

# **MDIA5022**

7 cfdcfUhY 'UbX' + bhYfdYfgcbU`'7 ca a i b] WUh]cb

HYfa 'H\fYY''##'&\$&\$

# 7ci fgY'Cj Yfj ]Yk

#### GHJZZ7cbHJWh8YHJ]`g

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUhjcb	D\cbY
A ]WXUY`?Ybh	n' ) &( \$- ) 4 i bgk "YXi "Ui			

#### GWXcc~7cbhJWh~abZcfa Uhlcb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@YjY`" žFcVYfhK YVghYf'6i]X]b['f) %(Ł

D\cbY. f\$&Ł'-',) '(,) \*

9a U]` gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GW.cc``cZh,Y'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'th,Y'HfUX]tjcbU``7i gtcX]Ubg\cong hY'6YXY[U'``fPYbg]b[tcb'WJa di g\cong \cong \cong \cong mUbX'5fti/'8Yg][b'7Ua di gYg\cong \cong \con$ 

## 7ci fgY 8YHJ] g

#### 7 fYX]h'Dc]bhg'\*

### Gi a a UfmcZh\Y 7ci fgY

DfcZ/gg]cbU`cf[Ub]nUh]cbU``Waa i b]Wh]cb']g'i bXYfd]bbYX`Vmh\Y`UV]`]lmhc'Zcfa 'dcg]hj Y`fY`Uh]cbg\]dg
UbX`bUj ][UhY`X]ZYfYbW'g']b'h\Y`k cf\_d`UW\"'K \]`Y`k Y`cZhYb'Uggi a Y`h\UhVi g]bYgg'Wa a i b]WUh]cb']g'U
fUh]cbU``dfcW'ggž'k cf\_d`UW\g'UfY`U`gc']a dUW\YX`cb'VmYa ch]cbg'h\UhYa d`cmYYg'WUffmk ]h\`h\Ya "`K \]`Y
]ggi Yg`cZ`Ub[i U[YžW`hi fY`UbX`Ya ch]cb'i bXYfd]b'U```]bhYfdYfgcbU``Waa i b]WUh]cbžghi XYbhg']b'h\]g
Wai fgY`UfY`dUfh]W``Uf`mYbWai fU[YX`hc`Udd`mh\Ycf]Ygž'WabW'dhg'UbX`]XYUg'cZ]bhYfdYfgcbU``Waa i b]WUh]cb
hc']ggi Ygž'W\U``Yb[Ygž'UbX`cddcfhi b]h]Yg']b'WafdcfUhY`UbX`ch\Yf'cf[Ub]nUh]cbU``WabhYl hg"'H\Y`Wai fgY`k]`
ZcW g'cb'Udd`]WUh]cb'cZ]bhYfdYfgcbU``Waa i b]WUh]cb'WabW'dhg'UbX`dfcWygyg'hc'cf[Ub]nUh]cbU``WabhYl hgž
cddcfhi b]h]Ygž'W\U``Yb[Yg'UbX`]ggi Yg'''? Ymhcd]Wg']b'h\]g'Wai fgY']bWi XY'cf[Ub]nUh]cbU``Waa i b]WUh]cb'UbX
X]gWai fgYž'bY[ch]Uh]cbž'dYfgi Ug]cbž'UXj cWUWhž'a UbU[]b[`W\Ub[Yžj U`i ]b[`X]j Yfg]hmžYh\]WO``XYW]g]cb!
a U ]b[`UbX`WabZ]Whifyqc`i h]cb''

