

7ci fgY⁻Cj Yfj]Yk

GHJZ:7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
9X[Uf'; ÏaYn'7fin	Y"[ca YnWfi n4 i bgk "YXi "Ui		&'%; FcVYfh KYVghYf 6i]`X]b[

GWkcc~7cbhJWh~abZcfa Uh]cb

GWXcc``cZhXY`5fhg`UbX`AYX]U

Fcca ' %&ž@Yj Y`' žFcVYfhK YVghYf6i]X]b[fl %(Ł

D\cbY. f\$&Ł'-',) '(,) *

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'k ci`X``]_Y'tc'FYgdYWM\cong ``m5W_bck``YX[Y'thY'HfUX]tjcbU``7i gtcX]Ubgž thY'6YXY[U'`fPYbg]b[tcb'WJa di gŁż'; UX][U'`fP]tmUbX'5fth/'8Yg][b'7Ua di gYgŁ'UbX'thY'B[i bbUk U` dYcd'Y'fbi gtfU']Ub'8YZYbWY': cfWY'5WJXYa m']b'7UbVYffUŁ'cZthY'`UbXg'k \YfY'YUW\cong WJa di g'cZl BGK ']g `cWJtYX''$

5ggYgga Ybh

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
±b!7`Ugg'Dfc'YWfi	&\$ı	\$&#%\$#&\$&\$`%%) - `DA</td><td>%</td></tr><tr><td>G\cfh'9ggUm</td><td>' \$ı</td><td>&' #%\$#&\$&\$`%%) - `DA</td><td>%ž&</td></tr><tr><td>A U'cf'Dfc YWh</td><td>) \$1</td><td>&\$#%#&\$&\$`%%) - `DA</td><td>%ž&Ž</td></tr></tbody></table>	

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5ggYgga Ybh% ±0!7`Ugg Dfc YWh

GhUfh'XUhY.

8YHJ]`g.

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 $Gh XYbhg k] ``fYWY] j Y j YfVU ``ZYYXVUW_ i] b `WUgg `UbX k f]hhYb `ZYYXVUW_ ij]U `A ccX`Y/gcW]U ``a YX]U `ghUh]gh]Wg UbX`gcW]U ``a YX]U `ZYYXVUW_ k] ``]a dUWih\Y [fUXY"$

5XX]h]cbU`XYhU]`g.

7\YW_AccXYZcfacfY]bZcfaUh]cb

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5ggYgga Ybh& G\cfh9ggUm

GhUfh'XUhY.

8 YHJ] g.

%\$\$\$ k cfXg"

Ghi XYbhg k]``fYWY]j Y k f]hhYb ZYYXVUW_ij]U Hi fb]h]b"

5XX]h]cbU`XYhU]`g.

7\YW_'AccX`Y'Zcf'acfY']bZcfaUh]cb

5ggYgga Ybh' . 'A U'cf'Dfc YWh

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&) \$\$`k cfXg/cfž`U`a YX]U`Zcfa Uh`fU`j]XYcž`U`V`c[ž`U`dcXWUghž`YhWłż`k \]W\`bYYXg`hc`VY`Z|fgh`Uddfcj YX`Vmh\Y Wcbj Ybcf"

H\]g`]g`h\Y`Z]bU``UggYgga Ybh'hUg_"

Ghi XYbhg k]``fYWY]j Y k f]hhYb ZYYXVUW_j jU'Hi fb]h]b"

5XX]h]cbU`XYHJ]`g.

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 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ $H \ fb \end{tabular} $$ fo \end{tabular} $$ Hi & fb \end{tabular} $$ fo \end{tabular} $$ fo \end{tabular} $$ fo \end{tabular} $$ Hi & fb \end{tabular} $$ fo \en$

5 https://bxubwww.Fyei]fya Ybhg

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\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'''=\mathred{Image: Jmci UfY'i bUV'Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hYbg]cbz'Vi hmci a i ghfYhJ]b'mci f'h]\W_Yhbi a VYf'Zfca '9! hYfbU''Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY'Yj UbhXcW a Ybhgt'hc
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; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUhjj Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b['h\Ya `Ug`mci f`ck b"`±hWJb`hU_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b['hc`UWW]XYbhJ``mWcdm]b['Zfca `U`gci fWY`k]h\ci hUW_bck `YX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$

