

ARTS3047

7 cbhYa dcfUfm7 f]h]WU``UbX`7 i `hi fU``H\Ycfm

HYfa 'H\fYY''##'&\$&\$

7ci fgY'Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWJhjcb	D\cbY
9`]nUVYh\'A WA U\cb	Y"a Wa U\cb4 i bgk "YXi "Ui	: f]XUm'&!'	K YVghYf"&&"	I %%* (

GWXcc~7cbhJWh~abZcfa Uhlcb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@Yj Y`" žFcVYfhK YVghYf'6i]`X]b[ff %(Ł

D\cbY. f\$&Ł'-',) '(,) *

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'``fPYbg]b[tcb'WJa di g\cong \cong \cong \cong mUbX'5fti/'8Yg][b'7Ua di gYg\cong \cong \con$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y'7ci fgY

 $H\ [g'WcifgY'] b ffcXi \ W'g'mci' fc gca Y'cZh\Y'WbffU'hYl fg'UbX'WcbW'dhg'] b fh\Y'_Ym'] b fh\Y'YWh U'a cj Ya Ybfg UbX'h\Ycf] Yg'Zfca' fh\Y'YUf'mfk Ybh]Yfh 'W'bhi fmfc'h\Y'dfYgYbh'Ug'h\YgY'VYUf'cb''] hYfUfmUbU'mg]g'''Mci'k]'`Yl Ua]bY'U'fUb[Y'cZfY`Uh'XX']ggi Yg'] b Wi X]b[.$

- %"\ck 'h\Y'j Yfm'Uggi a dhjcbg'cZ`Ub[i U[Yž'Wca a i b]\W\hjcb'UbX'a YUb]b['\Uj Y'VYYb'ei YghjcbYX'\Zfca YUf'mk YbhjYh\'W\bh fm'rc'h\Y'dfYgYbh'
- &" h\Y fY!WcbWdhcb cZ]XYbhlmiUbX]hq]a dUWicb ``]hYfUfmWcbqlfi Whcb/
- '" h\Y`k Umg`dc`]h]WU`ghfi WhifYg`Wcbghfi Whih\Y`fY`Uh]cbg\]d`VYhk YYb`\i a Ub`gi V^YWg`UbX`cV^YWg`gi W. Ug`Wca a cX]h]Yg"

