

# **IEST5004**

9bj ]fcba YbhJ``A UbU[ Ya YbhF YgYUfW\ \*\bhYfbg\]d

HYfa 'H\fYY''##'&\$&\$

# 7ci fgY'Cj Yfj ]Yk

## GHJZ:7cbHJWh:8YHJ]`g

### 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D∖cbY
HYa U'A ]`ghY]b	hYa U"a ]`ghY]b4 i bgk "YXi "Ui	6miUddc]bha Ybh	A cfj Yb <sup>.</sup> 6fck b	
		Xif]b['Wcbgi`HUH]cb	' *( 'fbf'cb`]bY	
		\ci fg	j]U'HYUa g'Xi Y	
		fK YXbYgXUmgž'‱	hc:X]ghUbWY	
		U"a "`!`%&`d"a "Ł	`YUfb]b[ 'cf	
			7 cj ]XŁ	

## GWXcc`7cbHJWhi±bZcfa Uh]cb

GWkcc``cZ<iaUb]h]Yg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b[ '<ci fg. 'A cbXUm!': f]XUmž'- Ua '!') da

D\cbY.'Ž\*%'&'-',)'%\*,%

: UI . Ž\*%'&'-',) ', +\$)

9a U]`. \<u>U`4 i bgk "YXi "Ui</u>

# 7ci fgY'8YW]`g

### 7 fYX]hDc]bhg\*\*

### Gi a a UfmcZh\Y 7ci fgY

H\Y'cVYWIJ Y'cZh\Y'9bj ]fcba YbhU``A UbU[ Ya Ybh]bhYfbg\]d']g'hc'dfcj ]XY'mci 'k ]h\'h\Y'cddcfh b]lmhc'k cf\_k ]h\'Yl dYf]YbWX'Ybj ]fcba YbhU``dfUWJh]cbYfg']b'U'dfcZYgg]cbU``gYhJb[ "'Mci 'k ]``i gY'h\Y'\_bck `YX[ Y [ U]bYX']b'dfYj ]ci g'A 9A 'Wci fgYgž'UbX'hc'Udd`m]hhc'dfc YWhUg\_g'h\fci [ \ci hh\Y']bhYfbg\]d"'8YhU]`g'cZh\Y HUg\_g'k ]``'VY'UffUb[ YX']b'Wcbgi `HUh]cb'k ]h\'h\Y'Wci fgY'WccfX]bUhcf'cf'a i hi U``mU[ fYYX!i dcb'UWUXYa ]W ghUZgi dYfj ]gcf'UbX'mci f'cb!g]hY'gi dYfj ]gcf'fUb'Uddfcj YX'ghUZa Ya VYf'k ]h\]b'df]j UhY'cf'di V`]WgYWcf ]bgh]h h]cb'cf'cf[ Ub]gUh]cbgž'h\Y'Wci fgY'WccfX]bUhcf'k ]``k cf\_k ]h\ mci 'UbX'h\Y'U[ YbW]Yg'hc'Ybgi fY'mci 'UfY'Z ``mdfYdUfYX'UbX'h\Uh'h\Y'dfc'YWh]g'Uddfcdf]UhY'"

### 7ci fgY @YUfb]b[ 'Ci hWca Yg

- %" Ghi XYbhg``YUfb`\ck 'hc`XYj Y`cd`U`dfcZYgg]cbU`#YgYUfVX`dfc^YWhh\fci [\``]U]gcb`k]h\`U`dUfhbYf cf[Ub]gUh]cb
- &" Gh XYbhg'a i ghXYj Y`cd'dfc YWhU]a g'UbX'gWcdY
- '" K ]h\n\Y`dUffbYf`cf[Ub]gUf]cbžghi XYbhg`a i ghXYj Y`cd`UbX`Udd`miUddfcdf]UhY`a Yh\cXc`c[]Yg`hc UXXfYgg`h\Y`U]a g`cZh\Y`cf[Ub]gUf]cb`UbX`h\Y`ghi XYbh'dfcYVh
- ("Ghi XYbhg`YUfb'hc'WcbXi Wh`]hYfUhi fY'fYj ]Yk g`]b'hc'UXXfYgg'gdYW[Z]WfYgYYUfWX'hcd]Wg"
- ) " Gh XYbha UbU`maYžfYdcfhUbX ]bhYfdfYhZbX]b[ a
- \*" Gh XYbhg`YUfb`hc`XY`]j Yf`WcbWi g]cbg`UbX`fYWca a YbXUhJcbg`h\fci [\`fYdcfh]b[`UbX`cfU``XY`]j Yfm

