

7ci fgY Cj Yfj]Yk

GHJZZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWJhjcb	D\cbY
DYHYf'FcVYfhg	5 5	J		B <i>#</i> 5
		&"\$\$da!("\$\$da	KYVghYf"&" %D	

Hi hcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWJhjcb	D\cbY
A]f]Ua '7 UVY``c	a "WUVY``c4 i bgk "YXi "Ui	B#5	B#5	B#5
5g\`YmiA W fUh\	Ug\`Ym'a W[fUh\4 i bgk "YXi "Ui	B <i>#</i> 5	B#5	B <i>#</i> 5

GWXcc`7cbHJWh =bZcfa Uh]cb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " &ž@YjY`" žFcVYfhK YVghYf'6i]`X]b[fl %(Ł

D\cbY. 'f\$&Ł'-',) '(,) *

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'hc'FYgdYWM\cong \cong m5W_bck`YX[Y'h,Y'HfUX]hjcbU``7i ghcX]Ubgž h,Y'6YXY[U'`fPYbg]b[hcb'WJa di gŁž'; UX][U'`fP]hmUbX'5fhi/'8Yg][b'7Ua di gYgŁ'UbX'h,Y'B[i bbUk U` dYcd'Y'fbi ghfU`]Ub'8YZYbW': cfW'5WJXYa m]b'7UbVYffUŁ'cZh,Y'`UbXg'k \YfY'YUW.'WJa di g'cZl BGK ']g `cWJhYX''$

7ci fgY 8YW] g

7 fYX]hDc]bhg *

Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`dfcj]XYg`U`Wca dfY\Ybg]j Y`]bffcXi Wf]cb`fc`Di V`]WF Y`Uf]cbg`UbX`5Xj Yff]g]b['Df]bW]d`Yg`]b`Ub 5i gffU`]Ub`UbX`]bhYfbUf]cbU``WcbhYl ff`Mci `k]``Yl Ua]bY`fh\Y`\]gfcfmž[fck fh\žUbX`WcbfYa dcfUfm]bZi YbWY`cZ Vch\`X]gW]d`]bYg"'H\Y`Zci bXUf]cbg`cZDF 'UbX`UXj Yff]g]b['UfY`ghi X]YXž`Ug`UfY`fh\Y]f`]bZi YbWY`k]h\]b`UbX ci fg]XY`cf[Ub]gUf]cbgž'UbX`fh\Y``]b_U[Yg`Vch\`X]gW]d`]bYg`\Uj Y`]b`fh\Y`"Wca a i b]WUf]cbg`a]l "\"''Mci `k]```VY UV`Y`fc`i bXYfgfUbX`g]a]`Uf]fmUbX`X]ZYfYbW'`VYfk YYb`DF`UbX`UXj Yff]g]b[']b`fY`Uf]cb`fc`Wca a i b]WUf]cb gffUfY[mž'fYgYUfW.ž'a YX]U`fY`Uf]cbgž']bXi gffmUbX`dfcZYgg]cbU``dUf\k Umg''Mci `k]```U`gc`[U]b`Ub UddfYW]Uflcb`cZfh\Y`Yfh\]WU``X]a Ybg]cbg`UbX`]qqi Yg`]b`U``]YX`Wca a i b]WUflcb`X]qW]d`]bYq"

7 ci fgY @YUfb]b[Ci hWca Yg

- %" 9I d`U]b h\Y`Z bXUa YbhU``bUh fYg`cZDF 'UbX`UXj Yfh]g]b[ž`]bWi X]b['h\Ycf]YgžWcbhYa dcfUfmdfUWf]WY lb`U`k]XY`j Uf]YhmcZUfYbUgž'UbX`Ya Yf[]b['hfYbXg"
- &" 9j U`i UhY 'WcbWYdhg'cZdi V`]Wg#Ji X]YbWYgžfY`Uh|cbg\]dg'UbX'Yh\]WU`'dfUWf|WY"
- ' " 5bU`mgY``c\WJ``UbX`]bhYfbUhJcbU``DF`\WJgY`gh X]Yg"
- ("8Yd`cm_bck`YX[Y`cZUbX`g_]``g`]b`dfcZ/gg]cbU``k f]h]b["

HYUW]b['GhfUhY[]Yg'

5ggYgga Ybh

 $H\ [g.7ci\ fgY.Ci\ h]bY.dfcj\]XYg.cb\ mh\ Y.a\ cghVUg]W]bZcfa\ Uh]cb.fY[\ UfX]b[\ 'h\ Y.UggYgga\ Ybhg''.8YhJ]`YX \]bghfi\ Wh]cbg.Zcf.U``UggYgga\ Ybhg.UfY.Uj\ U]`UV.Y.cb.'A\ ccX.Y''$