7 ci fgY @YUfb]b[Ci hWca Yg

HYUWX]b[GlfUhY[]Yg

H\]g`Wci fgY`]g`]bZcfa YX`VmUb`UddfcUVX`hc`hYUVX]b[`UbX``YUfb]b[`h\UhZcghYfg`]bhYfUWfj]]mžghi XYbhU[YbWh UbX'h\Y'd'YUqi fY'cZ_bck 'YX[Y" \(\delta\) dfca chYg'bYk k Umg'Zcf'gh XYbhg'hc XYj Y`cd'dfcXi Whj Y'dcgg]V]]h]Yg Zfca 'h\Y'X]qW|d`]bUfmVUqYq'cZh\Y]f'qh XmUbX'hc'fYWc[b]qY'h\Y'dchYbh|U`]b']bhYfX]qW|d`]bUfmYb[U[Ya Ybh]b h\Y`\ia Ub]h\Yg":5g'U'h\]fX'mYUf'Wci fgYž']hdfca chYg']bXYdYbXYbh`YUfb]b['Ug'fYZYWhYX']b'gh XYbhZcW ggYX qYa]bUfgžh\YYa d\Uq]q`cb`fYqYUfW\ZUbX`h\YWti fqY`qma dcq]i a "`H\YfY`k`]```VY`U`cbY!\ci f``YWfi fY`YUW\ k YY_".5 "YWn fY]g'a cfY h\ub'U'dfYgYbhUhcb'cZ]bZcfa Uhlcb/]h]g'Ub'UWnj Y'UbX]bhYfUWnj Y'Zcfi a 'k \YfY [XYUq:UbX:fYUX]b[:dfUWfWg:UfY:fY\YUfqYXz:hYqhYX:UbX:W\U``Yb[:YX":\\=i]g:h\Y:q]hY:k\YfY:gh:XYbhq YbWci bhYf'h\Y'Wcbj YbhJcbg'cZdfYgYbhJb['UWUXYa]WfYgYUfW\'UbX'ghU[]b['UWUXYa]WXYVUhY"'@YWf fY UthYbXUbWY]grgi ddcfthYXVmtk crfYj]g]cbrbcc`g. YWkc' *\$rUbXrcb!`]bYra UthYf]U`rdcgthYXr]brtk,Yrk YY_rUZtyfrtk,Y `YWi fY"BY]h\YfcZh\YgY'gi ddcfhq`]q'U'fYd`UWYa YbhZcf'UhYbXUbWY"'H\Y'YW\c' *\$`]q'U'fYWcfX'cZUb'Yj Ybh bchuʻqi a a UfmcZ]bZcfa Utlcb"' \disp_YYd]b['k]th th Y cVYWlj Yg cZth]fX!mYUf gh Xmžth Y gYa]bUf a cXY dfcj |XYg'U'a cfY'|bXYdYbXYbhUbX'Wc``UVcfUhj Y ``YUfb]b['Ybj |fcba Ybh''K \]Y'Z ``mgi ddcfhYX'UbX'X]fYWhYXž h\Y`gYa]bUf`U``ckg`ghiXYbhg`hc`UggiaY`]bWfYUgYX`fYgdcbg]V]`]hmZcf`h\Y`X]fYWf]cb`cZYUW\`WUgg`UbX`Zcf`h\Y Zcfa i `Uhlcb cZfYqYUfW. HUq a"'H\YmUfY 'U`qc 'UV'Y hc WfhWU``mfYZYWhcb 'h\Y]f'ck b dfcWyqqYq cZ`YUfb]b[ž k\]WX']g'cZdUff\W'\Uf']a dcfrub\W']b'U'\Ci fqY']bfcXi Wb['bYk'\Wf\\W\TfYUX]b['dfU\W\\W'g"'H\Y'gYa]bUf fYei]fYg YUWX ghi XYbhhc i bXYfhU_Y. dfYdUfUh]cbž]bWi X]b[gca Y Wc ``UVcfUh]cb k]h\ ch\Yf ghi XYbhg/ $X]gWigg]cb"UbX"dfYgYbHUf]cbg/Wc"UVcfUf]jY'kcf_{]b"gaU""[fcidg"'HckUfXg"h\Y'YbX"cZh\Y'WcifqY'kY'k]"$ \c`X´U`h\Ya YX`Gma dcgli a `lb`d`UWY`cZqYa lbUfq"`DfY!UffUb[YX`[fci dq`k]```YUW\`[li Y´U`&`a lbi hY dfYgYblUlicb cb 'U'lcd]Wh\Uh'\Ug 'VYYb 'Uddfcj YX'Vmlh\Y'Wcbj Ybcf"'H\]g k]``YbUV'Y'gli XYblg lc 'Yb[U[Y k |h\ 'h\ Y'Z ```fUb[Y'cZhcd]Wa'UbX']bhYfYaha'fU]qYX']b'h\ Y'Wci fay'UbX'hc'a]hi UhY'h\ Y]f'ck b'dcg]hjcb'Ug'Ub]bhYfj YbhJcb]b Ub UWUXYa]WX]gW gg]cb" H\]g Wci fgY]g gi ddcfhYX Vm@YUfb]b[A UbU[Ya YbhGmghYa Wca dcbYbhgždfci [X]b['Wci fgY']bZcfa Uhcb'UbX'i dXUhYgžWcd]Yg'cZhYl hg'UbX'UXX]hcbU```]b g'hc'fY`UhYX qW.c`Uf`mfYqci fW'qžcb`]bY'X]qW'qq]cb'[fci dq'Zcf'[fci d'k cf_"

5ggYgga Ybh

A midfYZYffYX'fYZYfYbWJb['ghm`Y'Zcf'h\Y'A U^cf'YggUm']g'A cXYfb'@Ub[i U[Yg'5ggcWJUh]cb'fA @5'ghm`YŁ'': cf Yl Ua d'Yg'cZ\ck 'hc'i gY'h\]gžgYY

\hdg.#ghm\Y"a \U"cf[#k cf g!W|hYX!U!ei]W![i]XY#

\hd.#|\VfUfm'\Vcb\VcfX|\U"\VU#\Y`d#\\h|b[#a `U"d\d

.

