# 7ci fgY'Cj Yfj ]Yk

#### GHJZZ7cbHJWh8YHJ]`g`

#### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWJhjcb	D\cbY
F YVYVWU 7 c ``]Y	fYVYVWW"Wc``]Y4 i bgk "YXi "Ui	-	Fa'% %Z'>c\b ; ccXgY`` 6i]`X]b[	\$&`-' , ) -' %+

#### GWkcc~7cbhJWh~abZcfa Uhlcb

GWXcc``cZ9Xi WUhjcb 5fhg`UbX`GcWjU``GWjYbWYg

@Y|Y``%ž'>c\b'; ccXgY```6|]`X]b[ 'f! &\$Ł

H. Ž\*%f&Ł'-',) '%++

9. YXi WUhcb4 i bgk "YXi "Ui

K . YXi WUNcb"Ufhg"i bak "YXi "Ui

#### 5 hryb XUb WY F Yei ] fYa Ybhg

#### 5XX]h]cbU``GWXcc``cZ'9Xi WUh]cb'UhhYbXUbWY`fYei ]fYa Ybhq

#i]gYl dYWYX'h\UhUbmghi XYbhYbfc``YX']b'Ub'98GH'Wci fgY'k]h\'h\Y'GW\cc``cZ9Xi WUhjcb'k]``a U\_Y'U Wca a ]ha Ybhhc'h\Y]f``YUfb]b['UbX'UhYbX'U```WUggYg']b'Z ```k\YfY'WcbhYbh]g'XY`]j YfYX']b'U'ZJ\W!hc!ZJ\W'a cXY'Yl Wrdh]b'WfW]a ghUbWrg'k\YfY'UVgYbW']g'Xi Y'hc']``bYggža ]gUXj Ybhi fY'cf'i bZcfYgYYb WfW a ghUbWrg'VYmcbX'h\Y'ghi XYbhg'Wcbhfc`"'5'a ]b]a i a 'UhhYbXUbW'fYei ]fYa YbhcZ, \$i ']g'fYei ]fYX'Zcf YUW. 'Wci fgY''' \delta \windfallo WfWa a ghUbWrgžU'ghi XYbha Uma ]gg'i d'hc'&\$i 'cZU'Wci fgY'k]h\ci h\Zcfa U` Udd`]\WUhjcb'UbX'i d'hc'' 'i 'cZU'Wci fgY'k]h\'Zcfa U``XcW a YbhUhjcb'fgYbhj ]U'Ya Uj`'hc'Wci fgY'Wcbj Ybcfb dfcj ]X]b['U``h\Y'fYei ]fYa Ybhg'XYhU]'YX'VY`ck 'UfY'a Yh'''Ghi XYbhg'bcha YYhjb['h\Y'UhhYbXUbW'fYei ]fYa Ybhg'ci h]bYX'UVcj Y'k]``VY'Uk UfXYX'Ub'I bgUhjgZJ\Wcfm: U]`fI : b'Zcf'h\UhWci fgY'fY[UfX`Ygg'cZh\Y]f'dYfZcfa UbW']b'h\Y'UgqYgga YbhhUg\_q'cf'ch\Yf'fYei ]fYa Ybhg'Zcf'h\Y'Wci fqY''

: cfiZ fh\Yfi]bZcfa Uh]cbicbi9Xi WUh]cbiWci fgYiUhhYbXUbWifYei ]fYa YbhgžidiYUgYifYZYfihcih\YifYgdYWf]j Y 98GHiA ccXiYia cXi `YfgŁ"

### 5 WUXYa ]W = 1 Zcfa Uh]cb

: cf`YggYbh]U``ghi XYbh`]bZcfa Uh]cb`fY`Uh]b[`hc.`fYei Yghg`Zcf`Yl hYbg]cb/fYj ]Yk `cZa Uf\_g/cWW dUh]cbU``\YU`h\UbX`gUZ/hm'Yl Ua ]bUh]cb`dfcWXi fYg/gdYW]U``Wcbg]XYfUh]cb`]b`h\Y`Yj YbhcZ]``bYgg`cf`a ]gUXj Ybhi fY/ghi XYbhYei ]hmUbX`X]gUV]`]hm'UbX`ch\Yf`YggYbh]U``UWUXYa ]W]bZcfa Uh]cbž qYY`\hdg.#k k k "Ufhq"i bak "YXi "Ui #W ffYbhqhi XYbhq#UWUXYa ]W]bZcfa Uh]cb#dfchcWc`a![i ]XY`]bYq#

