

ARTS1211

5i ghfu`]Ufg`5g]Ub`7cbhYlh

HYfa 'H\fYY''##'&\$&\$

7ci fgY 8YHJ] g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y'7ci fgY

f5i gffU`]U'gʻ5g]Ubʻ7 cbhYl ffi]bffcXi Wygʻmci 'hc'lfUbgX]gW]d`]bUfmk UmgʻcZYl Ua]b]b['a cXYfbʻ5i gffU`]UfgʻfY`Uh]cbg\]d k]h\ 'h\Yʻ5g]UbʻfY[]cb"'=fiYl Ua]bYgʻh\YʻWcbZ]Wf]b['dYfWdh]cbgž']a U[YgʻUbXʻfYgdcbgYgʻh\Uh 5i gffU`]Ubgʻ\Uj Yʻhck UfXgʻh\Y]fʻfY[]cb'UbXʻh\Yʻj Uf]ci gʻYj Ybhgʻh\Uh`\Uj Yʻ]a dUWhYXʻi dcbʻh\Ya "'A UhYf]U`g Yl Ua]bYX`]bWi XYʻ\]ghcf]WU``XcW a Ybhgž`]hYfUhi fYžUfhžV`c[gž'WUfhccbgžXcW a YbhUf]Ygža cj]YgʻUgʻk Y``Ug UWUXYa]WVcc_gʻUbX`UfhJWYg"'Hcd]WgʻWcj YfYX`]bWi XY.`5g]UʻUbXʻh\YʻZcfa Uh]cbʻcZ5i gffU`]Ubʻ]XYbhJhJYgžh\Y f]gYʻcZ5g]UʻUbXʻ5i gffU`]U'gʻg\]ZhJb['gffUhY[]WfY`Uh]cbg\]dgž'h\Yʻ]a dUWhcZ5g]Ubʻa][fUh]cbʻUbX a i `hJW` hi fU`]ga žʻ5g]Ub!5i gffU`]UbgfiYl dYf]YbWygžYXi WUh]b['UVci hUbXʻZcf'h\Y`5g]Ubʻ7Ybhi fmža]`]hUfm Yb[U[Ya Ybh]bʻ5g]Užʻhci f]ga 'hc`5g]UžYWcbca]Wh]Ygʻk]h\`5g]Už'UbX`W``hi fU`]bhY[fUh]cbˈk]h\`ci fʻfY[]cb"

7 ci fgY @YUfb]b['Ci hWca Yg

%" 8 Ya cbglfUlY 'U'VfcUXž'Wfcgg!W 'hi fU'' bck 'YX[Y'cZ5g]U'

- &" ⇒XYbhZm_Ymh\Ya YgžXYVUhYg UbX WcbWdhg]b 5q]Ub Gh X]Yg
- ' " 7 cbglfi WhUf[i a Ybhgʻi g]b[ʻ]bhYfX]gW]d`]bUfmgci fWYgʻUbXʻa Yh\cXgʻ]bʻk f]hhYbʻUbX#cfʻj YfVU``Zcfa (" 91 d`U]bʻh\Y`X]j Yfg]hmcZ5i glfU`]UbʻfYgdcbgYgʻhcʻ5g]UbʻgcW]Yh]YgʻUgʻh\Ym\Uj Y`XYj Y`cdYX`h\fci [\h]a Y

HYUWX]b[GlfUhY[]Yg

H\Y`Wci fgY`]g`hUi [\hh\fci [\`k YY_`m`YWh fYg`UbX`h hcf]U`g"`\dots`YWh fYgžgh XYbhg``YUfb`h\Y`WcfY Uf[i a Ybhg`UbX`XYVUhYg`Uvci hgdYW]Z]W]ggi Yg`]b`5i ghfU`]Ufg`Yb[U[Ya Ybh]b`h\Y`5g]Ub`fY[]cb"`\dots'h hcf]U`gžgh XYbhg`Yb[U[Y`k]h\`h\YgY`XYVUhYg`]bZcfa YX`Vmdcgh`YWh fY`a UhYf]U`g`XfUk b`Zfca `U`j Uf]YhmcZX]ZZYfYbh a YX]U`UbX`[YbfYg"

 $7 \cgY'cbY'ghUbXUfX'fYZYfYbW]b['UbX'V]V`]c[fUd\]WghmY'UbX'Udd`m]hWcbg]ghYbhmh\fci[\ci hh\Y'YggUm']hWcbg]ghYbhmh\fci [\ci hh\X'YggUm']hWcbg]ghYbhmh\fci [\c$

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ H' \end{tabular} $$ $$ H' \end{tabular} $$

5ggYgga Ybh' . 'Cb`]bYa i `h]!WXc]WY hYgh

GhUfh:XUhY. "%" #%%#&\$&\$ "\$% \$\$ "DA

@Yb[h\.) \$ a]bi hYg

8YHJ]`g.

