Uf_`Zcf`h\Y`Wci~fgY''`$

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh' . 5i fU`K cf_g\cd fHi hcf]U`Ł

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`q.

 $\label{eq:definition} DcfhZc`]c`cZ5i~fU``9l~YfW]gYg.`cbY`UbU`mh]WU``Yl~YfW]gYžcbY`Ui~fU``hYghZcbY`g][~h!g]b[~]b[~hYghZcbY`Ui~fU``hYghZcbY`g][~h!g]b[~]b[~hYghZcbY`Ui~fU``hYghZcbY`g][~h!g]b[~]b[~hYghZcbY`Ui~fU``hYghZcbY`g][~h!g]b[~]b[~hYghZcbY`Ui~fU``hYghZcbY`g][~h!g]b[~]b[~hYghZcbY`Ui~fU``hYghZcbY`g][~h!g]b[~]b[~hYghZcbY`Ui~fU``hYghZcbY`g][~h!g]b[~]b[~hYghZcbY`Ui~fU``hYghZcbY`Ui~fU``hYghZcbY`g][~h!g]b[~]b[~hYghZcbY`Ui~fU``hYghZcbY`Ui~fU``hYghZcbY`Ui~fU``hYghZcbY`Ui~fU``hYghZcbY`g][~h!g]b[~]b[~hYghZcbY`Ui~fU``hYghZcbY`Ui$

K f]hhyb Zyyxvuw_dfcj]xyx vmh hcf"

5XX]h]cbU`XYhU]`g.

 $\label{eq:higher_problem} \mbox{Hi fb]h]b "gYhh]b[. "H\]g"]g"bch"U"Hi fb]h]b"Ugg][\mbox{ba Ybh}$

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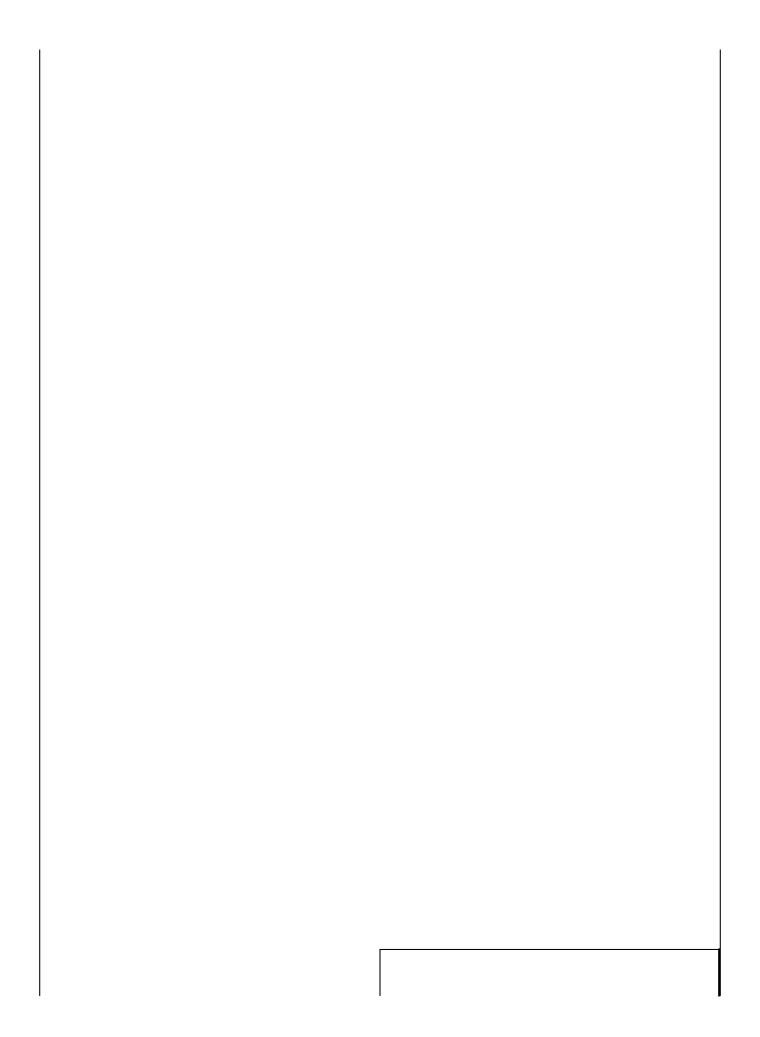
Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj]Yk '`YWhi fY`fYWcfX]b[g"

