

ARTS3283

F ca Ub '9a dYfcfg.': fca '5i [i ghi g'hc'BYfc

HYfa 'C bY''##'&\$%

7ci fgY 8YHJ] g

7 fYX]h'Dc]bhg'*

Gi a a UfmcZ'h\Y'7ci fgY

Gi VYWh5fYU.: [efack

 $\label{thm:linear} $$HY'Wc'i fgY'YI d`cfYg'thY'\ghcfmcZF ca Yfg'ZfghXmbUghmUbX'\ck ']ha UbU[YX'thY'hfUbg]hcb'Zfca 'fYdi V']Whc'a cbUfWkm' FZWDa_S` 7_bWdade$

5ggYgga Ybh

H\]g`Wci fgY`dfcj]XYg`U`i b]ei Y`cddcfhi b]mm'c`]a a YfgY`cbYgY`Z]bhc`h\Y`k cf`X`cZh\Y`7 UYgUfg"'Ghi XYbhg UfY`YbWci fU[YX`hc`XY`j Y`]bhc`h\Y``]j Yg`cZ]bX]j]Xi U``Ya dYfcfg`UbX`]a dYf]U``k ca Ybž'k \]`gh'Uhih\Y`gUa Y ha Y`fY`Uh]b[`h\Y]f`]bhja UhY`ghcf]Yg`hc`h\Y`W\Ub[]b[`bUhi fY`cZh\Y`F ca Ub`dck Yf`ghfi Whi fY"'Hc`\Y`d`i g`UhtU]b U`gYbgY`cZk \Uh``]ZY`k Ug``]_Y`]b`h\Y`]a dYf]U``F ca Ub`ZJa]`mž'k Y`k]```k UhW\`UbX`UbU`mgY`gY`YWf]cbg`cZ hY`Yj]g]cb`UbX`Z]`a `XfUa Ug`UVci hh\Y`>i `]c!7`Ui X]Ubgž'dUfh]W``Uf`mh\Y`667`hY`Yj]g]cb`UXUdhUh]cb`cZF cVYfh; fUj Ygfif\\T``\Ui X]i gf'i5h`YUghcbY`cZh\Y`ei Yghjcbg`]b`h\Y`WUgg`hYghik]``VY`cb`f\\T``Ui X]i gf'i

5ggYgga Ybh'HUg_g'

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX

GhUfhXUhY. Bch5dd`]WUV`Y

8 YHJ]`g. '9 UWX 'ghi XYbhik]```Wa d`YhY`cbY`]b! WUgg`dfYgYbHJhJcb`cj Yf`h\Y`Wa fgY`cZh\Y`gYa YghYf"`H\Y dfYgYbHJhJcb`k]```VY`ghf]Wimbc``cb[Yf`h\Ub`%\$`a]bi hYg`]b``Yb[h\":H\Y`dfYgYbHJhJcb`k]```VY`UggYggYX`cb`h\Y ei U`]hmcZ]hg`WabhYbhz`UbX`h\Y`Ua ci bhcZfYgYUfW\`h\Uhk Ybh]bhc`]hg`dfYdUfUhJcb"`9 UWX`ghi XYbhik]```gi dd`m U`[i]XY`hc`h\Y`dfYgYbhJhJcbz`VYhk YYb`+) \$!%\$\$\$`k cfXgz`Zcf`YUWX`ZY``ck `ghi XYbh'`=hk]```ghJdi `UhY`h\Y`a U]b dc]bhg`UbX`]bWi XY`U`V]V`]c[fUd\m'K f]hhYb`ZYYXVUW_UbX`a Uf_k]```VY`dfcj]XYX"

5XX]h]cbU`XYhU]`g.

H\]g'UggYgga YbhWca Yg']b'lk c'dUffg"'H\Y'ZJfgh]g'h\Y'dfYgYbhUh]cb"'9UW\ghi XYbh]g'fYei]fYX'hc'[]j Y'Ub cfU`'dfYgYbhUh]cb"'Mci 'k]``'VY'Ug_YX'hc'bca]bUhY'mci f'h\fYY'ZUj ci f]hY'hcd]Wg'UbX'mci 'k]``'VY'h\Yb'V U``cWUhYX'cbY'cZh\Ya "'Mci f'dfYgYbhUh]cb'k]``'h\Yb'fYZYWhcb'cZh\Yei Ygh]cbg'cb'h\Y'hcd]Wdfcj]XYX'Vmh\Y Wcj Ybcf']b'hY\'h hcf]U``[i]XY"'Mci 'k]``'gdYU_Zcf') 'a]bi hYg'VYZcfY'h\Y'WUgg'UbX'Ubgk Yf'Ubmei Ygh]cbg Zca 'mci f'Ui X]YbW'''Mci 'k]``'cb'mVY'a Uf_YX'cb'gi VghUbW.'h\Uha YUbg'mci 'Xc'bchbYYX'hc'k cffmUVci h mci f'gdYU_]b['j c]W'žXY`]j Yfmcf'h\Yei U`]hmcZmci f'dck Yfdc]bhf]Zmci 'W\ccgY'hci gY'cbY\!"H\]g Ugg][ba Ybh]g'k cfh\ '&\$'ci hcZ' \$"