### 7 ci fgY @YUfb]b[ Ci hWca Yg

- %" 5dd`m`]bhYfdYfgcbU``Wcaaib]WUh]cb`WcbWYdhg`UbX`dfcWYggYg`hc`cf[Ub]gUh]cbU``WcbhYl hg`UbX dfcV`Yag
- &" 8Ya cbgffUhY \_bck `YX[Y`cZcf[Ub]qUhcbU``Wca a i b]\WUhcb `Wcb\Wdhq
- ' " 5dd`mbY[ch]Uh]cb'UbX'dYfgi Ug]cb'g\_]``gž'UbX'h\Ycf]Yg'cZa UbU[]b['W\Ub[Y
- ("9|Ua]bY'h\Y']adcfrUbWY'cZjU'i]b['X]jYfg]lmžYh\]WU`XYW]g]cb'aU\_]b['UbX'WcbZ]WifYgc'ih]cb

### HYUWX]b[ 'GhfUhY[]Yq'

#### HYUW ]b[ GhfUhY[ ]Yg.

7 cfdcfUhY 'UbX' +bhYfdYfgcbU` '7 ca a i b]\Wh]cb']g'XYg][bYX' hc'YbWci fU[Y'UbX'WXU``Yb[Y'mci 'hc'fYZYWhcbUbX']a dfcjY'mci f'ck b'Wca a i b]\Wh]cb'dfU\Wh]Wh"

H\Y`Wci fgY`]g`XY`]j YfYX`]b`U`gYa ]bUf`Zcfa ž`k \YfY`X]gWi gg]cbgž`UbU`mg]g`UbX`Udd`]WUh]cb`cZ\_YmWcbWidhg UbX`fYWibhfYgYUfW\`Z]bX]b[g`UfY`i gYX`hc`XYj Y`cd`U`XYYdYf`i bXYfghUbX]b[`cZghfUhY[]WWca a i b]WUh]cb" H\Y`gi V^YWh]g`XYg][bYX`Ufci bX`U`a]l `cZUWhj ]hJYg`hc`Vi ]`X`dUfhJWJdUbhgfi]bhYfYghUbX``YUfb]b["`H\Y`hk c\ci f`WUgg`dYf]cX`YUW\`k YY\_k ]```]bWi XY.

- U'gYWjcb'cb'\_Ymdf]bWjd'Ygž'k \]W\'k]'``]blfcXi W\'h\Y'fY'Yj UbhWcbW\'dlg'UbX'h\YcfmcZWcfdcfUlY UbX']bhYfdYfgcbU'`Wca a i b]WUljcb"
- U'gYf]Yg'cZX]gW gg]cbg'UbX'UWfj ]h]Yg'U]a YX'UhUdd'm]b[ 'h\Y' Ymdf]bWd'Yg"

Hc cVHJ]b h\ Y Z ``VYbYZ]hg Zca h\ ]g Wci fgYzgh XYbhg UfY YbWci fU[ YX hc YI hYbX mci fgY`ZVYmcbX h\ Y]f Wca ZcfhncbY" H\ Uha YUbg UWlj Y`mdUff]W]dUlljb[ i]b h\ Y Wcbj YfgUlljcbg UbX X]gW gg]cbg i]b WUgg Ug Vch 'U `]ghYbYf UbX gdYU\_YfzVY]b[ cdYb hc UbX k ]``]b[ hc WcbhYa d`UhY U fUb[ Y cZdYfgdYWlj Yg UbX hfmbYk k Umg cZWca a i b]WUlljb[ zfYZYWljb[ XYYd`mcb h\ Y]f ck b Wca a i b]WUlljcb ghfYb[ h\ g UbX k YU\_bYggYgz UbX VY]b[ k ]``]b[ hc []j Y Wcbghi Wlj Y ZYYXVUW\_hc ch Yfg ]b cfXYf ZcghYf dcg]hj Y UbX YZYWlj Y Wca a i b]WUlljcbz dUff]W `Uf m]b X]j YfgYz hYUa !VUgYXz k cf\_!fY`UhYX WcbhYl hg" K Y \cdY h\ Uhigh XYbhg k ]```ZYY``h\ Uhih\ ]g Wci fgY c ZYfg U gUZY Ybj ]fcba Ybhhc hfmci hbYk 'ghmYg c ZWca a i b]WUlljcb k \]WX h\ Yma Um`UhYf Udd`m]b h\ Y k cf 'd`UWY"