Mci a i gh'Wca d`YhY'UbX'gi Va]h'U```h\fYY'UggYgga Ybh'hUg_g']b'cfXYf'hc'dUgg'h\Y'Wci fgY"

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
7 Ua dU][bʻ5bU`mg]g DfYgYbHJHjcbʻ	' \$1	K YY_'' `]b`WUgg	%ž&ž
7`]YbhD]hW\`8YjY`cdaYbh A]`YghcbYg`	&\$ı	K YY_gʻ) žʻ+žʻ, '/ '- ']bʻWUgg	% <u>ž&</u> ž(
7`]YbhD]hW.) \$1	K YY_'%\$	%ž&ž(

5ggYgga Ybh8YHJ]`g

5ggYgga Ybh% 7Ua dU][b5bU`mg]gDfYgYbhUh]cb

GhUfh'XUhY.

8YW]`q.

±0 [fci dgˈcZ(žˈghi XYbhgˈXY`]j Yfʻ%) la]bi hYˈdfYgYbhUh]cbgˈUWWca dUb]YXʻVmgi Va]gg]cbˈcZg`]XYgʻhc A ccX`Yž'k]h\ k f]hhYbʻZYYXVUW_dfcj]XYX"

5ggYgga Ybh& 7`]YbhD]hW 8Yj Y`cda YbhA]`YghcbYg

GhUfh'XUhY.

8 YW] g.

5ggYgga Ybh' . 7`]YbhD]hW

Ghufh'XUhY.

8 YW] g.

 $5 \cdot \&z) $\$! k cfX \cdot dfUWf)WU \cdot k f]h]b[\cdot hUg_ \cdot h \cdot Uh]bWi XYg \cdot U \cdot W]Ybhid]hW \cdot UbX \cdot WcbhYbhYl Ua d`Yg \cdot gi Va]hhYX \cdot hc Hi fb]h]bz \cdot k f]hhYb \cdot ZYYXVUW_"$

5 hm/b XUb VW F Yei]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] } \\$

7ci fgY GWX YXi `Y

J]Yk 'WUgg'hja YhUV'Y

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l .	I

	Hi hcf]U`]bZcfa YX'Vf]YZZfca 'h\Y'W]YbH'H\]g'k YY_žk Y'`cc_'Uh Vf]YZg'UbX'h\Y'Vf]YZ]b['dfcW'ggž'h\Yb'hi fb'hc'\ck fYgYUfW.']g'WcbXi WhYX'UbX'Wcbg]XYf']hg'd'UW'k]h\]b h\Y'dfcW'gg"'F YgYUfW.'Z Y`g'[ccX'd'Ubb]b[ž'k \]W\ dfcj]XYg'h\Y'ghfi Wi fY'cZUbmWJa dU][b"' K Y']bhfcXi W''U'bYk 'a cXY``Zcf'h\Y'fYa U]b]b[WUggYg.'h\Y'5[YbWm''5ZhYf'gca Y'Vf]YZX]gW gg]cb'cZ fYUX]b[g'UbX'WcbW'dhgžmci 'k]``VY[]b'k cf_'cb'h\Y a U'cf'UggYgga YbhcZh\Y'Wci fgY'!'h\Y'W]Ybhd]hW."
K YY_').'%&'CWcVYf'!'%* CWcVYf	@YWi fY	7`]Ybh6f]YZ]b[H\]g``YWifY']g`Wca di `gcfmUbX`k]```bchVY`fYWcfXYX* \$\din\]g`kYY_'g``YWifYzmci `k]```VY`Vf]YZYX`cb 5ggYgga Ybh' "'Mci `k]```\YUf`X]fYWimZca `h\Y`7`]Ybh UVci hh\Y]f`cf[Ub]gUh]cb`UbX`]hg`dfca ch]cbU``bYYXg" H\YfY`k]```VY`Ub`cddcfhi b]hmhc`Ug_`ei Ygh]cbgz XYdYbX]b[`cb`h\Y`Uj U]`UV`Y`h]a Y"
	Hi hcf]U` 5ggYgga Ybh	#D'hi hcf]U`gžk Y`k]```ZcW g`cb 5ggYgga Ybh' . '7`]Ybh D]hWk" K Y`k]```U`gc a YYhh\Y`Z]fghcZci f XYj Y`cda Ybha]`YghcbYg`Zcf'h\Y`d]hWkžZcf'k \]Wk mci gi Va]hk cf_!]b!dfc[fYgg`UbX`fYVW]j Y`Zcfa Uhjj Y ZYYXVUW_" 5ggYgga Ybh& '7`]YbhD]hWk`8Yj Y`cda Ybh A]`YghcbYgž]b`WUgg`
K YY_'*.'%-'CWcVYf'!'&' CWcVYf	FYUX]b[BC @97HI F9 CF 'HI HCF = 5 @G '5 G 'K 99? '* '= 6 '5 F958= B; 'K 99?'
K YY_'+.'&*'C WcVYf'!'' \$ C WcVYf	@YWi fY	9`Ya Ybhgʻf\L``GhfUhY[mUbX`HUWh]Wg Di V`]WfY`Uh]cbgʻUbX`UXj Yfh]g]b[`dfcZYgg]cbU`gʻbYYX hc`\$% %o