H\Y`gYWcbX`dUfh]g`U`\UbXci hih\UhfYei]fYg`mci 'hc`UddfU]gY`U`df]a UfmXcW a YbhfY"["'U'dUggU[Y`Zfca HUW]h g`cf`Gi Yhcb]i gž'Ub`Yd][fUd\Ł'h\UhfY`UhYg`hc`mci f`dfYgYbhUh]cb`hcd]W`H\]g`]g`k cfh\`%\$`ci hcZ' \$" A cfY`XYhU]``UVci hih\]g`Ugg][ba Ybhk]```VY`X]gW ggYX`]b`h\Y`Z[fghWUgg"

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FYgci fWyg

DfYgWf]VYX F Ygci fWYg

Gi Yhcb]i gžH\Y Hk Y`j Y 7 UYgUfgžDYb[i]b 6cc_q

HUWh gžH\Y 5bbU gžDYb[i]b 6cc_q

FYWca a YbXYX FYgci fW/g

< "< "'GW'``UfXž'<u>: fca 'h\Y': fUVW\]'hc'BYfc</u>ž'Fci hYX[Yž'UbmYX]h]cb

DYhYf; UfbgYm'UbX'F]W\UfX'GU``Yfž<u>H\Y'F ca Ub'9a d]fY</u>ž6`cca gVi fmž&bX'YX

A Uffijb; ccXa Ubž<u>H\YFca UbK cfX'(('67'!58'% \$</u>žFci hYX[Yž&bX'YX]hjcb

7 c`]b'K Y``qž'<u>H\Y'F ca Ub''9a d]fY</u>ž: cbfUbUž'&bX'YX]f]cb

Hca <c``UbXž<u>'8mbUghm</u>ž'8ciV`YXUm

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7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Ghi XYbhgʻk \c`k]g\ hc`[Yhgca YʻgYbgYʻcZ\ck 'h\YmUfYʻZJf]b[ʻ]bʻh\YʻWti fgYʻa][\hUg_'hc`XcʻcbYʻcZh\Y YUf`mdfYgYbhUh]cbʻhcd]WgʻZcfʻYUf`mZYYXVUW_"ʻ=Zmci 'WkccgY`cbY`cZh\Y``UhYʻhcd]WgʻVi hbYYXʻZYXVUW_'a i Wk YUf`]Yfž'd`YUgYʻgYYʻh\YʻWtbj Ybcf"`GYbX`\]a 'UʻgUa d`YʻcZUbmcZmci f`k f]hhYbʻk cf_'VYZcfY\UbX`UbX gWkYXi `Y`Ub`Uddc]bha YbhZcfʻqca YʻZYYXVUW_" Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

_Zmci YbWci bhYf'U'dfcV`Ya 'k \Yb'UhYa dhJb['hc'gi Va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]hJbž'd`YUgY
hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±iWUbihU_Yia Ubm Zcfa qžZfca iXYi]VYfUhYiW\YUh|b[ihciUVW]XYbhU`mWcdm]b[iZfca iUigci fWik]h\ci hiUW_bck iYX[Ya Ybh'i

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci žch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@&9'k]``\Y`d'mci 'a U_Y'h\Y'a cghcZmci f'gh X]Yg'UhI BGK "'\\hd.##qi V'YWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W = b Zcfa Uh cb.

: cf`YggYbhJU``gh XYbh`]bZcfa UhJcb`fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg`[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU`h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua]bUhcb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a]gUXj Ybhi fY/
- gh XYbhYei]lmUbX X]gUV]`]lm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa]W]bZcfa Uf]cb#dfchcWc`g![i]XY`]bYg#

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∍a U[Y'7fYX]h

GmbYf[]Yg^]b^Gci bX^&\$%*

7F = 7CG

7 F = 7 C G 'Dfcj]XYf'7 cXY. '\$\$\$-,;