

ARTS4247

 $F YgYUfWX G_]`g']b'hXY'< i a Ub]h]Yg$

HYfa 'C bY''##'&\$%

7ci fgY[·]Cj Yfj]Yk

GHJZZ:7cbHJWh:8YHJ]`g`

7 cdg jy V brogf

B Ua Y	9a U]`		

7ci fgY'8YW]`g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

H\]g`Wci fgY`k]``\Y`d`mci `hc`XYj Y`cd`U`fUb[Y`cZ\][\Yf!`Yj Y``fYgYUfW\`g_]``g"`\#ik]```Wcj Yf`gi W\`UfYUg`Ug fYgYUfW\`a Yh\cXc`c[mz̃fYgYUfW\`ei Ygh]cbgz`h\Y``]hYfUhi fY`fYj]Yk z`dfYgYbhUh]cb`g_]``gz`UbX`UWUXYa]W k f]h]b["'\#ik]``\Y`d`mci `hc`]XYbh]zmUbX`XYj Y`cd`mci f'fYgYUfW\`dfcdcgU`z`Uff]W``UhY`]hg`dc]bh'UbX`di fdcgYz`UbX`dfYgYbh]h]b`cfU``UbX`k f]hhYb`zcfa "'\#ik]``h\YfYVmgi ddcfhmci `]b`h\Y`g_]``g`mci `bYYX`hc`dfcXi W\`U gi ghU]bYX`fYgYUfW\`dfcYWigi W\`Ug`h\Y`<cbci fg`h\Yg]g"`H\Y`WcbhYbh]g`XY`]j YfYX`h\fci [\`gYa]bUfg`UbX k cf_g\cd"

7 ci fgY @YUfb]b[Ci hWca Yg

%" 9j U`i UhY h\Y`_Ymdf]bW]d`Yg`cZ\][\!ei U`]hmei U`]hmei U`]hmfYgYUfW\`]b`U`fY`Yj UbhX]gW]d`]bY &" @cWUhYž`UbU`mgY`UbX`Yj U`i UhY`fYgYUfW\`fY`Yj Ubh'hc`U`gdYV]Z]Whcd]W \" 7 cbghfi WhUbX`Uff]W\``UhY`U`WYUf`UbX`Wc\YfYbhfYgYUfW\`dfcdcgU` (" 5fh]W\`UhY`h\YYh\]WU``cV`][Uh]cbg`[YbYfUhYX`VmfYgYUfW\`]b`h\Y`\i a Ub]h]Yg

HYUWX]b[GlfUhY[]Yg

H\]g'Wci fgY'Wca df]gYg'U'nk c!\ci f'gYgg]cb'Zfca 'K YY_'%'nc'K YY_'-z'Zc``ck YX'VmU'g]l !\ci f'<cbci fg Gma dcg]i a ']b'K YY_'% "'H\YfY']g'bc'WUgg'Zfca 'K YY_'%'hc'K YY_''&z'k\]W\"UfY'Zcf'gY'Zlghi XmUbX]bX]j]Xi U`'Wcbgi `hUhjcbg'cb'UggYgga Ybng"'H\Y'nk c!\ci f'gYgg]cbg'UfY'XYg][bYX'nc'YbUV'Y'ghi XYbhg'nc 'YUfb'nk\Y'g_]``g'cZWcbW']j]b['UbX'a UbU[]b['U'fYgYUfWk'dfc'YWz']bWi X]b['`cWUhjb['nk\Y]f'fYgYUfWk' k]h\]b fY'Yj Ubn'UWXYa]W']hYfUhi fYz'a UbU[]b['nk\Y'fYgYUfWk' dfc'W'ggz'Wcbg]XYf]b['Uddfcdf]UhY'fYgYUfWk a YhkcXgz'UbX'Wca a i b]WUhjb['fYgYUfWk' YZZYWhj Y'm'H\Y'gYa]bUfg'k]``'Wcj Yf'ncd]Wg'gi Wk'Ug'nk\Y'`]hYfUhi fY fYj]Yk z'cb`]bY'XUhUVUgYgz'fYgYUfWk' a YhkcXc`c[mz'UbX'Ui hkcf]b['U'hkYg]g"'5hhkY'k cf_g\cdgz'ghi XYbhg'k]`` k cf_'k]h\'dYYfg'Zfca 'a i 'hjd'Y' X]gW]d']bYg'cb'gdYW]Z]WhUg_g'gi Wk'Ug'g\UfdYb]b['fYgYUfWk'ei Ygh]cbgz' VfU]bghcfa]b[']XYUgz'i bXYfghUbX]b['hkY]f'a YhkcXc`c[]Ygz'Z]bY'hi b]b['hkY]f'ha Y`]bYgz'UbX'dfYgYbh]b['U'Wf]hjei Y'cZWca d'YhYX'hkygyg'''''

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8 YHU]`g. Mci f`fYgYUfW. dfYgYbHUh]cb`k]```VY`%) a]bi hYg`]b`Xi fUh]cbž'UbX`k]``dfcj]XY`U`gi a a UfmcZmci f fYgYUfW. dfcdcgU`ž`Wcj Yf]b['h\Y`fYgYUfW. 'hcd]WžgHUhYa Ybhž'fUh]cbU`Yž``]hYfUhi fY`fYj]Yk ž`UbX`a Yh\cXc`c[m'\ #ik]```VY`dfYgYbhYX`]b`h\Y`Zcfa UhcZUb`UWUXYa]WWcbZYfYbWY`dUdYf"`Mci `k]```fYWY]j Y`cfU``ZYYXVUW_`Uhh\Y dfYgYbhUh]cb`UbX`k f]hhYb`ZYYXVUW_`UZhYfk UfXg"

5ggYgga Ybh& H\Yg]g Wf]hjei Y

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. %) \$\$ k cfXg

8 YHU]`g. '%) \$\$`k cfXg"`Mci f`HUg_`]g`hc`gi Va]hU`Wf]h]WU``Yj U`i Uh]cb`cZcbY`fYWYbh<cbci fg`h\Yg]g`]b`mci f X]gW]d`]bY"`Mci f`Wf]h]ei Y`g\ci `X`Wcbg]XYf`h\Y`Zc``ck]b[.`UE'H\Yg]g`hcd]\WVE'@]hYfUhi fY`fYj]Yk `WEH\Yg]g ghfi Whi fY`XE`5f[i a YbhYEE i U`]hmcZk f]h]b["`Mci `k]```\Uj Y`h\Y`cddcfhi b]hmhc`dfYgYbhU`g\cfhih\Yg]g`Wf]h]ei Y hc`mci f`Wc``YU[i Yg"`Mci `fYWY]j Y`k f]hhYb`ZYYXVUW_cb'h\Y`Wf]h]ei Y"

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5ggYgga Ybh' . F YgYUfWX 'dfcdcgU`

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. &ž) \$\$ k cfXg

8 YHU]`g. '&) \$\$`k cfXg`YI Wi X]b[`fYZYfYbWYg"`Mci f`fYgYUfWX`dfcdcgU``a i ghWcj Yf`h\Y`Zc``ck]b[.`UŁ`HYbHUh]j Y h]hY`cZh\Y`dfc^YWhVŁDfc^YWhgHUhYa YbhWŁ'F Uh]cbU`Y`XŁ'@]hYfUhi fY`fYj]Yk `YŁ'A Yh\cXc`c[miZ:HYbHUh]j Y WXUdhYf`ci h]bY`[Ł'H]a Y`]bY`\Ł'F YZYfYbWYg`]Ł'9h\]WU``]a d`]WUh]cbg"`Mci `k]```fYWY]j Y`k f]hhYb`ZYYXVUW_"`H\]g]g`h\Y`Z]bU``UggYgga YbhZcf`UhYbXUbWY`di fdcgYg"

Hi fb]hjb gYhjb[. H\]g Ugg][ba Ybh]g gi Va]hYX h\fci [\ Hi fb]hjb UbX gh XYbhg WUb gYY Hi fb]hjb g]a] Uf]hm

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7 ci fgY '9j U'i Uh]cb 'UbX '8Yj Y`cda Ybh

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hY`Yd\cbY'9| hYfbU``Gi ddcfhcb'-',) '' ' ' %cf'Ya U]`'h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-') .\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft'' =Zmci
UfY'i bUV`Y'hc'gi Va]hmci f'Ugg][ba YbhXi Y'hc'U'ZUi 'hk]h\'Hi fb]hJb'mci 'a Um'Udd'mZcf'Ub'Y! hYbg]cbžVi hmci
a i ghfYhU]b'mci f'hJW_Yhbi a VYf'Zfca '9! hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf'fY`Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW'hc'gi ddcfhmci f'Y! hYbg]cb'Udd`]WUhJcb'' =Zmci 'Ya U]`'9! hYfbU``Gi ddcfhmci 'k]``

5 WUXYa | W < cbYghmUbX D`U[| Uf]ga

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IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U`gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k\YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_]b['k]th 'cth Yfg'Vi hidUgg]b['cZZth Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'thc'Ubcth Yf'ghi XYbh'VYZcfY'th Y'Xi Y'XUHYZcf'Zcf'th Y'di fdcgY'cZth Ya

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

5 WUXYa]W±bZcfa Uhjcb

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