### HYUWX]b[ 'GhfUhY[]Yg'

HYUW\]b['glfUhY[]Yg']bWi XY'Wcbgi `HUh]cb'UbX'gi dYfj ]g]cb'a YYh]b[g'k]h\'mci f'Wci fgY'Wc!cfX]bUhcf'cf a i hi U``mU[fYYX!i dcb'UWUXYa]WgHUZZgi dYfj ]gcfz'U'\UbXg!cb'k cf\_]b['fY`Uh]cbg\]d'k]h\'mci f'\cgh cf[Ub]gUh]cbz'UbX'fYdcfh]b['i]U'h\Y'Wci fgY'UggYgga Ybhg"

# 5ggYgga Ybh

H\Y`Zcfa Uhz``Yb[h\z`UbX`gWcdY`cZU```Ugg][ba Ybhg`]g`Zcf`bY[ch]Uh]cb``VYhk YYb`mci z`mci f`U[fYYX!i dcb ]bhYfbg\]d`gi dYfj ]gcfz`UbX`mci f`\cghicf[Ub]gUh]cb"`A cfY`[i]XUbW\k]```VY`[jj Yb`]b`Wcbgi `hUh]cb'a YYh]b[g YUf`m]b`h\Y`HYfa `k]h\`mci f`gi dYfj ]gcf`UbX`\cghicf[Ub]nUh]cb"

### 5ggYgga Ybh'HUg\_g'

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
: ]bU`F Ydcfh	* \$1	Bch5dd`]WUV`Y	' <u>ž(</u> ž) ž*
Cfu``dfYgYbrUh]cb`cZfYgi`hg UbX`Z bX]b[g`cZ]bhYfbg\]d dfcYVhi	&\$ı	Bch5dd`]WUV`Y	) ž*
±bhYfbg\]d`fYgYUfWX`Vf]YZ	%\$ı	Bch5dd`]WUV`Y	%ž&
=bHYfbg\]d'8]Ufm' %		Bch5dd`]WUV`Y	%

### 5ggYgga Ybh8YHJ]`g

5ggYgga Ybh%: ]bU`FYdcfh

GhUfhXUhY. Bch5dd`]WUV`Y

 $8 YHJ]^*g.: ]bU`^fYdcfhXYHJ]^]b[ ^UJa gž'gWcdYž'a Yh\cXgž'UWfJj ]h]Yg^WcbXi WhYXž'fYgi ^hg#JbX]b[ gž'fY^Yj Ubh ^]hYfUhi fYž'UbX^fYWca a YbXUh]cbg^Zcf^h\Y^ccf[ Ub]nUh]cb"$ 

F Yj ]Yk 'Vm'Wci fgY'Wcbj Ybcf'UbX'Wca a Ybhg'Zfca 'cf[ Ub]gUh]cb'gi dYfj ]gcf"

#### 5XX]h]cbU`'XYhU]`q.

D`YUgY`WcbHJWiU`[ccX!Z]hi9bj]fcba YbhiUbX`GcW]YhmiUWJXYa]WgHJZa Ya VYf`hc`fYei Yghh\YmiWc!gi dYfj]gY mci f`k cf\_k]h\`mci f`\cghcf[Ub]gUh]cb"

Gi Va ]gg]cb bch/g.: i ```XYhU]`g`hc`VY`bY[ch]Uh/X`k]h\`\cgh'cf[Ub]gUh]cb`UbX`]bh/fbg\]d`gi dYfj ]gcf

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb & fb & gYh &$ 

5ggYgga Ybh& CfU`dfYgYbhJhcbcZfYgi`hg UbXZ]bX]b[gcZ]bhYfbg\]ddfcYWh

GhUfhXUhY. Bch5dd`]WUV`Y

 $@\textbf{Yb[h}. : i ```XYHJ]`g`hc`VY`bY[ch]UhYX`k]h. `\cghcf[Ub]gUh]cb`UbX`]bhYfbg\]d`gi dYfj]gcf$ 

 $8YHU]`g. C fU``dfYgYbHUH]cb``cZZ]bX]b[g`hck UfXg`YbX`cZ]bHYfbg\]d \\ DUffbYf`UbX`Wcbj Ybcf`Wca a YbhUbX`h\]g`dfcj ]XYg`Ugg]gHUbW'`k ]h\`Z]bU``fYdcfhWca d`Yh]cb$ 

5XX]h]cbU`XYhJ]`g.

H\Y`Zcfa Uh'UbX`WcbhYbhk]```VY`XYhYfa ]bYX`Vmih\Y`]bhYfbg\]d`gi dYfj ]gcf`UbX`h\Y`\cghicf[ Ub]gUh]cb"

Gi Va ]gg]cb bch/g.: i ```XYhU]`g`hc`VY`bY[ch]Uh/X`k]h\`\cghicf[Ub]gUh]cb`UbX`]bh/fbg\]d`gi dYfj ]gcf

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh' . \*bhYfbg\]d fYgYUfW Vf]YZ

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.G\cfhVf]YZk]h\`dfc^YWiU]a gžigWcdY`UbX``YHYf`cZU[fYYaYbhik]h\`cf[Ub]gUh]cb`]bWiXYX" FYi]Yk`VmWcifqY`WcbjYbcf

#### 5XX]h]cbU`XYhJ]`g.

H\Y`Vf]YZZcf`h\Y`\bhYfbg\]d`]g`YghUV`]g\YX`]b`Wcbgi``hUh]cb`VYhk YYb`h\Y`]bhYfbg\]d`gi dYfj ]gcf`UbX`h\Y\cghcf[Ub]gUh]cb"`Bcfa U``mh\Y`Vf]YZk]```VY`XYhYfa ]bYX`UbX`k f]hhYb`k ]h\]b`h\Y`Z]fghihk c`k YY\_g`cZh\Y HYfa"

Gi Va ]gg]cb'bch/g.: i ```XYhU]`g'hc'VY'bY[ch]Uh/X'k]h\ \cghicf[Ub]gUh]cb'UbX']bh/fbg\]d'gi dYfj ]gcf

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh( . +bhYfbg\]d 8]Ufm

GhUfhXUhY. Bch5dd`]WUV`Y

**8 YHJ]`g.** ♣ `cfXYf`hc`Ugg]ghk ]h\ `ghi XYbhdfc YWha UbU[ Ya Ybhg\_]``gžghi XYbhg`a i gh\_YYd`UbX`gi Va ]hU`X]Ufm fYdcfh]bX]WUh]b[ `UWNjj ]h]Ygž'dfc YWhd`Ubb]b[ `UbX`h]a Y`]bYg" 5 ggYggYX`Vmh\Y`Wcbj Ybcf

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

# 5 https://bxubwww.Fyei]fYaYbhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`m'YbWc'i fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk'``YWhfY'fY'WcfX]b[g''' } \\$ 

# 7ci fgY'GWXYXi `Y

## J]Yk 'WUgg'hja YHUV'Y

# H]a YtUV`Y

8UhY	HmdY	7 cbhYbh
K YY_'%'%('GYdhYa VYf'! % 'GYdhYa VYf		5 k YY_`mgWkYXi `Y`]gʻbchdfcj ]XYX`\YfY"'8i f]b[ `cf VYZcfY`h\Y`Z]fghk YY_`cZHYfa žh\Y`gWcdY`cZh\Y ]bhYfbg\]dʻbYYXgʻhc`VY`XYhYfa ]bYX`]b`bY[ ch]Uh]cbg VYhk YYb`\cghcf[ Ub]gUh]cbžmci ž'UbX`mci f`U[ fYYX! i dcb`UWUXYa ]WghUZZ]bhYfbg\]dʻgi dYfj ]gcf" 5```ch\Yf gWkYXi`]b[ `UbX`fYei ]fYa Ybhg`Zck `Zfca `h\]g`YUf`m YghUV`]g\a YbhcZh\Y`]bhYfbg\]d"

## FYgci fWyg

### DfYgWf]VYX F Ygci fWYg

H\Y`fYgci fWrg`bYYXYX`Zcf`mci f`\bhYfbg\]d`k]```VY`XYhYfa]bYX`Vmh\Y`gWcdY`UbX`]bhYbhcZh\Y`dfcYWb`UbX`Vmh\Y`fYei]fYaYbhg`cZmci f`\cghicf[Ub]gUh]cb`UbX`mci f`U[fYYX!i dcb`UWUXYa]WgHUZZ]bhYfbg\]d gi dYfj]gcfž'fY`UhYX`hc`mci f`hcd]W

### FYWca a YbXYX FYgci fW/g

 $\label{thm:continuity} $$Mci 'UfY'YI dYWMYX'hc'YI YfW]gY'U'\][\XY[fYY'cZ]b]h]Uh]j Y']b'Z]bX]b['mci f']bhYfbg\]d'UbX'fYgYUfWX'a UhYf]U'g fY'Yj UbhZcf'mci f'dfc'YWf'$ 

### 7ci fgY 9j U'i Uhjcb UbX 8Yj Y'cda Ybh

@\_Y'U``A 9A 'Wci fgYgžh\Y'UddfcUW\'hc'h\Y'+bhYfbg\]d']g'XYj Y`cdYX'cb'h\Y'VUg]g'h\Uh]bXYdYbXYbhUbX Udd`]YX'`YUfb]b[']g'\][\`mj U'i YX'Vmgh XYbhgž'VUgYX'cb'dUghYl dYf]YbW\'UbX'dfYj ]ci g'ZYYXVUW\_" Gh XYbhg`\Uj Y'U`gc'Yl dfYggYX'ghfcb[']bhYfYgh]b'h\Y'Yl hYfbU` Wca a i b]m#[cj Yfba Ybh#B; C#bXi ghfmYl dYf]YbW\'h\Uh\\Y'+bhYfbg\]d'cZYfg"

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

# Hi fb]h]b Gi Va ]gg]cb

#mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dhJb[ 'tc'gi Va ]hmci f'Ugg][ ba Ybht\fci [ \'Hi fb]hJbž'd'YUgY
hY'Yd\cbY'9| hYfbU'Gi ddcfhcb'-', ) '''' %cf'Ya U]''h\Ya 'cb'Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft\"'#c'i'#c'i'
UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZUi 'hk ]h\ 'Hi fb]hJb'mci 'a Um'Udd'm\Zcf'Ub'Yl hYbg]cb\ZVi hmci
a i ghfYhU]b'mci f'hJ\W\_Yhbi a VYf'\Zfca '9| hYfbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj ]XYbW''hc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WUh]cb'''#mci 'Ya U]''9| hYfbU''Gi ddcfhmci 'k ]''

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=iWUbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'i

- i bXYfgHUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbhUhI BGK
   VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hz'I BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5VWWdhUV`Y'I gY'cZ IBGK =7 HF Ygci fWrg Dc`]Wh

## 5 WUXYa ]W = b Zcfa Uh cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg 'Zcf'Yl hYbg]cb/
- `UhY`gi Va ]gg]cbg`[ i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc`]\\\Yg/
- YI Ua ]bUh]cb dfcWYXi fYg/
- gdYWJU`Wcbg]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a ]gUXj Ybh fY/
- gh XYbhYei ]mUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Ufhg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWJXYa ]W]bZcfa Uh]cb#dfchcWc`g![i ]XY`]bYg#

### ⇒a U[Y'7fYX]h

D\chc Vm8Ub]Y`FcV]bqcb

#### 7F = 7CG

7F = 7CG Dfc| ]XYf 7cXY. \$\$\$-,;

#### 5W\_bck `YX[ Ya YbhcZ7ci bhfm

 $K Y : UW\underline{b} ck `YX[Y : h, Y : 6YXY[U : dY cd `Y : k \ c : UfY : h, Y : hfUX]h\underline{b} cb : `W ghcX]Ubg : cZh, Y ``UbXg : cb : k \ ]WX : I B G K ? Ybg]b[hcb : WUa di g : ]g ``cWUhYX''$