Ghi XYbhgʻUfYʻUg_YXʻhcʻfYZYWhicbʻh\YʻgYhfYUX]b[gʻVYZcfYʻYUWXʻWUggžʻUbXʻVf]b[ʻh\Y]fʻei Ygh]cbq UVci hh\YʻfYUX]b[gʻVUgYXʻcbʻh\Y]fʻck bʻi b]ei YʻYI dYf]YbWYgʻhcʻYUWXʻWUgg"	gʻUbXʻ]XYUg

# 5ggYgga Ybh

## 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
±b!7`Ugg'9l Ua '	' \$1	Bch5dd`]WUV`Y	%ž&Ž Ž(
; fci d'dfYgYbfUf]cb'UbX fYdcfhi	' \$ı	Bch5dd`]WUV`Y	%ž&
F YgYUfWX 'DUdYf'	(\$1	Bch5dd`]WUV`Y	%ž&ž ž(

## 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% ±17`Ugg 9l Ua

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

9ggUmlVUgYX`]b!WUgg`Yl Ua "`E i Ygh]cbg`k ]```VY`dfcj ]XYX`U\YUX`cZh]a Y/ei Ygh]cbg`k \]W\`gh XYbhg`k ]`` Ubgk Yf`]b`h\Y`Yl Ua `k ]```VY`fUbXca `mgY`YWhYX`cb`h\Y`XUmcZh\Y`Yl Ua "

K f]hhYb ZYYXVUW\_dfcj ]XYX cb Yl Ua gžk \ ]WX k ]``VY fYh fbYX hc gh XYbhg"

#### 5XX]h]cbU`XYhU]`g.

H\Y'YI Ua 'k]``'VY'U'ZUWY!hc!ZUWY'YI Ua ž'[]j Yb'j ]U'Ncca ž'YhW'

@Yb[h\.'&ž)\$\$Ž'kcfXg

8YHJ]`g.

&) \$\$`k cfXg`Zcf`k f]hhYb`fYZYWh]cb"`H\]g`]g`h\Y`Z]bU``UggYgga YbhhUg\_"

K f]hh/b ZYYXVUW\_dfcj ]XYX j ]U Hi fb]h]b"

 $\label{thm:continuous} \mbox{Hi fb]h]b$'gYhh]b$[.$'H\]g$'Ugg][ba Ybh]g$'gi Va ]hhYX'h\fci [\'Hi fb]h]b$'UbX'gh XYbhg`WUb'gYY'Hi fb]h]b$'g]a ]`Uf]hm fYdcfhg"$ 

Fygci fWyg

DfYgWf]VYX F Ygci fWYg

GYY gm`UVi g" K ]` VY dfcj ]XYX"

FYWca a YbXYX FYgci fWYg

BchUj U]`UV`Y

7ci fgY '9j U`i Uh]cb 'UbX '8Yj Y`cda Ybh

H\fci [\'h\Y'a UbXUh\X'\Zcfa i `U'h\Uh'U`fYUXmYI ]ghg"`5bX'j ]U']b\Zcfa U``Wcbj YfgUh]cbg"`

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g Hi fb]h]b Gi Va ]gg]cb

-Zmci 'YbWci bhYf'U'dfcV`Ya 'k \Yb'UhhYa dh]b[ 'hc'gi Va ]hmci f'Ugg][ ba Ybhh\fci [ \'Hi fb]h]bž'd`YUgY hY`Yd\cbY'9I hYfbU``Gi ddcfhcb'-',)'''' %cf'Ya U]`'h\Ya 'cb'YI hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$ 

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@-69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.#gi VYWn[i]XYg"`]VfUfm"i bgk "YXi "Ui #\`]gY#UVci hY`]gY