

MUSC2703

DYfZcfa UbWY @UVcfUhcfm6

HYfa "Hk c" ## "&\$&\$

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUhjcb	D\cbY
@Ui_fU^7\]g`Yhh	n' (- , &&- 4 i bgk "YXi "Ui	VmUddc]bha Ybh		
>cbYg				

GWkcc~7cbhJWh~abZcfa Uh]cb

GVXcc``cZhY`5fhg`UbX`AYX]U

Fcca ' %&ž@YjY`' žFcVYfhK YVghYf6i]`X]b[fl %(Ł

D\cbY. f\$&£'-',) '(,) *

9a U]` gUa 4 i bgk "YXi "Ui

K YVq]hY. k k k "Ufhq"i bak "YXi "Ui #qUa

 $\label{eq:hamiltonian} $$HY'GWKcc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck`YX[Y'h\Y'HfUX]hjcbU``7ighcX]Ubgžh\Y'6YXY[U'`fPYbg]b[hcb'WJadigŁž'; UX][U'`fP]hmUbX'5fh\/'8Yg][b'7UadigYgŁ'UbX'h\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cf\W'5\WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`c\WJhYX''$

7 ci fgY 8 YHJ]`g 7 fYX]hDc]bhg (' Gi a a UfmcZh\Y 7 ci fgY

5ggYgga Ybh

 $: i \ fh\ Yf']bZcfa \ Uf]cb'UVci \ h'fh\ Y'UggYgga \ Ybh'fUg_g'k \]```VY'[\]j \ Yb']b'WUgg'cf'cb`]bY''$

5 d[f[cgWe k]``VY`gi Va]lthYX`UWfcgg`H%Z`H&`UbX`H' "``H\YgY`k]``VY`fY`UthYX`tc`a UgthYfWUggYg`dfYgYbthYX`Vm j]g]lt]b[`YI dYftg`]b`U`fUb[Y`cZa i g]\WJ``gtm`Yg`UbX`UWfj]lt]Yg"``7 f]tf]ei Y`ei Ygtf]cbg`k]```VY`g\WJZc`XYX`tc

5 'fja YHJV`Y`Zcf`h\Y`dYfZcfa UbWY`dfYgYbHJfjcbg`k]``VY`YgHJV`]g\YX`VmkYY_`&`UbX`i d`cUXYX`hc`A ccX`Y"

5 'gWcfY'cZh\Y'k cf_'hc'VY'dYfZcfa YX'a i ghVY'Ya U]'YX'hc'a Y'VYZcfY'h\Y'XUhY'cZmci f'DfYgYbhUh]cb"

 $\label{thm:condition} $$HY'gi a a Uf]g]b[`XcWaYbhfk`]WX'U`gc`aighVY'YaU]`YX'hc`aYŁ'g\ci`X'VY'bc``cb[Yf'h\Ub'Ub'5(`dU[YžUbX']haUm'VY']b`XcHdc]bhZcfa"$

H\Y'dYfZcfa UbWY']g'h\Y'a U]b'dUfhcZmci 'dfYgYbHJh]cb"

•

•

.

Gi Va]gg]cb bchYg.]b WUgg dfYgYblUh]cbžcf cb`]bY

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5ggYgga Ybh (. DfUWh]WU Yl Ua]bUh]cb

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. &) a]bi hYg

8 YW] g.

: cf'dYfZcfa UbWY'ghi XYbhgž'WJ S_ [`Sf[a`e

5 https://bxubwww.Fyei]fYaYbhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj]Yk '`YWhi fY`fYWcfX]b[g"

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'h]a YhUV'Y

H]a YtUV`Y

8 UhY	HmdY	7 cbhYbh		
K YY_'\$.'&) 'A Um!'&- A Um	Gh X]c	#blfcXi Wijcb." 8 cbffiizcf[Yhhc Ybfc``]b'Ub'9bgYa VY'Xi f]b['C! K YY_ A I G7 &+\$' ']g'U'g_]``g!VUgYX'Wci fgY"' 5 fYUg'zcf'X]gW gg]cb'h\]g'hYfa 'UfY. 'Glfi Wi fY']b a i g]W'dYfgcbU``k Y``!VY]b['zcf'a i g]WjUbgz'UbX'ch\Yf UgdYWg'fY`Uljb['hc']bzcfa YX'a i g]WjUbg\]d"		
K YY_'%'%'>i bY'!') '>i bY	GH X]c	H\i fgXUmghi X]c WUggYg UWWcfX]b['hc mci f'ck b ha YhUv'Y fl. \$\$!%\$.' \$zcf'%(.\$\$!%).' \$t"H& WUggYg k]``VY'cb`]bY'i bh]` Z fh\Yf'bch]WY" 5``dYfZcfa Yfg'UfY'hc dYfZcfa 'U'ghi Xm]b'h\Y'cb`]bY ghi X]c 'WUgg']b'k YY_'%" 7 ca dcgYfg.UZhYf`]ghYb]b['hc h\Y'Mci Hi VY'hfUW_V'ck 'cZ"DUgg]b['6]fX"UbX'"F cW_]b['A]ffcf" Zcf lk c'j]c`]bg'VmHcfi 'HU_Ya]ghi z'dfYdUfY'U'Vf]YZ dYfgcbU' Wca a YbhUfmUVci hmci f'fYUW]cbg'hc'h\Y d]YWg'ffY"["'Xc'mci ']_Y'h\Ya z'k \Uh]g`\UddYb]b[ghfi Wi fU'm]b'h\Y'a i g]W'Wb'mci 'fYWc[b]gY'Ubm a i g]WU`dfcWyggYg'cf']a U[Yfmzk \UhXcYg'h\Y'a i g]W a U_Y'mci 'h\]b_'cZz'k \Uh]g'h\Y'X]ZZYfYbW'VYhk YYb h\Y'lk c'd]YWgt" \hhdg.#mci hi "VY#7_< 5\[hG*E 5 J]c`]b]ghg. dfYdUfY'U'Vf]YZdYfgcbU''Wca a YbhUfmcb h\Y'lk c'dYfZcfa UbW'g *D'UXX]hjcbz'=k ci 'X`]_Y'Yj YfmcbY'hc'k f]hY'Xck b h\Y]f'_ [Wefa` WbWxad_S` UWbcSUf[UWUa_bae[f[a` S^YaS'e Zcf'h\Y'bYI hihYfa z'UbX'h\Yb'hc'Wca a Ybh']b WUgg"]b'k YY_'%cb'\ck 'mci 'k]``UW\]yj Y'h\cgY [cU'g''K Y'k]``fYj]g]hih\]g'dfcW'gg'cj Yf'h\Y'Wci fgY'cZh\Y'hYfa"		