7ci fgY'GWXYXi `Y

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K YY_`%`%`>i bY`!`) `>i bY	Hi hcf]U`	5i fU``Hi lcf]U`.`A cbXUm\$%'>i bYž%\$"' \$Ua '! %&"\$\$da žcb!`]bY"	
		Hi hcf."; Ycf[]U'@]_Ybg	
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		@YWhifYf.`>c\b'DYhYfgcb
K YY_'%\$.'' '5i [i gh!'+ 5i [i gh	Hi hcf]U`	5ifU`Hihcf]U`.`A cbXUm'\$' `5i [ighž'%\$"'\$Ua '! %&"\$\$da ž'cb!`]bY"
		Hi hcf."; Yc[]U'@i]_Ybg
		G][\hG]b[]b['HYghz'UbX'Z]bU``Wta a Ybhg'cb'A i g]W @]hYfUhi fY'fYdYfhc]fY"
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Ghi XYbhZYXVUW_']g'cb'h\]g'Wci fgY']g'[Uh\YfYX'dYf]cX]WU``mUbX']g'U`k Umg'WUfYZ ``mWcbg]XYfYX'k]h\'U j]Yk 'hc'UWf]b['cb']hWcbghfi Wf]j Y`mk \YfYj Yf'dcgg]V`Y"': YYXVUW_']g'[Uh\YfYX'Vmj Uf]ci g'a YUbg']bWi X]b[h\Y'cWWg]cbU``]bZcfa U``cb!`]bY'ei Ygh]cbbU]fY'UbX'j]U'h\Y'a cfY'Zcfa U``a m9I dYf]YbW'DfcWfgg"

#b^`][\hcZdfYj]cig^ZYYXVUW_Zfca `ghi XYbhg^]b^h\]g^WcifgY`a cfY`ZcW`g`\Ug^VYYb^[]j Yb^hc^i bXYfghUbX]b[\Ufa cbmUbX`\Ufa cb]WWcbWYdhg^Ug^h\YmfY^UhY`hc^Unn^UbX`ch\Yf^a i g]WghmYg^"G]a]`Uf^mžU^VfcUXYf^fUb[YcZa i g]WU^ghmYg^]g^fYZYfYbWYX`UbX`YI Ua]bYXžYgdYWJU^mk]h\]b^h\Y^Ui fU^UbX^A i g]W@]hYfUh fY Wca dcbYbhgž]b^cfXYf^hc^Z fh\Yf^Yb\UbW^UbX`Wcbgc^]XUhY`h\Y^ghi XYbhg^i bXYfghUbX]b[~cZh\cgY^WcbWYdhg"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

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HY'Yd\cbY'9| hYfbU''Gi ddcfhcb'-',) "''' %cf'Ya U]''h\Ya 'cb'Y! hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui ""Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'm\mathred{Image: Wa Jhmci f'Ub'Y! hYbg]cb\mathred{Image: Wa Jhmci h\Yf'f'Y'y JUbhXcW a Ybhg\mathred{Image: Wa Jhmci f'Hy\Mybg'\mathred{Image: Whybg\mathred{Image: Wa U]''9! hYfbU''Gi ddcfhmci 'k]'`
Ui hca Uh\W\mathred{Image: Wa Jhmci h\Yf\mathred{Image: Wa Jhmci 'k]\mathred{Image: Wa U]''9! hYfbU''Gi ddcfhmci 'k]'`
Ui hca Uh\W\mathred{Image: Wa Jhmci h\Yf\mathred{Image: Wa Jh\Mathred{Image: Wa U]''9! hYfbU''Gi ddcfhmci 'k]'`
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5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

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- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

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