

HUMS2007

FYgYUfWX 'D`UWYa Ybh'=ž'5fhg'UbX'GcWJU`'GWJYbWYg

HYfa 'H\fYY''##'&\$&\$

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUhjcb	D\cbY
BUca]'DUf_]bgcb	b"dUf_]bgcb4 i bgk "YXi "Ui	6miUddc]bha Ybh	' ' * A 6	

GWXcc`7cbHJWhi±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc`C\\Z|\\\žA cfj\Yb'6fck\b'6i]\X]b[\\ž@Yj\Y``&\\z\&),

CdYb]b['<ci fg. 'A cbXUm! ': f]XUmž - Ua '!') da

D\cbY.'Ž*%'&'-',)'%*,%

: UI . Ž*%'&'-',)', +\$)

9a U]`. <u>\U`4 i bgk "YXi "Ui</u>

7ci fgY 8YHJ] g

7 fYX]hDc]bhg '%'

Gi a a UfmcZh\Y 7ci fgY

±0.1h\]g`Wci fgYžmci ik]```Yb[U[Y`]b`U`fYU`!`]ZY`UWUXYa]WfYgYUfW\`YI dYf]YbW'cb`U'a U^cf`dfc^YWf`YX`Vm I BGK 'gW\c`Ufg`]b`5fhg/ 'GcW]U``GW]YbW'g"'Mci ik]```k cf_'k]h\`U``YUX]b['gW\c`Ufž'Wca d`YhY`cf][]bU`fYgYUfW\`UbX`k f]h\`U`dddYf`UVci hmci f`UfYU`cZfYgYUfW\"'

7 ci fgY @YUfb]b[Ci hWca Yg

%" 91 YW hY 'Ugg][bYX'fYgYUfW. 'HUg_g']b'U'Wca dYhYbhž'dfcZYgg]cbU``UbX'h]a Y`ma UbbYf &" 7 f]h]WU``mfYZYWicb'mci f'fYgYUfW. 'HUg_gž'XYa cbghfUh]b['mci f'i bXYfgHUbX]b['cZmci f'fYgYUfW. dfc YWi]b'`][\hcZgca Y'fY`Yj Ubh`]hYfUhi fY'

' "FYZYWricbimci fiYl dYf]YbWYicZk cf_]b[icbiUifYgYUfWXidfcYWh

HYUW]b[GlfUlY[]Ya

Mci f`gi dYfj]gcf`k]```[]j Y`mci `U`Wc\YfYbhiga U``!gWU`Y`dfc^YWhitc`k cf_`cb"`#na Um`]bWi XY`df]a UfmfYgYUfW\`#XUHU`UbU`mg]g`UbX#cf`gYWcbXUfmfYgYUfW\": cf`Yl Ua d`Yžmci `a Um\Y`d`hc`fYgYUfW\`U`Vcc_`cf`ci fbU`Ufh]WY`Zcf`h\Ya ž`Xc`gWcd]b[`fYgYUfW\`Zcf`U`bYk `dfc^YWz`cf`\Y`d`hc`[Uh\Yfž'hfUbgWf]VYž']bXYl `UbX`UbU`mgYdf]a UfmfYgci fWygž']bhYfj]Yk g`cf`ch\Yf`XUhU"

± k YY_`mcf'Zcftb][\hma YYh]b[gžmci f'gi dYfj]gcf'k]```gi [[YghfYUX]b[g'Zcf'mci 'hc'XcžX]gW gg'mci f dfc[fYggž'hU`_'hc'mci 'UVci hmci f'Yj c`j]b['i bXYfghUbX]b['cZh\Y'fYgci fWrg'mci '\Uj Y'VYYb']bghfi WhYX'hc k cf_'cb'UbX'\Y`d'mci 'd'Ub'\ck 'hc'a cj Y'h\Y'dfc YWh'Zcfk UfX"'5 ZhYf'YUW\ 'a YYh]b[žmci 'g\ci `X'gYbX'U Zc``ck 'i d'Y!a U]`'WcbZ]fa]b['mci f'i bXYfghUbX]b['cZh\Y'hUg_'Ugg][bYX'UbX'fYWcfX]b['mci f'dfc[fYgg"'H\YgY Wca a i b]WUh]cbg'k]``'dfcj]XY']a dcfhUbh'Yj]XYbW'Zcf'mci f'dcfhZc`]c"

 $\label{thm:linear} Wci f'a U cf'k f]h]b['HUg_g'UfY'U'fYgYUfW.'dcfhZc`]c'h\Uh[Uh\Yfg'Yj]XYbWf'cZUbX'fYZYWf]cbg'cb'mci f fYgYUfW.'YI dYf]YbWf'UbX'U'fYgYUfW.'fYdcfhih\UhXYa cbghfUhYg'mci f'WJdUW]mmhc'Xc']bXYdYbXYbhigW.c`Uf'm k cf_']b'h\Y'UfYU'cZ'mci f'fYgYUfW.'''$

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'hUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
FYgYUfV% 'DcfhZc`]c'	(\$1	%*#%%\$&\$`\$).\$\$`DA	%2
F YgYUfVX 'F Ydcfh'	(\$1	%*#%%\$&\$`\$).\$\$`DA	&
F YgYUfW. Gi dYfj]gcf 5ggYgga Ybh	&\$ı	Bch5dd`]WUV`Y	%

5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh'% F YgYUfW DcfhZc`]c

GHUfhXUhY. K YY_'CbY

@Yb[h\."\$\$\$!)\$\$\$

8 YHJ]`g.

#blYfbg\]d'dcfhZc`]c'fl \$\$\$!) \$\$\$ k cfXgL'H\Y'dcfhZc`]c'g\ci `X'VY'UggYa V'YX'h\fci [\ci hh\Y'hYfa "'#ig\ci `X XcW a Ybhimci f'Yl dYf]YbW'gž'dfc[fYgg'UbX'fYZYWjcbg'cZmci f'F @dfc[fUa "'8cW a YbhUhjcb']bWi XYg Zcfhb][\hmfYdcfhgž'UbX'Yj]XYbW'cZfYgYUfW\i bXYfhU_Yb"'#ig\ci `X'U'gc']bWi XY'U'Wf]hJWU`fYZYWjcb'cb mci f'dfcWyggž'dfc[fYggž'UbX'k\UhWcbgh]hi hYg'U'gi WWyggZ\`fYgYUfW\'dfc YWh]b'h\]g'ZYYX"'Wci 'k]`` k cf_g\cd'mci f'dcfhZc`]c'k]h\'mci f'Wcbj Ybcf'cf'XY'Y[UhY'UbX'fYWY]j Y'Zcfa Uhjj Y'ZYYXVUW_cb'mci f'dfc[fYgg Uhh\Y'a]Xdc]bhcZh\Y'hYfa "'H\]g'dcfhZc`]c'k]``VY'[fUXYX'Vmh\Y'Wcbj Ybcf'cf'XY'Y[UhY']b'Wcbgi `hUhjcb'k]h\ mci f'gi dYfj]gcf": YYXVUW_k \]``VY'[j] Yb'j]U'U'a Uf_]b['fi Vf]WUbX']b'k f]hb["

Gi Va]gg]cb bchYg.J]U 9a U]

Hi fb]hb gYhb[. H\]g]g bchU Hi fb]hb Ugg][ba Ybh

5ggYgga Ybh& FYgYUfW FYdcfh

GhUfhiXUhY. K YY_')

@Yb[h\.'&\$\$\$

8YHJ]`g.

F YgYUfW. F Ydcfhf&\$\$\$ k cfXgŁ Mci f fYgYUfW. fYdcfhdfcj]XYg Ub cddcfhi b]mhc k f]hY U fYgYUfW. dudYf VUgYX cb Ub UgdYWicZmci f fYgYUfW. h\ Uh\ Ug dUfh]W `Uf mgdUf_YX mci f]bhYfYghz cf hc fYZYWicb mci f fYgYUfW. d`UW a YbhYl dYf]YbW" Mci 'g\ ci `X'dfcdcgY mci f dfc YWi]b k f]h]b['UbX`\Uj Y]hUddfcj YX Vmmci f gi dYfj]gcf'UbX h\ Y Wci fgY Wcbj Ybcf'VYZcfY h\ Y a]XX`Y cZh\ Y hYfa "'=ha Um\ Uj Y Ub cfU` dfYgYbhUh]cb Wca dcbYbh—'hc 'VY XYhYfa]bYX`]b Wcbgi `hUh]cb k]h\ mci f gi dYfj]gcf"'H\]g k]```VY Xci V`Y a Uf_YX`Vmmci f Wcbj Ybcf'cf'XY`Y[UhY UbX mci f gi dYfj]gcf"': YYXVUW_k] ```VY []j Yb j]U'U a Uf_]b['fi Vf]WUbX`]b k f]h]b["

Gi Va]gg]cb bchYg.J]U 9a U]`

 $\label{eq:higher_problem} \mbox{Hi fb]h]b$^{-}gYh]b[.^{-}H\]g$^{-}]g$^{-}bchU$^{-}Hi fb]h]b$^{-}Ugg][$ba^{-}Ybh^{-}$

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

Bch'Uj U]`UV`Y

FYWca a YbXYX FYgci fWYg

Bch'Uj U]`UV`Y

7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

 $Ghi \ XYbhZYXVUW_k \] \ ``VY'[UhYfYX'] \] U Ya U] \ `UhhYfYX'] \ JUYa U] \ ``UhhYYYbX'cZhY'Wci fgY'WhiY'Wci fgY'Wcbj Ybcf''Ghi XYbhg k] \ ``VY'fYei YghYX'hc'dfcj]XY'ZYYXVUW_cb'hX'] f Ugg] [bYX'dfc'YWhf]bWi X]b ['ei U`] mcZ gi dYfj]g]cb/k cf_`cUX/hfUbgZyfUV] \ [mcZfYgYUfWk'g_] \ `gy'\ck'Yb [U[]b['h\YmZci bX'h\Y'Ugg] [bYX'dfc'YWh'' H\Ymk] \ ``U`gc'VY'fYei YghYX'hc'Wca a Ybhcb'h\Y'cj YfU``ghfi Whi fY'cZh\Y'Wci fgY'UbX'] hg'UggYgga Ybhg'' \ YYXVUW_cb']bX]j]Xi U``dfc'YWhg'k] \ ``VY'fYZYffYX'hc'gi dYfj]gcfg'hc'[i]XY'h\Ya']b'ghfi Whi f]b['a UhYf]U``Zcf Z hi fY'< | A G&$$+'fYgYUfWX']bhYfbg'': YYXVUW_cb'h\Y'Wci fgY'k] \ ``VY'i gYX'hc'UX'\ gha cXYg'cZ UggYgga Ybh''$

Gi Va]gg]cb cZ5ggYgga YbhHUg_g Hi fb]h]b Gi Va]gg]cb

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b['h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX'dfYgYbh]b['h\Ya 'Ug`mci f`ck b"'=h\Wb'hU_Y`a Ubm Zcfa qžZfca 'XY`]VYfUhY'W\YUh|b['hc`UWW]XYbhU`mWcdm]b['Zfca 'U'gci fW'k]h\ci h'UW_bck 'YX[Ya Ybh'

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'] Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`h\I hcf']XYU'k]h\ci hUW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U'gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfc'YWgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

⇒ Uddfcdf]UhY 'dUfUd\fUg]b[...7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k \]'Y 'a cghmfYhJ]b]b['h\Y'cf][]bU` ghfi Wh fY 'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa Uh]cb'k]h\ci hUW_bck `YX[Ya Ybh'H\]g'U`gc Udd`]Yg']b'dfYgYbhUh]cbg'k \YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci hWfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk 'k \c`Yžk]h\ci hUddfcdf]UhY'fYZYfYbW]b["

7 c``i g]cb. 'k cf_jb['k]h\ 'ch\ Yfg'Vi h'dUgg]b['cZZh\ Y 'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\ Yf'ghi XYbhVYZcfY'h\ Y 'Xi Y 'XUhYzcf'Zcf'h\ Y' di fdcgY'cZh\ Ya d`U[]Uf]g]b['Uh'Ubmh]a Yz'dUm]b['Ubch\ Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_z'ghYU']b['cf'UWei]f]b[Ubch\ Yf'dYfgcb'g' UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d`YhY'Ubch\ Yf'dYfgcb'g' k cf_'cf'gYY_]b[dUma YbhZcf'Wca d`Yh]b['UWUXYa]Wk cf_"

±bUddfcdf]UhY`W]hUh]cb.`7]h]b[ˈgci fW/gˈk \]WX`\Uj Y`bchVYYb`fYUXžk]h\ci hUW_bck`YX[]b[ˈh\Y`″gYWcbXUfm″gci fW/Zfca ik \]WX`_bck`YX[Y`cZh\Ya`\UgʻVYYb`cVhU]bYX"

8 i d`]\WUh]cb`ffgY`Zd`U[]\Uf]ga "Ł`gi Va]\h]b[`mci f`ck b`k cf_ž]b`k \c`Y`cf`]b`dUfhžk \YfY`]h\Ug`dfYj]ci g`m VYYb`dfYdUfYX`cf`gi Va]\hYX`Zcf`Ubch\Yf`UggYgga Ybhcf\Wci fgY`UhI BGK `cf`Ubch\Yf`i b]j Yfg]\m'

7 cffYWhfYZYfYbWb[dfUWhWyg.

- DUfUd\fUq]b[žgi a a Uf]q]b[žYggUmk f]h]b['UbX'h]a Y'a UbU[Ya Ybh
- 5ddfcdf]UhY'i gY'cZUbX'Uhff]Vi h]cb'Zcf'U'fUb[Y'cZa UhYf]U`g']bWi X]b['hYl hž']a U[Ygž'Zcfa i `UY'UbX WbWYdhg"

±0X]j]Xi U``Ugg]ghUbWr`]g`Uj U]`UV`Y`cb`fYei YghZfca `H\Y`@YUfb]b[`7 YbhY`f\hd.#k k k "`Wi bgk "YXi "Ui #£" Ghi XYbhg`UfY`U`gc`fYa]bXYX`h\UhWUfYZ ``h]a Y`a UbU[Ya Ybh']g`Ub`]a dcfhUbhdUfhcZghi XmUbX`cbY`cZh\Y]XYbh]Z]YX`WUi gYg`cZd`U[]Uf]ga `]g`dccf`h]a Y`a UbU[Ya Ybh'`Ghi XYbhg`g\ci `X`U``ck `gi Z]W]Ybh'h]a Y`Zcf fYgYUfW\z`XfUZh]b[`UbX`dfcdYf`fYZYfYbW]b[`cZgci fW/g`]b`dfYdUf]b[`U```UggYgga Ybh']hYa g"

IBGK @]VfUfmU`gc`\Ugʻh\Y`9@&9`hcc``UjU]`UV`Y`hc`Ugg]ghmci`k]h\`mcif`ghiXmUhIBGK"`9@&9`]g XYg][bYX`hc`]bHcXi WY`bYk`ghiXYbhg`hc`ghiXm]b[`UhIBGK`Vih]hWUb`U`gc`VY`U`[fYUhfYZfYg\Yf`Xif]b[`mcif

- i bXYfghUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'ghi XYbh'Uhil BGK
 VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hžI BGK 'Ghi XYbh'7cXY'cZ7cbXi Wh'UbX'5WW'dhUV`Y'I gY'cZ I BGK '=7 H'F Ygci fWfg'Dc`]Wh

5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['lc.