5ggYgga YbhHUg_g

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
6`c['FYZYW]cb'HJg_'	&\$I	\$, #%#&\$% `\$).\$\$`DA	
Gma dcg]i a 'DfYgYblUljcb'	' \$ı	%#%%\$% `\$+.\$\$`DA	
A U⁄cf'YggUm) \$1	&-#%#&\$% `\$).\$\$`DA	

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh'% '6`c['FYZYWh]cb 'hUg_

@Yb[h\.'%\$\$\$

8 YHJ]`g.

%\$\$\$`k cfXg`]b`hchU`"`@cWUhY`U`W`hi fU``cf``]hYfUfmhYI h`UbX`k f]hY`U`fYZYWf]cb`cb`]hžYI d`cf]b[`\ck`]hi]``i ghfUhYg cbY`cZh\Y`h\YcfYh]WU``WcbWYdhg`Zfca `h\Y`Wci fgY"`8c`h\]g`Zcf`h\fYY`X]ZZYfYbhk YY_`mihcd]Wg"`Mci `a Umbch V`c[`cb`h\Y`hcd]WcZmci f`Gma dcg]i a `dfYgYbhUh]cb`Vi ha Um'V`c[`cb`h\Y`h\YcfYh]WU``WcbWYdhmci `XYU``k]h\]b`mci f`a U'cf`YggUm"

Gi Va]gg]cb'bchYg. GhU[[YfYX'gi Va]gg]cbg'cb'Wci fgY'fYUXYf'V`c['Zfca 'k YY_'&"

K f]hhYb ZYYXVUW_ UbX bi a Yf]wU` [fUXY j]U @A G"

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh& Gma dcg]i a DfYgYbhJh]cb

GhUfh'XUhY. '&%#%\$#&\$&\$ '\$-.\$\$ '5A

8YHJ]`g.

H\Y'Gma dcg]i a ']g'U'a]b]!Wcb: cf

kci[fUXYg, cbY Zcfih,Yi[fci dikcf_'Ug'U'k\c`Y'UbX'cbY Zcfih,Y]fi]bX]j]Xi U``Wcblf]Vi h]cb"

K f]hhyb ZYYXVUW_ UbX bi a Yf]wU` [fUXY j]U @A G"

5XX]h]cbU`XYhU]`g.

 $H\g'mYUf\bar{z}'k\ Y'k\]^{\c}c\ X'kc\ C'7f]h]WU'H\Ycfm'Gma\ dcg]U''$

 $\label{thm:condition} D`YUgY`bchYz`gh`XYbhg`UfY`cb`mfYei]fYX`hc`dfYgYbhUhcbY`cZh\Ya~z`Y]h\Yf`Uhh\Y`gYa~]bUf`cZh\`C~WhcVYfz`cf`Uhh\Y`gYa~]bUf`cZ\(h\`Bcj~Ya~VYf'')$

Hi fb]hjb'gYhjb[.'H\]g'Ugg][ba Ybh']g'gi Va]hhYX'h\fci [\'Hi fb]hjb'UbX'gh XYbhg'Xc'bchgYY'Hi fb]hjb q]a]`Uf]hmfYdcfhg"

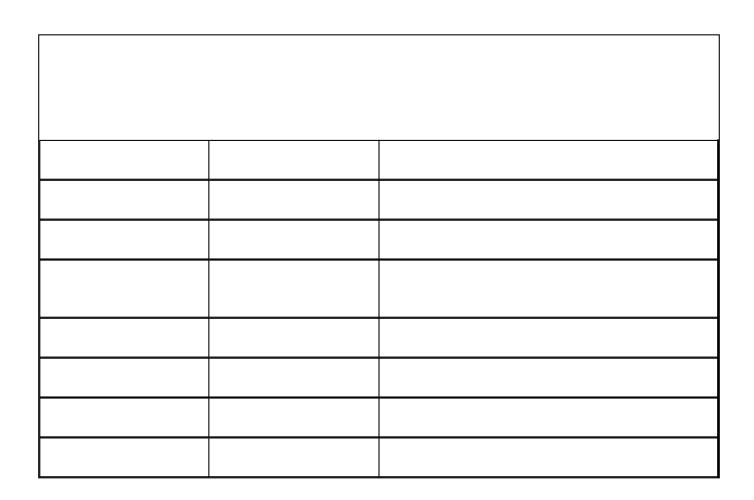
5ggYgga Ybh' . A U'cf YggUm

8 YW] g.

&) \$\$`k cfXg"`H\]g`]g`h\Y`ZbU``UggYgga YbhHJg_"

Ghi XYbhg`k]``fYWY]j Y`k f]hhYb`ZYYXVUW_'UbX`U`bi a Yf]WU``[fUXY"

 $\label{thm:limit} \begin{array}{ll} \mbox{Hi fb]h]b"gYhh]b" . "H\]g"Ugg][ba Ybh"]g"gi Va]hhYX"h\]fci [\" 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{array}$



Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['lc.