GU``mK U` Yf'#A cXYfb'/ '<]ghcf]WU` : `Ui hjgh
H\Y'7 f]hjei Y'ei Yghjcb'k]``'VY'dcghYX'hc'A ccX'Y VYZcfY'h\Y'`YWhi fY"'7 f]hjei Yg'UfY'Xi Y']b'k YY_'%\$'cb : f]XUm'

FYgci fW/g

DfYgWf]VYX F Ygci fWYg

K]``]Ua cbž5'&\$\$('f9X"Ł? ge[US^7j UW'W UW, efdSfWY[We S` V fWUZ` [cgWe fa W ZS` UWbWdYad_ S` UWC | ZcfX | b]j Yfg]miDfYggžC | ZcfX

FYWca a YbXYX FYgci fWYg

 $6 \ UWX \ Z'7 \ "'D"'9"'fW+)' \ E'7 \ eeSk \ a` fZWF \ dyW3 \ d'aXB'Sk[`Y = WkTaSdV;`efdy_ W' fe'HfUbg''K]``]Ua'A] \ HWXY``'' BYk' \ Mcf_. 'K "Bcfhcb' UbX'7 c"'% (-$

6Yfa Ubž6":f&\$\$\$L":@afWe Xta_ fZWb[S` [efe TW UZ":BYk '< Uj Yb.:MUY:I b]j Yfg]lmiDfYgg"

6YfbUWZD"f% +* L"FZW[`fWbdWfSf[a` aX8dW UZ ea`Y"'@cbXcb.'; c``UbWn"

6i h½'>"ž/ '8 fYm½ gž'@''f9Xg"Ł"'f&\$\$%Ł" B 'Sk[`Yi [fZ: [efack, FZWZ[efac[US^SbbcaSUZ fa? ge[US^BWbAc_S`UW'7 Ua Vf]X[Y.'7 Ua Vf]X[Y'I b]j Yfg]hmDfYgg"

7 cY\`cžJ "'fPX"Ł"'f&\$\$' Ł"'5 S_ TqVYWUa_ bS` [a` fa fZWYg[fSd"7 Ua Vf]X[Y. 7 Ua Vf]X[Y'I b]j Yfg]lmiDfYgg"

7 c ZZ]b ž 6 " 'f 16 * \$! & 2 " F Z We [Y Wde dWb Wdfa [dW

 $< i \ Xgcb\check{z}F "`f\% - + \&"`E fa `W f[_W fZWZ[efa ck a XfW_ ba \ dgTSfa"`7 \ Ua \ Vf]X[Y.`7 \ Ua \ Vf]X[Y$

 $\label{eq:hufig_bar} HUfig_]b\check{z}F]W\&ufX\check{z}f\%-)\&"FWJfS`VSUf, 7eeSke~a`~ge[US`V~bWb\&d_S`UW\&CIZcfX.'CIZcfX'I~b]j~Yfg]lm~DfYgg"$

K YghbYmžK "'f&\$\$' Ł"'FZWbWdWUfi da`Y`afW'BYk 'Mcf_.'5a UXYi g'DfYgg"

\hdg.#k k k "\]VfUfm'i bgk "YXi "Ui #gh Xm

ł: cf'g]b[Yfg.

\hd.#kkk"UhUbhUjc]WYggcbg"Wca#cWUhYWXb]eiY"\ha

\hd.#kkk"UighfU`]Ubaig]WWYbhfY"Wca"Ui#

\hd.#kkk&"g]\VU"Z|\Uf^c]\hY`i #bXYI "d\d3]X1&/`U1Yb

\hdg.##gcWf/maig]\wh\Ycfm'cf[#gcWf/hf/g#bh/f/gh#dYfZcfaUb\wfUbU`mg]g#/]\v`]c[fUd\m

\hdg.#kkk"`YUfb\UnnghUbXUfXg"Wca#

: cf'k f]h]b['hYWXb]ei Y. '\UbXVcc_g'Zcf'k f]hYfg'Uj U] 'UV'Y'Uh'h\Y'I BGK '@]VfUfm

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Mci 'k]```VY`]bj]hYX`hc`Wca d`YhY`h\Y`A m9I dYf]YbWY`gi fj YmUhih\Y`WcbWi g]cb`cZh\Y`hYfa "`H\Y`ZYYXVUW_Zfca 'h\Y`gi fj Ymk]```VY`hU_Yb`]bhc`Wcbg]XYfUh]cb`Xi f]b['Zcfk UfX`d`Ubb]b['Zcf`bYI hihYfa "

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) "''' %cf'Ya U] hya 'cb'Yl hy'fbu'hy'ngi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Yl hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9| hy'fbu''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Yl hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9| hy'fbu''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hy'Yd\cby'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cby''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY