

5W_bck `YX[Ya YbhcZ7ci bhfm

I BGK '5frgž8Yg][b'UbX'5fW,]hYWi fY'? Ybg]b[hcb'UbX'DUXX]b[hcb'WJa di gYg'UfY'Vi]'hcb'5Vcf][]bU' @UbXg"'K Y'dUmci f'fYgdYWg'hc'h\Y'6]X'][U''UbX'; UX][U''dYcd'Yg'k \c'UfY'h\Y'7 i ghcX]Ubg'cZh\YgY'`UbXg" K Y'UW_bck 'YX[Y'h\Y'5Vcf][]bU''UbX'HcffYg'G'HU]h=g'UbXYf'dYcd'Ygž'h\Y':]fgh5i ghfU']Ubgžk \cgY'`UbXgž k]bXg'UbX'k UhYfg'k Y'U``'bck 'g\UfYž'UbX'dUmfYgdYWhc'h\Y]f'i b]ei Y'j U'i Ygž'UbX'h\Y]f'Wcbh]bi]b['UbX YbXi f]b['W 'h fYg'k \]W. 'XYYdYb'UbX'Ybf]W. 'h\Y'`]ZY'cZci f'bUh]cb'UbX'Wca a i b]h]Yg'''

5ggYgga Ybh

< YfYžmci fi`cb`mZjbX'j YfmVUg]W]bZcfa Uh]cb`fY[UfX]b['mci f`5ggYgga Ybh'' Mci 'WUb`ZjbX'XYhU]`YX
]bZcfa Uh]cb`cb`A ccX`Yž]b`h\Y`"5ggYgga Ybhg`"gYWh]cb"' +b`UXX]h]cbžmci fi`VY`Vf]YZYX`cb`h\Y`5ggYgga Ybhg`]b
gdYWJZJW`YWh fYg`Xi f]b['gYa YghYfžk \]W\"UfY`]XYbh]ZJYX`]b`h\Y`k YY_`mci h]bYg`cb`A ccX`Y"</pre>

D`YUgY`Z|fghfU]gY`ei Ygh]cbg`]b`mci f`h hcf]U`ž`Ug`ch\Yfg`a Um`\Uj Y`h\Y`gUa Y`ei Ygh]cb"`8]gW gg]cb`Zcfi a g ZcfYUW\5ggYgga Ybhk]```VY`WfYUhYX`cb`A ccX`Yž`gc`mci `k]```U`gc`VY`UV`Y`hc`fU]gY`ei Ygh]cbg'h\YfY hcc"`9a U]``g\ci `X`cb`mVY`i gYX`]Zmci f`ei Ygh]cb`fY`UhYg`hc`U'dYfgcbU``a UhYfž`gi W\`Ug`Ub`YI hYbg]cb"`

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UHY	7 ci fgY @YUfb]b[Ci hWca Yg 5ggYggYX
%" ⁻ 9ggUm	(\$1 &) #%\$#&\$&%*\$\$\$*5A		%ž`&ž`¹
&" ⁻ 91 Ua]bUh]cb	*\$1	%+#%#&\$&%`%%) - `DA	%ž'&ž''

5ggYgga Ybh'% '9ggUm

8i Y'XUhY. '&) #%\$#&\$&%'\$-.\$\$'5A

%) \$! k cfX YggUmcb 'U 'hcd] Whc 'VY 'UXj]gYX"

Ghi XYbhg'k]``fYWY]jY'U'fi Vf]W

: YYXVUW_'j]U'@A G"

 $\label{thm:linear} $H\setminus [g^*Ugg][\ ba\ Ybh]g^*gi\ Va\]hhYX^*h\ fci\ [\ \ 'Hi\ fb]h]b^*UbX^*gh\ XYbhg^*Xc^*bchgYY^*Hi\ fb]h]b^*g]a\]^*Uf]hmfYdcfhg''$

5ggYgga Ybh& 9l Ua]bUh]cb

GHJfhXUHY. %) #%#&\$&%\$-.\$\$*5A8iY*XUHY. %+#%#&\$&%%%) - *DA

&) \$\$!k cfX`cdYb!Vcc_`HJ_Y!\ca Y`Y| Ua ":: YYXVUW_'j]U`@A G"`

H\]g`Ugg][ba Ybh]g`gi Va]h\X`h\fci [\'Hi fb]h]b`UbX`ghi XYbhg`Xc`bchgYY`Hi fb]h]b`g]a]`Uf]hmfYdcfhg"

5 hmy b X Ub WY F Yei] f Ya Yb hg

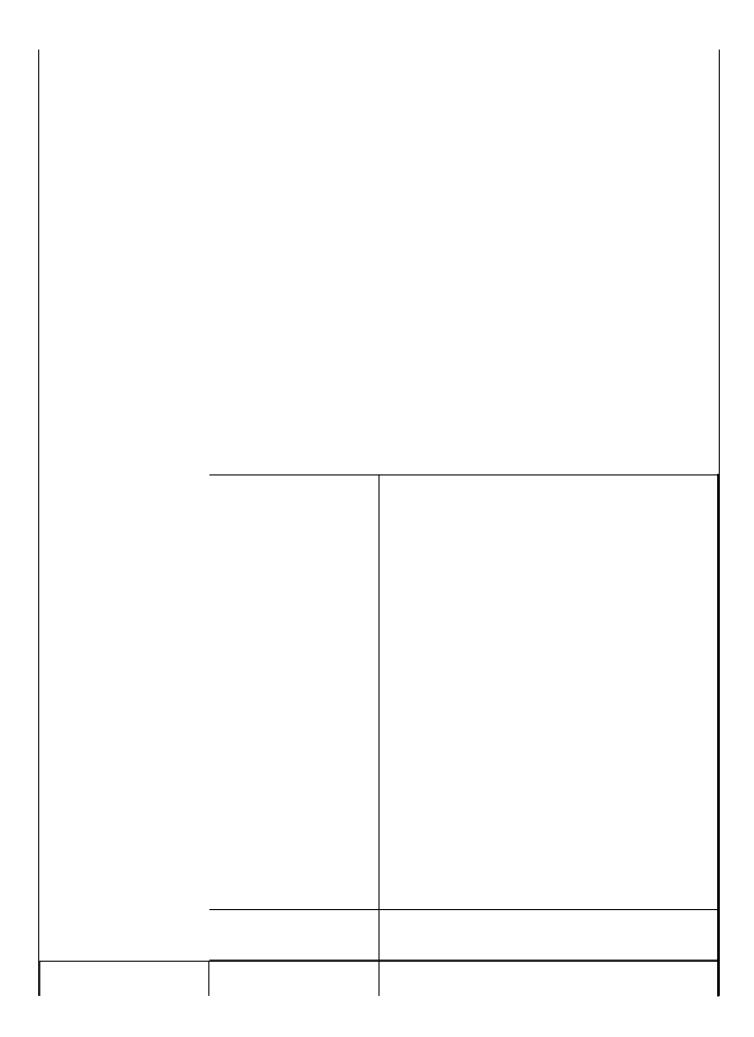
 $\hbox{Ghi XYbhg'UfY'ghfcb[`m'YbWc'i fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk'``YWhi fY'fYWc'fX]b[g''' } \\$

7ci fgY'GWXYXi `Y

J]Yk 'WUgg'ha YhUV'Y

H]a YtUV`Y

8UhY	HmdY	7 c bhYbh
8UhY K YY_'% '% 'GYdhYa VYf !'%+'GYdhYa VYf	Hmd Y K YV	7cbhybh K 99? '% '7CI FG9' BHFC8I 7H CB. 'GC7-5@ A 98-5 D@5H: CFA GZDF57H=79GZ5B8 5: : CF85B79G K Y'VY[]b'h]g'Wci fgY'VmYI d'cf]b['gcW]U'a YX]U h\fci [\'U'W]h]W'\]ghcf]WU''YbgZVcbg]XYf]b['\ck K YY_'%'% 'GYdhYa VYf



%) CWcVYf	5 HH9BH€B'97 CBCA ⇒G H\]g'k YY_'k Y'Wcbg]XYf'\ck ']a dcftUbhX][]tU` UthYbh]cb'\Ug'VYWca Y'Ug'U'gcW]U`fYgci fW'''K Y
	k]```YId`cfY`h\Y`dfUWf]WYg`cZgcW]U``aYX]U]bZiYbWYfgž`UbX`h\Y`WcbWYdhg`cZa]WfcWY`YVf]hm UbX`dYfgcbU``VfUbX]b["
	B6.': f]XUm']g'U'di V`]W\c`]XUm
	?YmWcbWYdhg∴ ∍ZiYbWYfgža]WfcWY`YVf]hmž

1		FYUX]b[.
		A]`fbYfž?"'A "ž/ '<][\Z]Y`Xž'H"'f&\$%+Ł"'BYj Yf; cbbU'; ≑ 'Mci 'I d.'5bU'mn]b['fh\Y'7i 'fi fU' G][b]Z]WJbW'cZfh\Y'5b]a Uff\X'; ≑ "' `TZR]? VUZRŁ `TZVġł%f1 Łž'%l%%"
		Gruf_ž@ _Yž'UbX'? UrY'7 fUk ZcfX"'&\$%) "'"H\Y 7 cbgYfj Urjga 'cZ9a c1; 'K cf_ž'5 ZZYWrž'UbX 7 ca a i b]WUrjcb""` `TZR]? VUZR Ł `TZV-g <`fc_R] %f&Ł"
	K YV	Hi hcf]U``hcd]W'9I d`cf]b['\ck 'dYcd`Y'VcbX'cb`]bY i g]b['fYgci fWg'gi W.'Ug'Ya c']"
K YY_'%\$.'%) 'Bcj Ya VYf !'% 'Bcj Ya VYf	K YV	K 99? '%\$. 'BH9FB9H'A 9A 9G'5B8 D5FH=7 D5HCFMDI 6@=7G
		#bhYfbYha Ya Ygʻ\Uj YʻVYWca YʻUʻWca a cbʻZYUhi fY]bʻdi V`]WX]gWci fgYʻUVci hdcʻ]hJWgʻʻH\]gʻk YY_ʻk Y Wcbg]XYfʻa Ya YgʻZfca ʻUʻa i ʻhJa cXUʻdYfgdYWfJj Yž `cc_]b[ʻUh\ck 'h\YmZ bWf]cbʻUgʻUʻgYa]chJW fYgci fWYʻZcfʻgcWJUʻVcbX]b[ʻUbXʻgcWJUʻ Wca a YbhUfmʻ
		? YmWcbWrdhg. '±bhYfbYha Ya Ygž']a U[Y'a UWfcgž d\fUgU`'hYa d`UhY'a Ya Ygž'UggYa V`U[Ygž a i `h]a cXU`]hmž'dc`]h]WU`'X]gWci fgY
		FYUX]b[.
		Fcggž·5"·G"ž/·F]j Yfgž·8"·>"·f&\$%+Ł"·8][]hU` W`hi fYgʻcZdc`]hJWU`dUfhJWJdUhJcb.:\bhYfbYhia Ya Yg UbX`h\Y`X]gW fg]j Y`XY`Y[]hJa]nUhJcb`cZh\Y`&\$%* I G`DfYg]XYbhJU``WUbX]XUhYg"·6 ZdT`f \aV\5`_\b\i e` ? VUZRž·%* ž·%I %%"
		HYI Mcc_fYUX]b[.
		7\UdhYf`- 'fi̯H]bmHfi a d. '∌hYfa cXU`'7 ci d`]b['UbX J]gi U`' < Ug\hU['A Ya Ygfi]b ']b 'NUddUj][bU f&\$% Ł" VRcTYRS]V R] : RdYeRXd R_U `Tæ] ? VUæ? VeRUæT` f cdVž'@cbXcb. '6`cca gVi fm'
	Hi hcf]U`	Hi hcf]U``hcd]W%\$. '9I d`cf]b[']bhYfbYha Ya Yg`UbX dUfh]W]dUhcfmdi V`]Wg"

FYgci fW/g

DfYgWf]VYX F Ygci fWYg

 $H\g`WcifgY`igYg`U`hYIhVcc_R_U`ch\Yf`fYUX]b[g"$

H\Y'hYIhVcc_']g.

 $\label{eq:local_bound} $$ NUddUj][bUZA "`f&$% L''`GYUfW`UV`Y`HU`_. `< Ug\HU[g`UbX`GcW]U``A YX]U`A YHX]gWci fgY"`@cbXcb. 6`cca gVi fm'$

#i]g'Uj U]`UV`Y']b'h\Y'I BGK 'Vcc_g\cd'UbX'h\Y'`]VfUfm'

5```ch\Yf`fYUX]b[g`WUb`VY`UWWggYX`j]U`h\Y`@Y[Ubhc`gmghYa``cb`AccX`Y"

FYWca a YbXYX FYgci fWyg

5bmZ fh\Yf'fYgci fWyg'k]``VY'dfcj]XYX`]b'h\Y'k YY_'Vmk YY_'a cXi 'Yg'cb'A ccX`Y"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\]g`Wci fgY`i bXYf[cYg`Wcbh]bi U``XYj Y`cda Ybhžj]U`A m9I dYf]YbW'`ZYYXVUW_`UbX`Wc``Y[]U``fYj]Yk "`K Y h\YfYZcfY`hU_Y`mci f`ZYYXVUW_j YfmigYf]ci g`m'`H\YfY`k]```U`gc`VY`cddcfhi b]h]Yg`]b`h\Y`hi hcf]U`g`Xi f]b[gYa YghYf`Zcf`mci `hc`X]gW gg`mci f`YI dYf]YbW'g`cZh\Y`Wci fgY"`H\Y`ZYYXVUW_`ZYUhi fY`cb`A ccX`Y`]g`U`gc Uj U]`UV`Y`Zcf`mci `hc`i gY`h\fci [\ci hh\Y`Wci fgY`a cXi `Yg"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

; YbYfU``mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\W)``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g''

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=iWUbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'i

5 WUXYa]W±bZcfa Uhjcb