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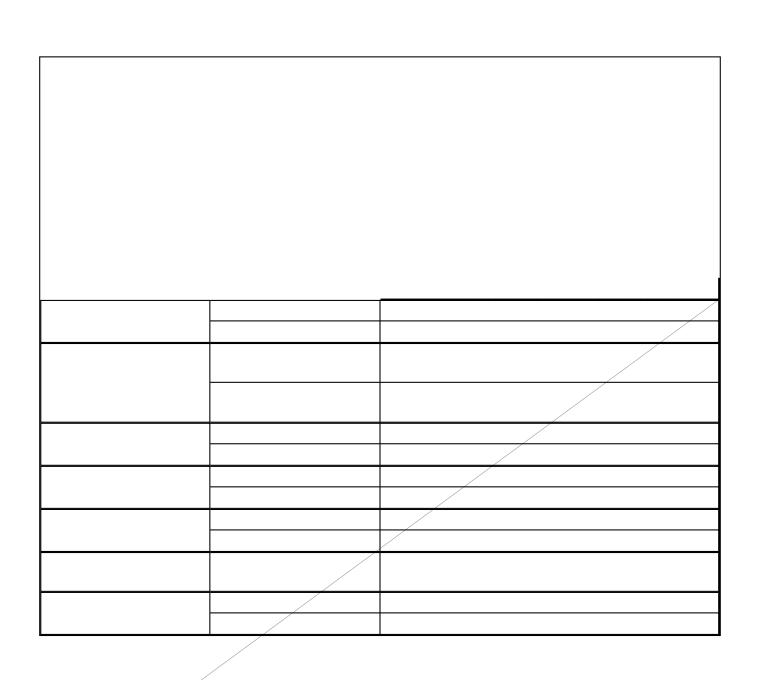
8 YHJ]`g. '(\$`a]b`hYghcb`hcd]\Wg`\Wcj YfYX`]b`h\Y``Y\Wi fYg`UbX`hi hcf]U`g"`\Mci `k]```fY\W]j Y`Z\YX\VU_`]bX]j]Xi U``m\UbX`h\fci [\`\WUgg`X]g\W gg]cb"

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