# 7ci fgY 8YHJ] g

7 fYX]hDc]bhg\*\*

K cf\_`cUX

%) \$`\ci fg`]bWi X]b[ 'WUgg'WcbHUWi\ci fgž'fYUX]b[ gž'WUgg'dfYdUfUh]cbž'UggYgga Ybhž'Zc``ck 'i d'UWhj ]h]Ygž YhW

### Giaa UfmcZh\Y 7ci fgY

#b 'h ]g'Wti fgYž'mci 'k ]````YUfb'UVci hih Y ]a dcfHubW' cZfYg]`]YbW' UbX'k Y``VY]b[ 'Ua cb[ 'YXi WJhcfg"'K ]h Zci bXUh]cbg']b 'h YcfmUbX'Ya d]f]WJ 'fYgYUfW Z'h ]g'Wti fgY']g'U'gc'\][\`mdfUW]WJ'"'Mci 'k ]``'UWei ]fY \_bck `YX[Y'cZYj ]XYbW!VUgYX'gHUhY[ ]Yg'Zcf'a U]bHJb]b[ 'UbX'dfca ch]b[ 'mci f'ck b'k Y``VY]b[ ž'Ug'k Y``'Ug fYgci fW'g'UbX'gHfUhY[ ]Yg'fY'Yj Ubh'hc gW.cc```YUXYfg'fY"[ "'df]bW]dU'gž'\YUX'hYUW.YfgŁ'U]a ]b[ 'hc'dfca ch'V VfcUXYf'W.Ub[ Y"'Mci 'k ]```YUfb'UVci hh\Y']a dcfhUbW'cZYXi WJhcf'k Y``VY]b[ 'UbX'fYg]`]YbW'Zcf'h\f]j ]b[ ghi XYbhgž'hYUW.YfgŁ'UbX'gW.cc`gž'Ug'k Y```Ug']ggi Yg'gdYW]ZJWhc'W'fhJ]b 'hmdYg'cZYXi WJhcfg'fY"[ "ž'df]bW]dU`gž YUf'mWJfYYf'hYUW.YfgŁ"``

# 5i ghfu`]Ub'DfcZYgg]cbU`; fUXi UhY'HYUWX]b['GhJbXUfXg

GMbXUfX	8YgWf]dh]cb
* "%"	5bU`mgY`h\Y`5i ghfU`]Ub`DfcZYgg]cbU``GhUbXUfXg`Zcf`HYUW\Yfg`hc`d`Ub`dYfgcbU` dfcZYgg]cbU``XYjY`cda Ybh[ cU`gž'gi ddcfhWc``YU[ i Yg`hc`]XYbh]ZmUbX`UW\]YjY`dYfgcbU` XYjY`cda Ybh[ cU`g`UbX`dfY!gYfj ]WY`hYUW\Yfg`hc`]a dfcjY`WUggfcca `dfUWf)WY"
*"("(	5Xj cWUHYždUfhjWjdUHY`]b`UbX``YUX`gHfUHY[]Yg`hc`gi ddcfh\][\!ei U`]hmdfcZYgg]cbU` `YUfb]b[`cddcfhi b]hjYg`Zcf`Wc``YU[i Yg`h\Uh'ZcW g`cb`]a dfcj YX`ghi XYbh`YUfb]b["

