

ARTS2096

A YX]U'F][\hgž'A YX]U'K fcb[g

HYfa 'H\fYY''##'&\$&\$

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWJhjcb	D\cbY
HUb1J:8fY\Yf	ĕ	H\ifgXUmg'%\$'!'% cf'Vm'Uddc]bhaYbh	FcVYfh KYVghYfž@YjY` '`žFcca'' ‰G	

GWkcc~7cbhJWh~abZcfa Uh]cb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca " %&ž@Yj Y`" žFcVYffiK YVgfYf'6i]`X]b['f) %(Ł

D\cbY. f\$&L'-',) '(,) *

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. 'k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck`YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7UadigYgŁ'UbX'thY'B[ibbUkU`dYcd'Y'fbigtfU]Ub'8YZYbW': cfW'5WJXYam]b'7UbVYffUŁ'cZthY'`UbXg'k\YfY'YUW\cong WJadig'cZIBGK']g`cWJtYX''$

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'tUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
7 ca a YbHJfmcb 'U'k YY_`m hcd]W	' \$I	('g\cfh'Wca a YbfUf]Yg'cb'('cZ h\Y'k YY_`mhcd]Wg'Xi f]b[K YY_g'' '! '%\$	%ž&
Dcgh/f ⁻	&\$ı	%'dcghYf'cb'%'cZh\Y'kYY_m hcd]Wg'Xif]b['KYY_g'''!'%\$	%ž&
A U⁄cf [·] 9ggUm) \$1	&' #%/#&\$&\$`\$).\$\$`DA	%ž&

5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% 7ca a YbhUfmcb U'k YY_`mhcd]W

GhJfhiXuhy. '('g\cfhiWca a YbhJf]Yg'cb '('cZh\Y'k YY_`mhcd]Wg'Xi f]b['K YY_g'' '! '%\$

@Yb[h\.(I)\$\$!kcfXWcaaYbhUf]Yg

8YW]`q.

&\$!dYfW\bh

('I ') \$\$!k cfX 'Wca a YbhUf]Yg''

Gh XYbhg k]``dfcXi W g\cfhWa a YbhJf]Yg]b fYgdcbgY hc dcghYfg dfYdUfYX Vmh\Y]f dYYfg"

: YYXVUW_'j]U'h\Y'Wti fgY'@A G"

5XX]h]cbU`XYhJ]`g.

: i ff\\Yf'XYfU]`g'cb'f\\]g'Ugg][ba Ybfik]``'VY'Uj U]`UV'Y']b'A ccX'Y

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h]b "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h "UbX"gh XYbhg"Xc "bchgYY"Hi fb]h "UbX"gh XYbhg"Xc "bchgYYh]h "UbX"gh XYbhg XYbhg"Xc "bchgYYh]h "UbX"Ybhg XYbhg XY$

5ggYgga Ybh& DcghYf

GhUfhiXuhy. %dcghyficb %cZh\Yik YY_mhcd]WgiXi f]b[iK YY_gii !i%

@Yb[h\.'5' DcghYf

8 YHJ] g.

'\$!dYfWbh

 $Ghi \ XYbhg'k] ``dfcXi \ WY'U'dcghYf'cb'cbY'cZh\Y'k YY_mhcd] Wg'fY`Uh]b['hc'a YX]U'Yh\] Wg'zgcWJU'' ^1 ghJWY'UbX \ia Ub'f][\hg''$

: YYXVUW_'j]U'h\Y'Wti fgY'@A G"'

5XX]h]cbU`XYhJ]`g.

: i ft\Yf'XYfU]`g'cb't\]g'Ugg][ba Ybhk]``VY'Uj U]`UV'Y']b'A ccX'Y

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5ggYgga Ybh' . A U'cf 9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\.'&)\$\$

8YHJ]`g.

) \$!dYfWbh

&) \$\$ k cfXg"

Gh XYbhg k] ``dfYdUfY Ub YggUmfY Uh]b[hc a YX]U Yh] Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A gh] WY Ub X `` a Ub f][hg A War a YX]U Yh Mg z gc W]U `` A War a War

: YYXVUW_'j]U'h\Y'Wti fgY'@A G"

H\]g`]g`h\Y`Z]bU``UggYgga Ybh'hUg_"

5XX]h]cbU`XYhU]`g.

: i ff\Yf`XYfU]`g`cb`f\]g`Ugg][ba Ybhk]``VY`Uj U]`UV`Y`]b`A ccX`Y

FYgci fW/g

DfYgWf]VYX F Ygci fWyg

DfYgWf]VYX`fYgci fWfg`k]``VY`a UXY`Uj U]`UV`Y`j]U`A ccX`Y"`DfYgWf]VYX`fYgci fWfg`k]```]bWi XY`gWkc`Uf`m`]hYfUh fYžYl dYfhWca a YbhUf]Ygž'dcXWUghg`UbX`j]XYcg"

FYWca a YbXYX FYgci fWfg

F YWca a YbXYX'fYgci fWg'k]``VY'a UXY'Uj U]`UV`Y'j]U'A ccX`Y"'F YWca a YbXYX'fYgci fWg'k]```]bWi XY gWkc`Uf`m`]hYfUh fYžYl dYfhWca a YbHUf]YgždcXWUghg'UbX'j]XYcg"'

7 ci fgY '9j U'i Uh]cb 'UbX '8 Yj Y'cda Ybh

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hUW_bck iYX[Ya Ybh'

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl hcf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U'gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfc YWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

± Uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]'Y 'a cghmfYhU]b]b['h\Y'cf][]bU' ghfi WhifY'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa UhJcb'k]h\ci hUW_bck 'YX[Ya Ybh''H\]g'U'gc Udd']Yg']b'dfYgYbhUhJcbg'k\YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk'k\c'Yž'k]h\ci h'Uddfcdf]UhY'fYZYfYbW]b['"

7 c``i g]cb.'k cf_jb['k]h\ch\Yfg'Vi hdUgg]b['cZZh\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'gh XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZh\Ya d`U[]Uf]g]b['UhUbmhja Yž'dUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_žghYU`]b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZZYf]b['hc'Wca d'YhY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[dUma YbhZcf'Wca d'Yh]b['UWUXYa]Wk cf_"

±DUddfcdf]UhY`W]hUh]cb. '7]h]b['gci fWYg'k \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck `YX[]b['h\Y`″gYWcbXUfm″ gci fWY`Zfca 'k \]WX`_bck `YX[Y`cZh\Ya`\Ug'VYYb`cVhJ]bYX"

5 WUXYa]W + DZcfa Uh]cb

: cf'YggYbhJU`'ghi XYbhi]bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va]gg]cbg '[i]XY `]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc']W]Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- •