

# **MUSC3706**

HYfa 'H\fYY''##'&\$&\$

# 7ci fgY'Cj Yfj ]Yk

### GHJZ:7cbHJWh8YHJ]`g

### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]m	@cWUh]cb	D\cbY
>c\b`BUd]Yf		Hi YgXUm%\$" \$ ! %%" \$žcfVm Uddc]bha Ybhžcf Xfcd]b	K YVghYf*%%	-',)*-)'

### GWXcc~7cbhJWh~abZcfa Uhlcb

GWXcc`cZh\Y'5fhg'UbX'AYX]U

Fcca " %&ž@YjY`" žFcVYfhK YVghYf'6i]X]b['f) %(Ł

D\cbY.'f\$&E'-',)'(,)\*

9a U]`. 'gUa 4 i bgk "YXi "Ui

K YVg]hY. k k k "Ufhg"i bgk "YXi "Ui #gUa

 $\label{eq:hamiltonian} $$HY'GWCC``cZhY'5ftg'UbX'AYX]U'kci`X``]_Y'tc'FYgdYWM\cong \cong m5W_bck``YX[Y'h\Y'HfUX]tjcbU``7igtcX]Ubgž h\Y'6YXY[U'`fPYbg]b[tcb'WJadigŁZ'; UX][U'`fP]tmUbX'5fthZ''8Yg][b'7UadigYgŁUbX't\Y'B[ibbUkU`dYcd'Y'fbighfU`]Ub'8YZYbW': cfWY'5WJXYam]b'7UbVYffUŁ'cZh\Y'`UbXg'k\YfY'YUW\'WJadig'cZIBGK']g`cWJhYX''$ 

## 7ci fgY 8YHJ] g

### 7 fYX]h'Dc]bhg'('

### Gi a a UfmcZh\Y'7ci fgY

 $A \ I \ G7' + \$^* \ DYfZcfa \ UbWr @UVcfUrcfm7' fA \ i \ g]W7 \ fYUh]i \ Y'DfUW]Wr / A \ i \ g]WDYXU[\ c[\ mŁ']g'h\ Y'h\]fX \\ DYfZcfa \ UbWr @UVcfUrcfmWci \ fgY' Zcf'ghi \ XYbhg'Ybfc``YX']b'h\ Y'A \ i \ g]W7 \ fYUh]i \ Y'DfUW]Wr'UbX'A \ i \ g]W \\ DYXU[\ c[\ ma \ U'cfg'cZh\ Y'6UW, Y'cf'cZA \ i \ g]Wdfc[\ fUa "'-\fi\Wcbgc']XUhYg'h\ Y'a \ ]'YghcbYg'UW, ]Yj \ YX']b'YUf]Yf \\ DYfZcfa \ UbWr'@UVcfUrcfmWci \ fgYgž'\ fh\ Yf'XYj \ Y'cd]b[\ 'h\ Y'ghi \ XYbhg'h\ W.b]Wb'z'WfYUh]i \ Y'UbX']bhYfdfYh]i \ Y \ g_]``g''H\ Y'Wci \ fgY'U'gc'Vi ]'Xg'cb'ghi \ XYbhg''Uk \ UfYbYgg'cZU'fUb[\ Y'cZa \ i \ g]Wb''[\ YbfYg'UbX'dfUW]Wrg \ YbWci \ bhYfYX']b'ch\ Yf'Wci \ fgYg']b'h\ Y'a \ i \ g]Wdfc[\ fUa "'Ghi \ XYbhg'`YUfb'hc'Udd`mWf]h]Wb''`]ghYb]b[''UbX \ fYUX]b[\ g'cZdYfZcfa \ UbW''']hYfUhi \ fY'UbX'[\ U]b'\ \ fh\ Yf'g_]``g'UbX'YI \ dYf]YbWr']b'h\ Y'XYj Y'cda \ YbhcZYbgYa \ V'Y \ dYfZcfa \ UbW'g"$ 

 $\textbf{BchY}. \text{ ``H\ ]g`]g`U`\%\&\text{`I} C7\text{ `WcifgY''}\text{Mcik}]`\text{``Ybfc}``]b`(\text{`I} C7\text{']b`YUW\ `cZh\fYY`gi\WW\gg]j\Y`hYfa\g`k\]h\`U`fYgi\`h\fYdcfhYX\VmU`g]b[\`Y`[\fUXY`Uhh\Y`YbX`cZh\Y`h\]fX`(\`I C7\WcifgY''$ 