# 5ggYgga Ybh

## 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'HJg_	K Y][ \h	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX	5 +HG@'DfcZYgg]cbU`; fUXi UhY HYUWX]b[ 'GHJbXUfXg'5ggYggYX	8iY 8UhY
K Y``VY]b[ 'UbX fYg]`]YbWY'dYfgcbU` fYZYW¶cb'	&\$I	%	*"%"	&+#\$, #& \$% \$).\$\$ DA
K_Y``VY]b[_'UbX fYg]`]YbWY'd`Ub'	(\$1	&Ž	*"("(	&&#%\$#& \$% \$).\$\$ DA</td></tr><tr><td>9ggUm</td><td>(\$1</td><td>%ž&</td><td>* "%"</td><td>\$) #%#& \$% \$) .\$\$ DA</td></tr></tbody></table>

 $\label{eq:continuous} D`YUgY`fYZYf`hc`3gefcS`fS`BcaXWee[a`S^EfS`VSoVe~XadFWSUZWcb`cb`h\Y`dfYj~]ci~g`dU[~Y].$ 

## 5ggYgga Ybh'8YHJ]`g

 $5\,ggYgga\ Ybh\%\ 'K\ Y``VY]b[\ 'UbX'fYg]`]YbWY'dYfgcbU``fYZYW¶cb$ 

GhUfhXUhY. Bch5dd`]WUV`Y

**8 YHJ]`g.** 'HUg\_'%! '9ggUm!'% \$\$`k cfXgGhi XYbhg`k]```fYWY]j Y`k f]hhYb`ZYYXVUW\_'k]h\]b`%\$`Vi g]bYgg`XUmg`cZgi Va]gg]cb

#### 5XX]h]cbU`XYhU]`g.

BSdf#, 6WeUd[bf[a`,

- H\]b\_'UVci hUb'YI dYf]YbW'mci '\Uj Y'\UX']b'hYUW\]b['k\YfY'mci f'k Y``VY]b[#Yg]']YbW'k Ug gi ddcfhYX'UbX'Ubch\Yf'k\\YfY'mci f'k Y``VY]b[#Yg]']YbW'k Ug'bchgi ddcfhYX''
- 8 YgWf]VY 'h\YgY 'lk c 'YI dYf]YbWrg 'fUfci bX'&\$\$ 'k cfXg 'YUW\Ł" 'H\YgY 'WUb 'VY 'cbY!cZYI dYf]YbWrg 'cf cb[c]b['cbYg"

#### BSdf \$, >[] e i [fZ >[fWdSfgdW]

7 cbg]XYf'h\Y'Zc``ck ]b[ 'ei Ygh]cbg'UbX'a U\_Y'Ui h\Ybh]W']b\_g'hc'h\Y'\_bck `YX[Y'mci '\Uj Y'[U]bYX Zfca 'h\Y'Ugg][ bYX'fYUX]b[ g"

 $\bullet \ \, \text{Hc'k \setminus Uh'YI hYbh'\setminus Uj Y'mci f'dYfWYdh]cbg'cZh\Y'YI dYf]YbWYg'W\Ub[YX'UZhYf'h\Y'fYUX]b[g3] }$ 

BSdf% 5a\_bSd(`Y7jbWd(W)UWe

BSdf\$z7\ccgY`cbY`UfYU`]b`k\]W\`mci f`kY``VY]b[#Yg]`]YbW\`Wci `X`VY`VYfhYf`gi ddcfhYX`fY"["žkcf\_`cUXž]bhYfUW]cbg`k]h\ gh XYbhg`cf`h\Y]f`dUfYbhgŁ"`\\Ybh]ZmgdYW]ZWdfUW]W\g\#JfUhY[]Yg`mci `Wci `X`di h]b`d`UW\`hcVYfhYf`gi ddcfhmci f`kY``VY]b[`UbX`fYg]`]YbW\"`AU\_Y``]b\_g`hc`h\Ycf]Yg\#\cbg\fi Wg\#JfUW]W\g`kY`\Uj Y\\circ \fi\Ycf]Yg\#\cbg\fi Wg\#JfUW]\Wg`kY`\Uj Y\\circ \fi\Yof]Yg\fi \fi\Y`Zc``ck]b[`ei Ygf]cbg.