 $\label{thm:conditional} $$H\YfY'k]``VY'cbY'cb`]bY'a i `f]WXc]WY'fYghhc'i bgYYb'ei Ygh]cbg'XfUk b'Zfca 'fh\Y'Wci fgY'fYUX]b[g'UbX'YWfi fYg'']$

: YYXVUW_. "♠X]j]Xi U``GW¢fY

Hi fb]h]b gYh]b[. H\]g]g bchU Hi fb]h]b Ugg][ba Ybh

5 hm/b XUb VW F Yei]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] }$

7ci fgY GWX YXi `Y

J]Yk 'WUgg'hja YhUV'Y

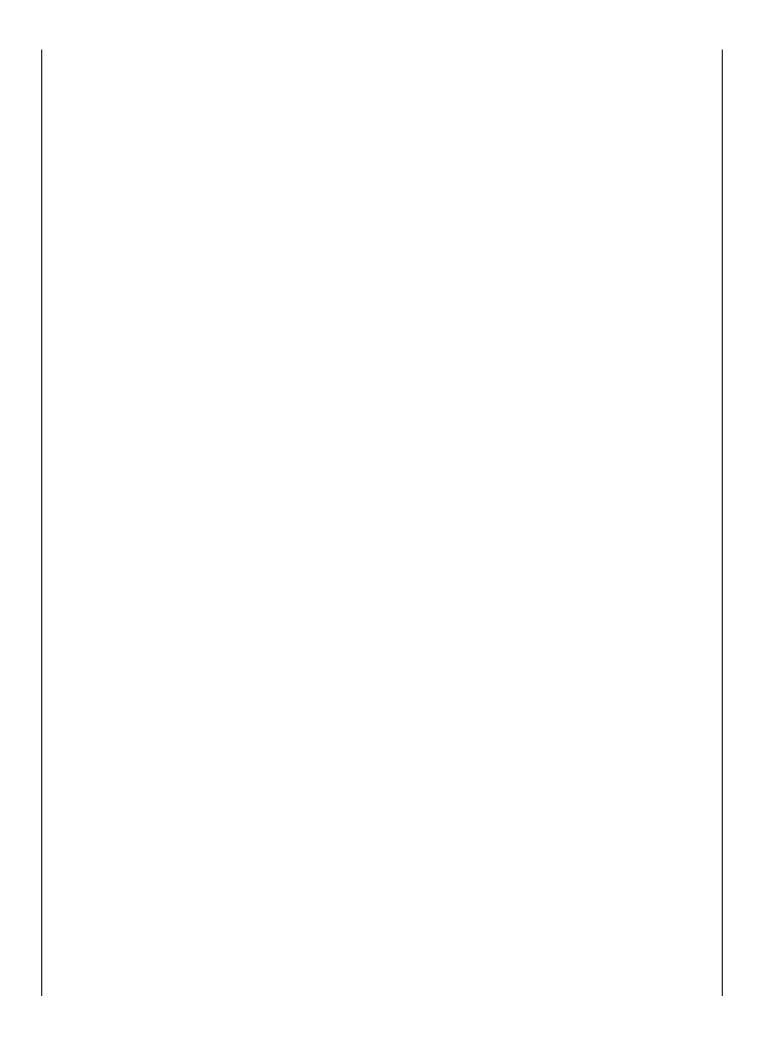
H]a YtUV`Y

8 UhY	HmdY	7 cbhYbh
C K YY', 'GYdhYa VYf'!'- GYdhYa VYf	@YWh fY	: fca ': YUf'hc'Cddcfhi b]hm
Gruira vii		7 cbhYbh ·
		 +bHfcXi Whjcb'hc'7ci fgY'5]ag +g'5i gHfU']U'Ub'5g]Ub'bUhjcb3
		DfY!`YWhi fY`UWhjj]hjYg
		K UHWX HXY'j]XYc'cb'A ccX'Y'ci h]b]b['HXY Wci fgY'U]a g'UbX'UggYgga Ybh]HYa g'f&\$ a]bgE'UbX'fHi hcf]U'FYUX]b['; i]XYfU]XYc f&\$'a]bgE

	DfY!`YWi fY`j]Yk]b[

	IZWoWKag5S_W8da_ž7\UdhYf%ždd" %!(("

	`@k. '5bX'k UHW 'UbmcZ\Yf'7ca a i b]hm 7\UbbY``j]XYcg'cb'Mci hi VY



	7j bae[Y BdafWUf[a` aXBWWabZ[Wei [fZ[fZW3dfe 7 (fWf&\$%) Ł •

		FYj]Yk z'% "%f&\$%&\z'dd"" +!*%" 5b[Y`U`7\Ybz'\H\Y`GYf]ci g'Gi V\Yl hcZ >UdUb'g'"7i \Y''7i `hi fYz'>GHCFz'>i `m%(&\\$%*"\\hdg.##XU]`m'\g\cf"cf[#\\Y!gYf]ci g! gi V\Yl hcZ\UdUbg!W\\Y!W\`hi fY#
		? YmE i Ygh]cbg.
		 8 cYg'YWcbca]Wdck Yf'Yei UhY 'hc'W 'hi fU' UbX'dc']h]WJ'']bZi YbW'3 K \UhUfY'h\Y'']b_g'VYhk YYb'dc']h]WgžW'hi fY UbX'YWcbca]Wg3 K \Uh']g'W'hi fU''dck Yf'UbX'\ck 'XcYg']h k cf_3
YY_'%\$.' [*] %^ Boj \$'Bcj Ya VYf		F]g]b['5g]U'UbX'h\Y']a d`]WUh]cbg'Zcf'5i ghfU`]U
		7 c bhYbh
	Ī	

	8 K	cYgʻ5g]Ufgʻf]gYʻaYUbʻh\YʻkYghfgʻXYW]bY3 \Uhʻ]gʻh\Yʻfc`YʻcZk\]hYbYggʻ]b (YghYfbYggf3
		Yunig riy rc y czk y jry bygg jb (Yghy fbyggfB

FYgci fW/g

DfYgWf]VYX F Ygci fWYg

 $H Y f Y ^{1}] g ^{1} b c ^{1} g Y h ^{1} H V c c ^{1} Z c f ^{1} h ^{1}] g ^{1} W c i ^{1} f Y Y ^{1} f Y U X] b [^{1}] g ^{1} U J ^{1} U V ^{1} Y ^{1} c b ^{1} Q Y [U b h c ^{1} c f ^{1} c b ^{1} A c c X ^{1} Y ^{1}] b _{Q} ^{1}$

FYWca a YbXYX FYgci fW/g

6fUk `Ymž`GYUb"`H\Y`K \]hY`DYf]`.: cfY][b`FY`Uh]cbg'UbX'5g]Ub`⊕aa][fUh]cb`hc'5i ghfU`Ug]U`UbX'Bcfh\5aYf]WJŽ'%%!%+, žI BGK `DfYggŽGmXbYmž'%-)"

6Yhgž?Uh\Yf]bY"°=XYc`c[mUbX′=a a][fUh]cbžA | DžA Y`VcifbYž°%,,"

6fc]bck g_]ž5`]qcb`fYX"Ł"8ci V`Y`J]g]cb. 5g]Ub`5WVci bhg`cZ5i ghfU`]UžDUbXUbi g`6cc_qž7UbVYffUž&\$\$("

6fc]bck g_]ž5`]gcb"'H\Y'MY``ck '@UXm'5i glfU`]Ub'a dfYgg]cbg'cZ5g]UžCI Dž'@cbXcbž'% - &"

6i f_Yž5bh\cbm' +b: YUf'cZGYW f]mm 5i ghfU`]U'g`+bj Ug]cb 5bl]YmžD`i hc DfYggžGmXbYm

 $\label{eq:continuous} 6mfbYgžA]WXUY`"`5i ghfU`]U`UbX`h\Y`5g]U`; Ua Yž`5``Yb`UbX`l bk]bž`GmXbYmž`% - ("$

:]m[YfU`Xž>c\b"`6][K\]hY`@]Y.7\]bYgY`5ighfU`]Ubg`]bK\]hY`5ighfU`]U"IBGK DfYggžGmXbYmž&\$\$+"

:]m[YfU`XžGhYd\Yb"`=g'5i ghfU`]U`Ub`5g]Ub`7 ci bhfm3ž'5``Yb`UbX`I bk]bž'GmXbYmž'% - +"

: fUgYfž'A U'Wc'a '8 Ub[Yfci g'5'`]Ygž'A I Dž'A Y'Vci fbY'&\$%"

: fY]ž< "D" >UdUb'g Gci h\k UfX 5 Xj UbWY UbX 5 i ghfU`]U Zfca 'h\Y G]I hYYbh\ 7 Ybh fmhc K cfX K Uf &žA I Dž A Y`Vci fbYž% - %"

; YfghYfžFcV]b"'<chY`5g]U.'5i ghfU`]Ub'@hYfUfmHfUj Y``]b['hc'h\Y''9Ugh'žDYb[i]bžA Y`Vci fbYž'%-)

; c`Xgk cfh\mž8Uj]X`flYXL'': UWJb[Bcfh\. 5 WYbhi fmcZ5i ghfU`]Ub'9b[U[Ya Ybhik]h\ 5g]UžA | Dž7Uf`hcb

Di b[ž'5`]WY'fYXŁ"; fck]b['I d'5g]Ub']b'5i glfU`]Už'6`UW_=bWZA Y`Vci fbYz'&\$\$, "Fc``gz'9f]W'Gc'ci fbYfg. 'H\Y'9d]WGlcfmcZ7\]bU'g'7Ybhi f]Yg!c`X'FY`Uh]cbg\]d'k]t\ '5i glfU`]Už'I E D.

6f]gVUbYž"% - &"

GU]Xž9Xk UfX" Cf]YbhU`]ga . K YghYfb '7 cbW/dh]cbg cZh\Y'Cf]Ybh' DYb[i]bž@cbXcbž'%+, "

GcVcW|bq_Uz~5[b]Yqn_U#J]q]h|b['h\Y'BY][\Vcifq.'5i qhfU`]Ubq']b'5q]Uz~BYk 'Gcih\z~GmXbYmz~&\$%("

HUI Ubž; k YbXU" H\Y @cb[žG`ck 8YUh\cZK \]hY 5i qhfU`]UžGWf]VYžA Y`Vci fbYž&\$\$)"

 $\label{localize} $$ HcfbYmDUf]W_]\ddot{z}DfiY"GcaYk\YfY]b'5g]U.'K Uf\ddot{z}>cifbU`]ga'UbX'5iglfU`]U'g'BY][\Vcifg % (%% +) "I BGK 'DfYggZ'GmXbYmz'&$$$"$

K U`_Yfž8Uj]X:/ 'GcVcW]bg_Už5[b]Ygn_U'f|YXgL''5i ghfU`]U'g'5g]U.': fca 'h\Y'MY``ck 'DYf]``hc'5g]Ub'7Ybhi fm' I K 5'Di V`]q\]b[ž7fUk 'YmžK 5ž'&\$\%"

 $K U '_Y f z ' 8 U j] X'' 5 b I] c i g 'B U f j c b . '5 i g f f U '] U 'U b X 'h Y 'F] g Y 'c Z 5 g] U z '%) $! \% ' - z 'I E D z G h @ W j U z '% - - / I K 5 D i V '] g \] b [z '7 f U k 'Y m z & $ \% ."$

K Yq`Ymž'A]\WUY`"'H\YfY'[cYq'th\Y'BY][\Vcif\ccX"'BYk 'Gcith\ž'GmXbYmž'&\$\%"

K Yg`Ymž'A]W\UY`"F YghYgg`7cbh]bYbh`K YU`h\žF]j U`fmUbX`5g]Ufg`BYk '; Ycdc`]h]Wg" 6`UW_ ±\WZ'&\$%) "

K YVg]hYg. 5g]U 9Xi WUh]cb': ci bXUh]cb/H\Y'8]d`ca Uh/H\Y'7cbj YfgUh]cbž5g]Ub'7i ffYbhgžH\Y'+bhYfdfYhYfž5DDG'Dc`]Wn: cfi a

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\]g`Wci fgY`]g`Yj U`i UhYX`i g]b['a m9I dYf]YbW'gi fj YmcdYfUhYX`Vmi BGK "'@]b_g`k]```UddYUf`cb`A ccX`Y k \Yb'h\Y`Yj U`i Uh]cb`dYf]cX`]g`cdYb"': cf`]bZcfa Uh]cb`cb`\ck 'h\Y`Wci fgY`k Ug`U`hYfYX`]b`fYgdcbgY`hc dfYj]ci g`gi fj Ymgž'gYY`h\Y`fY`Yj UbhgYWf]cb`cb`A ccX`Y"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hi fb]h]b Gi Va]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb['hc'gi va]hmci f'Ugg][ba Ybhh\fci [\ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-',) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg'fl *) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va]hmci f'Ugg][ba YbhXi Y'hc'U'ZJi 'hk]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb['k]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W_Yhbi a VYfZ'Vi h]Zmci 'hy'Yd\cby'mci 'k]``bYYX'hc'gdYW]Z]WJ'`mUg_'Zcf'cby''
Hi fb]h]b'U'gc'dfcj]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug_g'a i ghVY'gi Va]hhYX'Y`YVWfcb]\WJ``mij]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k]h\U`hYfbUh]j Y'gi Va]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va]hUgg][ba Ybhgicbi]bYij]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va]hUgg][ba Ybha ccXiY

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj]WYg'UbX ']bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+G9'k]``\Y`d'mci 'a U_Y'h\Y'a cghcZmci f'gh X]Yg'UhI BGK "'\\hd.#gi VYWn[i]XYg"`]VfUfm"i bgk "YXi "Ui #\`]gY#JVci hY`]gY

5 WUXYa]W + DZcfa Uh]cb

: cf'YggYbhJU`'ghi XYbhi]bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va]gg]cbg'[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc']W]Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- •