### 7 ci fqY @YUfb]b[ 'Ci hMca Yq

- % 5dd`mWf]r]Wjga 'UbX'UbU`mg]g`cZh\Y``]hYfUhi fY`cb`dYfZcfa UbWYž'UbX'cZa UghYf`WUgg`dfYgYbhUh]cbgž hc`h\Y`dfYdUfUh|cb`cZh\Y]f`ck b`dYfZcfa UbWY
- &": cfa i `UhY`Ufh]gh]WcVYWfj Yg`UbX`fYU`]gY`h\Ya`]b`U`fUb[Y`cZghm`Yg`]b`dYfZcfa UbWY`cf`Wca dcg]h]cb' "5dd`mdf]bW]d`Yg`cZhYUa k cf\_`hc`a i g]WYbgYa V`Y

## HYUWX]b[ GhfUhY[]Yg

Ghi XYbhgfidYfgcbU``g\_]``g']b'dYfZcfa UbWY'cf'Wca dcg]h]cb'k ]```VY'XYj Y`cdYX'h\fci [\'cbY'cb'cb'cbY Wcbgi `hUhjcbg'k ]h\ 'gdYWJU`]ghhi hcfg'Q 'I '%\ci fYUW\ 'hf]a YghYfQ'' +b'k cf\_g\cdg'Q 'I '%') '\ci fg'YUW\ hf]a YghYfQ' ]bghi a YbhU``UbX'j cWU``ghi XYbhg'k ]```[U]b'dYfZcfa UbWY'YI dYf]YbWYžk\]Y'Wca dcg]h]cb'ghi XYbhg k ]```VYWca Y'ZJa ]`]Uf'k ]h\ ']ggi Yg'UZZYWJb[ 'dYfZcfa UbWY'UbX'dYfZcfa UbWY'dfYdUfUhjcb'''K cf\_g\cd'gYhJb[ g k ]```U`gc'gi ddcfhiYb[ U[ Ya Ybhk ]h\ ']ggi Yg'WcbWYfb]b[ 'a i g]WJUbgfi\YU'h\ 'UbX'k Y``VY]b[ ž'h\Y'XYj Y`cda Ybh cZghU[ Y'WcbXi Wz'UbX'Ub'Y'Ya YbhcZdYYf'cVgYfj Uhjcb'Zcf'gdYWJU`]ghdYfZcfa UbWY'UbX'Wca dcg]h]cb [fci dg'''Ghi XYbhg'k ]```UhhYbX'a UghYfWUggYg'YX'Vmj ]g]h]b[ 'YI dYfhg']b'U'fUb[ Y'cZa i g]WU``]bghfi a Ybhgž j c]WYg'UbX'ghmYgžYbWci fU[ ]b[ 'ghi XYbhg'hc'XfUk '`]b\_g'UbX'Wca dUf]gcbg'k ]h\ 'h\Y]f'dYfgcbU`'ghi X]fYWYX YbgYa V'Ygž'W\ccg]b[ 'Zfca 'b]bY'a UghYfWUggYg'UWfcgg'h\Y'mYUfQ''5``ghi XYbhg'k ]``'dUff]W]dUhY']b'X]fYWYX YbgYa V'Ygž'W\ccg]b[ 'Zfca 'U'k ]XY'fUb[ Y'cZa i g]WU`'ghmYg'UbX'W' hi fYg'UbX'dUff]W]dUh]b[ ']b'U'Z]bU`'dYfZcfa UbWY']b'YUW\'hf]a YghYfQ''

# 5ggYgga Ybh

; YbYfU``]bZcfa Uh]cb'UVci hYl Ua ]bUh]cb'fYei ]fYa Ybhg']g'Uj U]`UV`Y'cb'h\Y'A ccX`Y'g]hY'Zcf'h\]g'Wci fgY''

# 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'HJg_	K Y][\h	8i Y <sup>.</sup> 8UhY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
9bgYa V`Y`Wcblf]Vi hjcbž dYfZcfa UbWYgždUfhW.YW_]b[ hYgh	&\$I	9bgYa V`Y`dUfhWXYW <u>]</u> b[ bcfa U``mihc`VY`Wca	‰



j]XYc[fUd\m'UbX'bchYg'cb'Uh`YUgh(a UghYf!WUggYgŁ"

Dcfh7c`]cg'k]```VY`gi Va]hhYX`h\fci[\`Hifb]h]b`UbX`ZYYXVUW<u>'</u>j]]U`@A G

5XX]h]cbU`'XYhU]`g.

FYZYf'hc'h\Y'bchYg'Zcf'h\]g'hUg\_'cb'AccX`Y

5ggYgga Ybh' . DfUWhJWU Yl Ua ]bUhJcb

GhUfh'XUhY.

8YHJ]`q.

cf'dYfZcfa UbW'gh XYbhgžYl Ua ]bUhjcbg'k ]``VY'i bXYffU\_Yb']b'H' "'Gh XYbhg'k ]``dfYgYbh'U'&) !a ]bi hY dfc[fUa a Y']bW X]b[') 'a ]bi hYg'cZhYW.b]WU`k cf\_žXYj ]gYX']b'bY[ch]Uhjcb'k ]h\h\Y]f'gdYWJU`]ghh hcfg'hc'gi ]h h\Y]f'dYfgcbU`']bhYfYghg'UbX'hfUYWcf]Yg"'Gh XYbhg']XYbhjZm]b['Ug'Unn'gdYWJU`]ghg'k ]``dfYgYbh'HVW.b]WU` k cf\_'Zfca 'h\Y'Unn'gm`UVi g'XYj ]gYX'Zcf'h\]g'Wci fgY"'H\Y'Yl Ua ]bUhjcb'k ]``U'gc']bW XY'U'g\cfhg][\h fYUX]b['hYghZcf'U``gh XYbhg"''H\]g'k ]``VY'h\Y'ZjbU``UggYgga Ybh'HUg\_'Zcf'dYfZcfa Yfg"

: cf`Wca dcg]h]cb`gh XYbhgždcfh7c`]cg`k ]```VY`gi Va ]hh/X`]b`H' "`Dcfh7c`]cg`a Um]bWl XY`Ubmbi a VYf`cZk cf\_gž Vi hh\Y`hchU``Xi fUh]cb`cZk cf\_g`g\ci `X`VY`%)!&\$`a ]bi h/g" `GWcfYg`g\ci `X`VY`gi Va ]hh/Xž'U`cb[ `k ]h\fY\WcfX]b[ g`cZU```k cf\_gž'UbX`cbY`cZh\Y`fY\WcfX]b[ g`a i ghVY`cZU``]j Y`dYf7cfa Ub\W''``H\]g`k ]```VY`h\Y`Z]bU` UggYgga YbhhUg\_`7cf`Wca dcgYfg"

: YYXVUW\_. K f]hhYb fYdcfhcb dfUWf)wU`YI Ua 'UbX Wca dcg]h]cb dcfh7c`]c"

H\] ]g h\Y ZbU`UggYgga Ybh''

.

5XX]h]cbU`XYhU]`g.

5 hh\]g dc]bhk Y Ybj ]gU[Y ``]j Y Yl Ua ]bUh]cbgž ZJWY hc ZJWY "

5ggYgga Ybh(. DYfZcfa UbWY#dfYgYbHJh]cb

GHJfh'XUhY.

8 YW] g.

 $H\fYY'dYfZcfa\ UbVW'#dfYgYbfUh]cbg'k]^``VY'WcbXi\ WhYX'Xi\ f]b[\ 'k\ cf_g\cd'gYgg]cbg']b'H\%'H\&'UbX'H' "'``$ 

 $Gdc\_Yb `ZYYXVUW\_'k \ ] ```VY `[ \ ] j \ Yb `Vm'h Y ``YW'i \ fYf `]b `WUggž `UbX `k \ f] thYb `ZYYXVUW\_'j \ ] U `@A \ G \ A$ 

 $5\,XX]h]c\,b\,U^{^{*}}\!XYhU]^{^{*}}\!g.$ 

# 5 hm/b XUbVW F Yei ]fYa Ybhg

D`YUgY`bchY`h`Uh`YWifY`fYWcfX]b[g`UfY`bchUjU]`UV`Y`Zcf`h`]g`WcifgY"`GhiXYbhg`UfY`ghfcb[`mYbWcifU[YX`hc] bchUjU]`UV`Y`Zcf`h`]g`WcifgY"`GhiXYbhg`UfY`ghfcb[`mYbWcifU[YX`hc] bchUjU]`UV`Y`Zcf`h`]g`WcifgY"`GhiXYbhg`UfY`ghfcb[`mYbWcifU[YX`hc] bchUjU]`UV`Y`Zcf`h`]g`WcifgY"`GhiXYbhg`UfY`ghfcb[`mYbWcifU[YX`hc] bchUjU]`UV`Y`Zcf`h`]g`WcifgY"`GhiXYbhg`UfY`ghfcb[`mYbWcifX]b[`g`UfY`bchUjU]`UV`Y`Zcf`h`]g`WcifgY`g`h`g`UfY`ghfcb[`mYbWcifX]b[`g`UfY`bchUjU]`UV`Y`Zcf`h`]g`WcifyY`ghfcb[`mYbWcifX]b[`mYbWcifX]b[`g`UfY`bchUjU]`uV`Y`Zcf`h`]g`UfY`g`h`g`UfY`ghfcb[`mYbWcifX]b[`g`UfY`bchUjU]`uV`Y`Zcf`h`]g`UfY`g`h`g`ufY`ghfcb[`mYbWcifX]b[`g`UfY`ghfcb[`mYbWcifX]b[`g`UfY`bchUjU]`uV`Y`Zcf`h`]g`UfY`g`h`g`ufY`ghfcb[`mYbWcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mYbwcifX]b[`g`UfY`ghfcb[`mybwcifX]b[`g`UfY`ghfcb[`mybwcifX]b[`g`UfY`ghfcb[`myb

FYgci fW/g

DfYgWf]VYX F Ygci fWYg

Bch'Uj U]`UV`Y

FYWca a YbXYX FYgci fWYg

BchUj U]`UV`Y

7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

: YYXVUW\_cb^h\Y`Wci fgY'k]``VY`a UXY`h\fci [\h\Y`a m9I dYf]YbWY`E i Ygh]cbU]fY"`H\]g^]g`U'bYk `Wci fgY"`#g WcbhYbhUbX`hYUW\]b[`a Yh\cXg`\Uj Y`VYYb`g\UdYX`dUfhm]b`fYgdcbgY`hc`Yj U`i Uh]cb`cZYUf`]Yf`DYfZcfa UbW\@UVcfUhcfm7ci fgYg`Uhh\]g``Yj Y`"'7\Ub[ Yg`]bWi XY`U`ga U``Yf`WUgg`g]nYžYb\UbWYX`]b!WUgg`dYfZcfa UbW\cddcfhi b]h]Ygž'h\Y`d`Ubb]b[`cZ]bZcfa Uh]cbU```YWi fYg`]bj c`j ]b[`dYfZcfa UbW\`]ggi Ygž'U`ghfYb[ h\YbYX\WcbXi Wh]b[`Wca dcbYbhzX]gW gg]cbg`cZ]bXi ghfmg\_]``gž'UbX`h\Y`fYh]fYa YbhcZh\Y`@ghYb]b[`HYgh'`\b\UXX]h]cbz`=\Uj Y`XYj Y`cdYX`U`a cfY`X]gW fg]j Y`UddfcUW\`hc`a UbU[]b[`ghi XYbh[cU`g`UbX`ZYXVUW\_"

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

## Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft'''=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a Um'Udd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$ 

## 5 WUXYa ]W = b Zcfa Uh cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg 'Zcf'Yl hYbg]cb/
- `UhY`gi Va ]gg]cbg`[ i ]XY`]bYg/
- fYi ]Yk 'cZa Uf\_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua ]bUh]cb dfcWYXi fYg/
- qdYWJU``Wcbq]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a ]gUXj Ybh fY/
- gh XYbhYei ]miUbX'X]qUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbz'gYY

\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa ]W]bZcfa Uh]cb#dfchcWc`g![i ]XY`]bYg#

.

### ⇒a U[Y'7fYX]h

H\]g^]a U[Y^]g^h\Y^dfcdYffmcZ>c\b^BUd]Yfz\*h\Y^Wci fgY^Wcbj Ybcf"

#### 7F = 7CG

7 F = 7 C G 'Dfc| ]XYf'7 cXY. '\$\$\$-,;

### 5W\_bck `YX[ Ya YbhcZ7ci bhfm

 $K_{Y'UW\underline{b}ck} YX[Y'hY'6YXY[U''dYcd'Y'k \c'UfY'hY'ffUX]h]cbU''W ghcX]Ubg'cZhY''UbXg'cb'k \]WX'I_BGK_{Ybg]b[hcb'WJa_di_q']q''cWJhYX'''$