- %" K \UhiUfY'h\Y'[ cU`g`cZh\Y'd`Ub3'K \m]g'h\]g']a dcffUbh'hc'UXXfYgg3
- &" K \UhWcbglfi Wfg#\Ycf]Yg`Wcj YfYX`]b`h\Y`Wci fgY`UfY`fY`Yj UbhifUfci bX`&!' /a U\_Y`gi fY`mci `XYZ]bY \_YmihYfa q`Vf]YZmL3`K \m3
- ' "K\UngdYWJZWUWJcbg'WJb'mci i bXYfhU\_Y'fYei]fYX'hc'a YYhih\Y'[cU'g'ffY`Yj Ubh'hc'fYgci fW/gž'h]a Yž UWJj]hJYgžYhWL3
- ("5fY'h\YfY'Ubmgiddcfhg'bYYXYX'hc'\Y'd'mcigiWWyggZ\`mibXYfhU\_Y'h\YgY'UWnjcbg'fZfca'mcif kcf\_d`UWYžZJa]`mžZf]YbXg'YhWt3
- ) " K \UhVXU``Yb[Yg'UfY'h\YfY'hc']a d`Ya Ybh]b['h\YgY'UWn]cbg3'<ck'\WUb'h\YgY'VY'UXXfYggYX3
- \*" K \UhYj ]XYbWY!VUgYX`ci hWca Yg`UfY`UggcW]UhYX`k ]h\`h\YgY`hmdYg`cZUWf]cbg`fY"[ "ž´cV`gUh]gZUWf]cbž gh`XYbh`UW\]Yj Ya Ybhž`ck Yf`UVgYbhYY]ga Ł3
- +" < ck 'Wci 'X'mci 'ZYUg]V'miUbX'dfUWfWU'`miUggYgg'th\Y'YZZYWfj YbYgg'cZth\Y'd'Ub3'K \Uhk ci 'X'mci 'VY

 $\label{thm:linear} $$ Mci_k = Mci_k$ 

% 7 fYUHY 'Ub 'UbbcHJHYX 'V]V`]c[fUd\m

# I BGK 'G7<CC@'C: '98I 75H€B'FI 6F = 7 # 998657? 'G<99H' 98GH)' %(

 $5\,ggYgga\ Ybh'HUg\_.\ 'K\ Y``VY]b[\ 'UbX'fYg]`]YbWY'dYfgcbU``fYZYWI]cb$ 

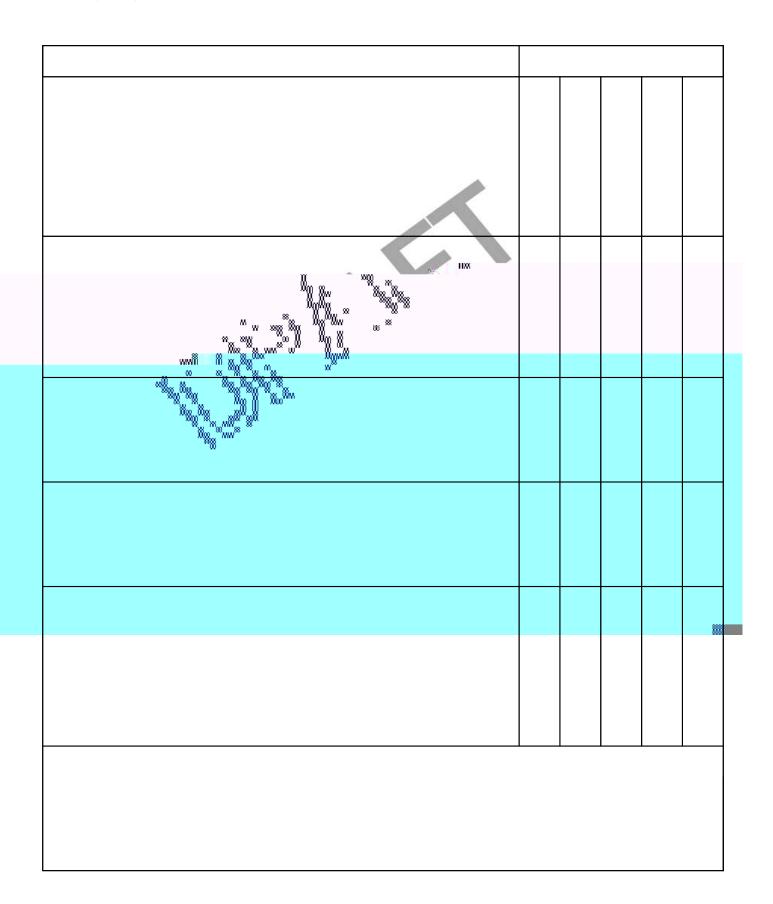
GdYWJZJW7f]hYf]U	f]Ł—		 2fŽŁ
I_bXYfgHJbX`cZ`h\Y`ei_Ygh]cb`cf`]ggi_Y`UbX`h\Y`_YmWcbWfdhg ]bj_c`j_YX			
<ul> <li>FYZYWJcbicbik cidf]cfiYl dYf]YbWgicZk Y``VY]b[i'UbXifYg]`]YbW</li> <li>7`YUf``]b_gihcimci fidf]cfiYl dYf]YbWgi]biDUfhgi%liicZh\Y 5ggYgga YbhHUg_</li> </ul>			
3Ydh\cZUbUmg]g'UbX#cf'Wf]heiY']b'fYgdcbgY'hc'h\Y'hUg_			
<ul> <li>Gcd\]gh]WUhYX'UbX']b!XYdh\rfYZYWf]cb'cb'df]cf'YI dYf]YbWYg</li> <li>         ⇒XYbh]Z]WUh]cb'cZgi ddcfhg'h\Uhk YfY'dfYgYbhcf''UW_]b[']b'h\Y'hk c</li> </ul>			

GdYWJZJW7 f]hYf]U	··flt2fŽt
GdYWJZJW7fJhYfJU	"flt2flt

@YWhi fYf

``B6.`h\]g`h]W\_g`]b`h\Y`j Uf]ci g`Vcl Yg`UbX`XYg][ bYX`hc`dfcj ]XY`ZYYXVUW\_`hc`gh XYbhg/h\YmUfY`bch`[ ]j YbYei U``k Y][ \h]b`XYhYfa ]b]b[ `h\Y`fYWca a YbXYX`[ fUXY"`8YdYbX]b[ `cb`h\Y`bUh fY`cZh\Y`UggYgga Ybh'hUg\_ž

I BGK 'G7<CC@'C: '98I 75H€B'FI 6F = 7 # 998657? 'G<99H' 98GH)' %(

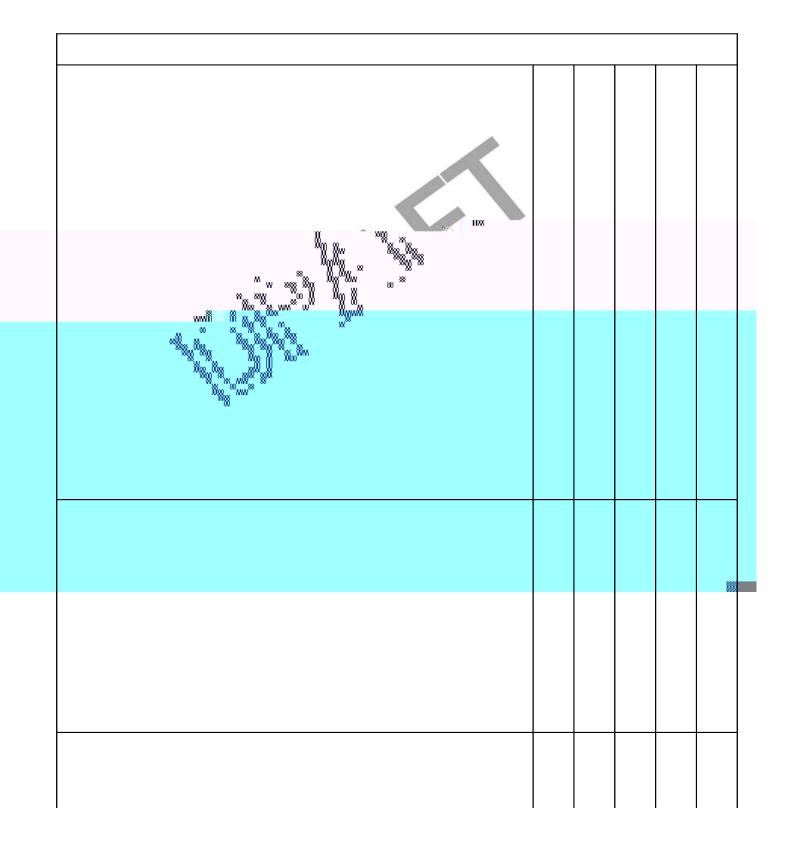


GdYW <mark>]Z</mark> ]W7 f]hYf]U	"flt2flt

@YWhifYf

 $FYWca \ a \ YbXYX. \qquad ```#&\$fl @`DG`7F`8B`<8Ł` \qquad \qquad k \ Y][\ \ h]b[\ \ . ``&\$i$ 

"B6."h\]g'h\W\_g']b'h\Y'j Uf]ci g'Vcl Yg'UbX'XYg][ bYX'hc'dfcj ]XY'ZYYXVUW\_'hc'ghi XYbhg/h\YmUfY'bch[]j Yb
Yei W"k\Y][\\h\h]b\X\\h\fatab]b[bh\Y'\\tau\]a YbXYXb[f\tau\X''\8YdYbX]b[ 'cb'h\Y'bUhi fY'cZh\Y'UggYgga YbhhUg\_ž b]b[ 'h\
'Y\Wi fYfg'a UmU'gc'\\cbh\I hi U`]nY'UbX\\chi Tua YbX'h\YgY'gdY\\J]\\Wf]l\Yf]U"'\\h\Y'fY\\can a YbXYX\[f\UXY']g
h\Ybh\Jh]j Y'cb`mžgi Va \( Y\Whic'gh\DX\UfX]gUh]cb'dfc\W'ggYg'UbX'Uddfcj U`'Vmh\Y'G\X\cc`'cZ'9Xi \WJh]cb



GdYWJZJW7 f]HYf]U	¨flŁ—		 2fŽŁ

lfb]b[ 'UbX'HYUV\\]b	<u>[ '7 ca a ]hhYY"</u>			

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

Gh XYbhgʻUfYʻYl dYWYXʻhcʻdi hh\Y]fʻbUa YgʻUbXʻgh XYbhbi a VYfgʻcbʻYj YfmdU[YʻcZh\Y]fʻUgg][ ba Ybhgʻ Hi fb]h]bʻGi Va ]gg]cb 7j S\_ b^W`5 'ghi XYbhigi Va ]hg`Ub`YggUm(, `\ci fg`UbX`%\$`a ]bi hYg`UZhYf`h\Y`gh]di `UhYX`XYUX`]bY"'H\Y`YggUm ]g`a Uf\_YX`ci hcZ%\$\$i "'H\Y`YggUmfYWY]j Yg`U`a Uf\_`cZ\*, "'H\Y`ghi XYbhg`a Uf\_`]g`h\YfYZcfY`\*, '-'%) `1`)'

- K cf\_'gi Va ]hh/X'%\hc'% 'XUmg'UZh/f'h\Y'gh]di `Uh/X'XYUX`]bY'k ]``'VY'UggYggYX'UbX'ZYYXVUW\_dfcj ]XYX'Vi hU'a Uf\_'cZnYfc'k ]``'VY'fYWcfXYX"'=Zh\Y'k cf\_'k ci `X'\Uj Y'fYWf]j YX'U'dUgg'a Uf\_'Vi hZcf h\Y'`UhYbYgg'UbX'h\Y'k cf\_']g'U'Wca di `gcfm'Wci fgY'Wca dcbYbhf\i fX`Y'fYei ]fYa Ybh\ZU'gh XYbhk ]``VY'XYYa YX'hc'\Uj Y'a Yhh\UhfYei ]fYa Ybh/
- K cf\_'gi Va ]httX:&\$'cf'a cfY'XUmg'UZhYf'h\Y'gh]di `UhYX'XYUX`]bY'k ]``bchVY'UWWdhYX'Zcf UggYgga YbhUbX'k ]``fYWY]j Y'bc'ZYYXVUW\_Z'a Uf\_'cf'[ fUXY"'=Zh\Y'UggYgga YbhHUg\_']g'U'Wca di `gcfm Wca dcbYbhcZh\Y'Wci fgY'U'gh XYbhk ]``fYWY]j Y'Ub'l bgUh]gZJWcfm: U]`fl : Ł'[ fUXY'Ug'U'fYgi `hcZ i bgUh]qZJWcfmdYfZcfa UbWY']b'Ub'YggYbh]U`'Wca dcbYbhcZh\Y'Wci fgY"

H\]q^]bZcfa Uhcb^]q^U`qc^Uj U]`UV`Y^Uh

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWUXYa \\W\]bZcfa Uhcb#dfchcWc`g![i \\XY`]bYg#

#### GdYWJU`7cbg]XYfUhJcb5dd`]WUhJcbg

 $\label{eq:linear_continuity} $$Mc_i = W_0 - W_$ 

G | W b Y g g ž'a | g U X j Y b h f Y 'c f 'c h \ Y f ' W f W a g h U b W g 'V Y m c b X 'm c i f 'W c b h f c ``a U m

ł DfYj Ybhmci Zfca Wca d`Yhjb[ 'U'Wci fgY'fYei ]fYa Ybhž

ł? YYd mci Zfca 'UthYbX]b[ 'Ub 'UggYggUV'Y 'UW] j ]tmž

ł Ghod moi gi Va ]hh]b[ 'UggYggUV'Y'k cf\_ Zcf'U'Wci fqYž

ł G][b]Zwubhmiuzzywhrci fidyfzcfa ubwhi]biuggygguviyik cf\_žvyi]hiuizcfa uniybx!czlgya yghyfiyl ua ]buh]cbžu wuggihyghžuniuvcfuhcfmhyghžuigya ]bufidfygybhuh]cbicfiubmich\yfizcfa iczluggygga Ybh'

# F Ygci fWyg

DfYgWf]VYX F Ygci fWYg

GYY'AccX`Y'g]hY'Zcf'fYei]fYX'fYUX]b[g"

FYWca a YbXYX FYgci fW/g

 $GYY^*A\ ccX^*Y^*g]hY^*Zcf^*fYgci\ fWYg^*UbX^*Z\ ffh Yf^*fYUX]b[\ g'''$ 

# 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

: cfa U`ghi XYbhZYXVUW\_`]g`cVhU]bYX`Uhh\Y`YbX`cZh\Y`Wci fgY`j ]U`A m9I dYf]YbWY`UbX`h\]g`]g`i gYX`hc a U\_Y`W\Ub[ Yg`hc`h\Y`Wci fgY`]b`h\Y`Zc``ck ]b[ `mYUf"`+b`UXX]h]cbž]bZcfa U``ghi XYbhZYXVUW\_`]g`Wc``YWMYX a ]Xk Umh\fci [\`h\Y`Wci fgY": cf`h\]gžghi XYbhg`UfY`Ug\_YX`hc`]bX]WUhY`W\Ub[ Yg`h\Uhh\Ymk ci `X``]\_Y`hc`gYY ]a d`Ya YbhYX`]b`h\Y`Wci fgY`f][ \h'Uk Umž`Ug`k Y```Ug`W\Ub[ Yg`hc`VY`]a d`Ya YbhYX`h\Y`bYI hh]a Y`h\Y`Wci fgY`]g fi b"`H\YgY`Wca a Ybhg`UfY`Wc``UhYX`UbX`a U^cf`h\Ya Yg`UfY`]XYbh]Z]YX"`H\YgY`W\Ub[ Yg`UfY`h\Yb`g\UfYX`k ]h\ghi XYbhg`]b`WUgg`UbXžk \YfY`dcgg]V`Yž`]a d`Ya YbhYX`]a a YX]UhY`m'`: cf`YI Ua d`Yždf]cf`]bZcfa U``ZYYXVUW\_