

ARTS2092

; `cVU``A YX]U. A Uf_Yhgž: `ck g'UbX'7i `hi fYg

HYfa 'Hk c''#'&\$&%

7ci fgY Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
GWcHiG\UbYf	0 0	C b`]bY'j]U'Ya U]` UbX'A ccX`Y	gYY ⁻ A ccX ⁻ Y	B#5

Hi hcfg

BUa Y	9a U]`	5j U]`UV]`] m	@cWJhjcb	D\cbY
5mYg\U' <ugub< td=""><td>UmYg\U"\UgUb4 i bgk "YXi "Ui</td><td>cb`]bY'j]U'Ya U]`'UbX</td><td>]b 'WUgg</td><td>B<i>#</i>5</td></ugub<>	UmYg\U"\UgUb4 i bgk "YXi "Ui	cb`]bY'j]U'Ya U]`'UbX]b 'WUgg	B <i>#</i> 5
		A ccX`Y		

GWXcc`7cbHJWhi±bZcfa Uhjcb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@Yj Y`" žFcVYfhK YVghYf'6i]`X]b[ff %(Ł

D\cbY. 'f\$&Ł'-',) '(,) *

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhq"i bqk "YXi "Ui #qUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5\Lock\`YX[Y'h\Y'HfUX]tjcbU``7i\gtcX]Ubg\cong h\Y'6YXY[U'`fP\Ybg]b[tcb'WJa\di\gt\cong \cong m5\Lock\`YX][U'`fP\Ybg]b[tcb'WJa\di\gt\cong \cong m5\Lock\`Y'B[i\bbUk\U'\dYcd'Y'fbi\gth\'Y'B[i\bbUk\U'\dYcd'Y'fbi\gth\'Y'\UW\\`WJa\di\g'\cong \cong \cong m5\Lock\Y'\UbXg'k\YfY'YUW\\`WJa\di\g'\cong \cong \cong\cong \cong \cong \cong \cong \cong \cong \cong \cong \cong \cong$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

fi `cVU``A YX]U. `A Uf_Yhgž: `ck g`UbX.7 i `hi fYgfiZcW gYg`cb`h\Y`fY`Uh]cbg\]d`VYhk YYb`a YX]U`UbX [`cVU`]gUh]cb"`K Y`k]```YI Ua]bY`h\Y`\]ghcfmcZ]bhYfbUh]cbU``lfUXY`Zck g`]b`Ui X]cj]gi U``[ccXg`UbX`gYfj]WYg UbX`YI d`cfY`\ck 'h\YgY`\Uj Y`V\Ub[YX`h\fci [\`[`cVU``lfYbXg`hck UfX`fY[]cbU`]gUh]cbž`Wci bhYf`Zck `UbX`h\Y]b\WYUg]b[`lfUbgbUh]cbU``dfcXi \W]cb`UbX`X]glf]Vi hJcb`cZa YX]U"`B Yk `mYa Yf[]b[`Zcfa g`cZ\W`hi fY`UbX]XYbh]lmvVugYX`cb`h\Y`YI dYf]Yb\W`cZ\cdot ga cdc`]h\Db]ga ž`\m\f]X]lmiUbX`X]UgdcfU`k]```VY`YI Ua]bYX`]b`XYhU]``]b fY`Uh]cb`hc`a YX]U`Zcfa g`gi \W`Ug`hY`Yj]g]cb`Zcfa Uhg`UbX`XfUa U`dfc[fUa gžbYhk cf_YX`a YX]UžZ]`a `UbX a i g]\WGh XYbhg`\cdot a d`Yh]b[`h\]g`\cdot fgY`k]```fY\W]j Y`U`h\cfci [\`[fci bX]b[`]b`h\Y`k Uma YX]U`\cdot bhf]Vi hY`hc h\Y`]bhYbg]Z]\WJh]cb`cZci f`YI dYf]Yb\W`cZh\Y`k cf`X`Ug`U`k \c`Yžk \]\W\k]``dUj Y`h\Y`k UmZcf`Z fh\Yfgh Xm]b]bhYfbUh]cbU``UbX`hfUbgbUh]cbU``a YX]U"

7 ci fgY @YUfb]b['Ci hWca Yg

- %" ⇒XYbhjZmUbX`Udd`m`_YmWcbW'dhg`UbX`h\Ycf]Yg`k \]W\`YI d`U]b`h\Y`fY`Uhjcbg\]d`VYhk YYb`a YX]U`UbX [`cVU`]qUhjcb
- &" 5 WWci bhiZcf'h\Y'fc`Y'[`cVU`a YX]U'd`Umg`]b'ZcghYf]b['bYk'dUhYfbg'cZWi`hi fY'UbX`]XYbh]Z]WUh]cbž k\]W\'UfY'bc'`cb[Yf'gc`Y`mVUgYX'cb'[Yc[fUd\mž'UbX']b'Xc]b['gc'UddfYW]UhY'X]j Yfg]miUa cb['UbX k]h\]b'W'hi fYq"
- "8]gWiggih\Y`VU`UbWiYVYlk YYb'h\Y'f[i`cVU`fiUbX'h\Y'ficWU`fi]b`X]ZZYfYbh'Zcfa g'cZa YX]U`]b`fY`Uh]cb'hc h\Y`WcbhYl h'cZh\Y]f`dfcXi Wh]cb'UbX'X]ghf]Vi h]cb"
- ("⇒XYbhjZm_YmhfYbXgʻUbXʻdUhhYfbgʻ]bʻh\YʻZck cZUi X]c!j]gi U``[ccXgʻUbXʻgYfj]WYgʻUbX`Yj U`i UhYʻh\Y]a dUWicZUhhYa dhgʻhcʻfY[i `UhY`h\Ya

HYUWX]b[GlfUhY[]Yg

FUh]cbU`Y.

5 X X] h] c b U` ` X Y h U] ` g.

: cf'U'Z ```XYgWf]dh]cb'cZh\]g'Ugg][ba Ybhd`YUgY'Wcbgi `hh\Y'5F HG&\$- &'7 ci fgY'A ccX`Y"'H\]g']g'U'[fci d Ugg][ba Ybh'

5 hm/b XUb VW F Yei]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] } \\$

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YtUV`Y

8 UHY	HmdY	7cbh/bh
K YY_"%" %"A Um"!"(">i bY	@YWhifY	I_bXYfgHJbX]b[; `cVU`]nUh]cb UbX A YX]U
	Hi hcf]U`	±blfc'hc'h\Y'WUgg/; `cVU`]gUh]cb'UbX'h\Y'A YX]U
		Hi hcf]U`g`UfY`U`W\UbWY`Zcf`mci hc`[U]b`gca Y`fYU` h]a Y`UbX`gdUWY`Zcf`mci hc`X]gW ggž[c`cj Yf`UbX`Ug_
		ei Ygh]cbgʻUVci hh\Yʻa UhYf]U`mci ʻUfYʻVY]b[UggYggYXʻcb"ʻB Uhi fU``mž]hg\ci `Xʻ[cʻk]h\ci hgUm]b[ž h\Ymk]``g][b]Z WUbhmVccghmci fʻVXUbWYgʻcZ
		gi WW ggʻ]bʻAYʻWci fgY" B chʻUhhybX]b[``UʻWUggʻ]gʻbch UbʻYI W gYʻZcfʻZU]`]b['hc'_bck 'k \UhhcʻXcʻcbʻh\Y UggYgga Ybhg"
	F YUX]b[7ci fgY F YUX]b[
		 : `Yk ž'HYffmf&\$% Ł'"; `cVU`]nUh]cb'H\Ycf]Yg"' +D'A ccX`Y .
K YY_`&.`+`>i bY`!`%% >i bY	@YWhi fY	Dc`]h]WU`YWcbca]WUddfcUW.Yg'hc'; `cVU`A YX]U UbX'h\Ycf]Yg'cZa YX]U'XYdYbXYbWh
	Hi hcf]U`	HcXUmk Yfi`X]gW gg;h\Y'dc`]hJWU`YWbba mcZf[i`cVU` a YX]UfiUbX``cc_'UhWUgY ghi X]Yg`k \JW\`fU]gY`[`cVU` fY[i`Uhcfmei Ygh]cbg`]b`fY`UhJcb'hc`h\Y'a YX]U
	F YUX]b[7ci fgY'F YUX]b[
		 5fm<u>ž</u> @"7\UdhYf". HfUbgbUh]cbU`_A YX]U fca '5fmž@"'9 'aTS^7` fWdfS[_ W f? WW[S, 3 5 d[f[US^;` fdaVgUf[a` K YghGi ggYl ž'l ? ž K]'Ym'6`UW_k Y``ž'&\$%) ž'd[g'+%], ("
K YY_'' .'%) '>i bY'!'% >i bY	@YWnifY	7 i `hi fU` =a dYf]U`]ga `UbX`8Y!K YghYfb]nUh]cb. 8cYg`h\Y`h\YcfmcZ'W``hi fU`]a dYf]U`]ga `gh]`` Udd`mhc`[`cVU` a YX]U`hcXUm3
	Hi hcf]U`	Hi hcf]U`'!'=g'W`hi fU`']a dYf]U`]ga 'gh]``fY`Yj UbhhcXUm3
	F YUX]b[GdUf_gž7""f&\$% Ł"""; `cVU``A YX]U'Gh X]Yg. +fg

		8 Yj Y`cda Ybh'UbX`8]`Ya a Ug"~`? WW[Sł 5 g fgdWS` V E a U[Wfk"'') fWzz'd[g"`%&%%%"
K YY_'(.'&%'>i bY'!'&)	@YWhifY	8]UgdcfU. 'A i `h]`cWU`]hm]b`U`[`cVU``k cf`X
>i bY	Hi lcf]U`	Hi hcf]U``!`A YX]U`UbX`Wca a i b]h]Yg`cZX]UgdcfU .
	F YUX]b[G]bWU]fž=UbX:7i bb]b[\Ua žG":f&\$\$%£"8]UgdcfUg UbX:h\Y:A YX]U"": fca '8'aSf[`Y >[hWe, FZW? WW[S S`V 3e[S` 6 [SebacSe"C ZcfXž ? ž F ck Ub 'UbX @]hhYZ]Y:Xg:Di V:]g\Yfgždd":%!'\$:CB@MfBchh\Y Ybh]fY:W\UdhYf!!Y Wi XY:Zfca 'h\Y:fDi V:]W Gd\Yf]W:YgfigYWf]cb cbk UfXgŁŁ"
K YY_').'&, '>i bY'!'&'>i `m	eYWhifY	7 cga cdc`]hUb]ga . '5 fY 'k Y`]bWfYUg]b[`m'W]h]nYbg cZh\Y'k cf`X3
	Hi hcf]U`	7 cga cdc`]hUb]ga '!'U'Zcfa 'cZW'hi fU`'Y`]h]ga 'cf Y[U`]hUf]Ub'['cVU`'W]j]`'gcW]Yhm8
	F YUX]b[J UbXYj ccffžF "'f&\$% Ł'"A cfU``Wcga cdc`]fUb]ga 'UbX h\Y'Yj YfmXUm`]ZY.`\ck 'gh XYbhg'YbWci bhYf'X]ghUbh ch\Yfg"'? VW[Sł 5 g fgdW` EaU[Wfk'&\$% žJ c`"'(\$f&Ł %)-&%\$
K YY_'*.") ">i `m'! "- ">i `m	±bhYbg]jY	: @9L 'K 99? '! 'H<9F9'5F9'BC'7'@5GG9G 81 F=B; 'K 99? '* 'f) '> @M!'- '> @MŁ
K YY_'+.''&\'>i `m'!''%* '>i `m	n@YWhifY	7 i `hi fU``\mVf]X]hmUbX`a i `h]W\`hi fU`]ga 'Ug [`cVU``W\`hi fY
	Hi kcf]U`	7 i `hi fu``< mVf]X]hm`UbX`A i `h]W\`hi fu`]ga
	F YUX]b[?fU]XmžA "'"7i `hi fU``< mVf]X]hmUbX =bHYfbUh]cbU` 7 ca a i b]WUh]cb": kTqV[fk, FZW5 g/fgd\$^>aY[U aX 9 'aTS/[eSf[a` "'D\]`UXY`d\]Už'HYa d`Y`I b]j Yfg]hmž &\$\$) ž'dd"'% %("
K YY_', .'%-'>i `m'! '&' '>i `r	n@YWhifY	±bhYfbUh]cbU``>ci fbU`]ga
	Hi hcf]U`	
I	I	l

		"5 h\cf]hUf]Ub`8ckb[fUX]b[žfGY`ZzWfbgcfg\]d`UbXBYk`AYX]U`5Wfj]ga`UZhYf`h\Y`5fUV`Gdf]b["`
K YY_'%\$.'&'5i [i gh'!'*	@YWhifY	7 cbWi g]cbg. F YgYUfWX]b['; `cVU`A YX]U
5i[igh	Hi hcf]U`	E#5 'UVci h'h\Y'fYgYUfW\'dUdYf

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

• 5 ```fYUX]b[g'UfY'Uj U]`UV`Y'j]U'@Y[Ubhc'UbX'h\Y'Wci fgY'AccX`Y

FYWca a YbXYX FYgci fWYg

DYf]cX]WU``mž`UXX]h]cbU``fYgci fWYg`k]```VY`a UXY`Uj U]`UV`Y`hc`ghi XYbhg`j]U`h\Y`Wti fgY`A ccX`Y''`Ghi XYbhg`UfY fYgdcbg]V`Y`Zcf`_YYd]b[`i d`hc`XUhY`k]h\`Ubm`]bZcfa Uh]cb`cf`fYgci fWYg`h\Uh'UfY`dfcj]XYX`j]U`h\Y`A ccX`Y`cb U`k YY_`Vmk YY_`VUg]g"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

DYf]cX]WJ``mghi XYbhYj U`i Uhjj Y`ZYYXVUW_`cb`h\Y`Wci fgY`]g`[Uh\YfYXži g]b['Ua cb['ch\Yf'a YUbgži BGK fig A m9I dYf]YbWf'DfcWfgg"'Ghi XYbh'ZYXVUW_`]g`hU_Yb`gYf]ci g`mž'UbX`Wcbh]bi U``]a dfcj Ya Ybhg'UfY`a UXY`hc h\Y`Wci fgY`VUgYX`]b`dUfhcb`gi W\`ZYYXVUW_"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.



5 WUXYa]W + DZcfa Uh]cb

: cf'YggYbhJU`ghi XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va]gg]cbg'[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/I BGK '<YU'h\ 'UbX'GUZYhmdc']W]Yg/