	a YUb]b[!a U_]b['UbX'dYfgi Ug]cb'UfY'fc'di V']W fY`Uh]cbg'UbX'UXj Yfh]g]b["'H\]g'k YY_ž'k Y'Wcbg]XYf \ck 'fc'WcbbYWfk]h\'Ub'Ui X]YbWf']b'dfUWf)Wf" #1g fUfY`mYbci [\'fc'\Uj Y'U'k]hmHU['']bY'cf'WYj Yf]a U[Y.'mci 'bYYX'fc'HY``'U'gfcfmUbX'mci 'bYYX'fc gHU[Y'h\UhHY``]b["'K Y'k]```cc_'Uh\ck 'gfcfmHY``]b[k cf_gž'Zfca 'ghfi Wfi f]b['bUffUh]j Yg'fc'dck YfZ' k f]h]b["'6i hk Y'k]``'U'gc'Ug_'\ck 'gfcf]Yg'UfY'ghU[YXž k]h\'U'dUff]W'`Uf'ZcW'g'cb'h\Y'hfYbX'fck UfXg YI dYf]Ybh]U`'WJa dU][bg"'

5 ggYg	ga Ybh 5gg &\$	gYgga Ybh' . 7`]YbhD]tWX 'Xi Y '%* "\$\$ž'Bcj Ya VYf
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FYgci fWyg

DfYgWf]VYX F Ygci fWYg

Bch'Uj U]`UV`Y

FYWca a YbXYX FYgci fWYg

BchUj U]`UV`Y

7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

 $K Y ``UfY ``Wcaa] '`thYX ``tc'] a dfcj]b[``A8 = 5 \% $$' ``UbX` \ Uj Y ``VYbYZ]tYX'[fYUthmZfca ``gti XYbthZYYXVUW_`cb'thXY Wci fgY"' <math>\pm$ 0 `UXX]t]cb'tc'thXY'A m9I dYf]YbW' gi fj Ymg' WcbXi WtYX'UththXY'YbX'cZYUWX gYaYgtYfZthXYfY' k]```VY' Ubcddcfti b]tmZcf'ZYYXVUW_'a]X!gYaYgtYf"' Mci '`UfY' U`gc'kY`WcaY'tc'YaU]``cf'j]g]tiaYZDYtYf'FcVYftgZ'UtiUbmdc]bttc'fU]gY'gdYVJZJWWcbWftbg"

Ghi XYbhiZYXVUW_`]g`Yj U`i UhiYX`]b`fY`Uh]cb'hc'h\Y``YUfb]b['cVYWIj Yg`cZh\Y`Wci fgY`UbX`W\Ub[Yg`UfY]a d`Ya YbhiYX`hc']a dfcj Y'h\Y'Yl dYf]YbW'`UbX`ci hWca Yg`Zcf'ghi XYbhg'': cf'Yl Ua d`Yž]b`&\$%+'h\Y'Z]bU`UggYgga Ybhik Ug`fYXi WfX']b`k Y][\h'UbX`Vfc_Yb`]bhc'hk c'Wca dcbYbhg`VUgYX`cb'ZYYXVUW_`Zfca 'ghi XYbhg`]b &\$%*"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

- i bXYfglUbX`mci f`f][\hg`UbX`fYgdcbg]V]`]h]Yg`Ug`U`gh XYbh'Uhi BGK
- VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hzil BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5WWdhUV`Y'I gY'cZ I BGK '= H'F Ygci fWfg'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wcaaib]hm
- `cWUHY 'gYfj]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm