

# **ARTS1570**

₺hfcXi WcfmGdUb]g\ 5

HYfa 'C bY''##'&\$&%

# 7ci fgY<sup>-</sup>Cj Yfj ]Yk

# GHJZ7cbHJWh8YHJ]`g

## 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWJhjcb	D\cbY
B UhʻU`]UʻC fhjn 7 YVYf]c	b"cft]nVWVYf]c4 i bgk "YXi "Ui	,	3 3 5	dfYZYffYX WcbHUWiVm Ya U]`

## GWXcc`7cbHJWhi±bZcfa Uh]cb

GW.cc``cZ<iaUb]h]Yg'UbX'@Ub[iU[Yg

@cWUhjcb. GWkcc`CZZJWYž'AcfjYb'6fckb'6i]X]b[ž'@YjY``&ž'&),

CdYb]b[ '< ci fg. 'A cbXUm! ': f]XUmž'- Ua '!') da

D\cbY. 'Ž\*%'&'-',)'%\*,%

: UI . Ž\*%'&'-' , ) ', +\$)

9a U]`. \<u>U`4 i bgk "YXi "Ui</u>

7 ci fgY 8 YHJ]`g 7 fYX]hDc]bhg \* Gi a a UfmcZh\Y 7 ci fgY  $[fcidX]gW gg]cbgžUbX'HJg_!cf]YbHYX'UWNjj]hJYg']b'dU]fg'UbX'[fcidg'''+b'mcif''YWNifYgž'hihcf]U'g'UbX''']jYWcUWN_jb['mci'k]'''VY'Yb[U[]b[']b.$ 

- GdYU\_]b[ UW]j ]h]Yg
- '; fUa a Uf'UWhjj]hjYg
- K f]h]b[ UWh]j ]h]Yg
- FYUX]b[ 'Wca dfY\Ybg]cb'UWfj] ]h]Yg'fY`UhYX'hc'h\Y'h\Ya Yg'gh X]YX
- 5 i X]c!j ]gi U``Wca dfY\Ybg]cb`UWfj ]h]Yg

# 5ggYgga Ybh

H\]g^]g^U^\*I c7 Wci fgY"Gh XYbhg^UfY^YI dYWYX`hc^]bj Ygh%) \$`\ci fg`cZ'gh Xmfl UWY`hc^ZUWY`cf`Uh\ca YŁ"

Mci 'bYYX'hc'Wca d'YhY'h\Y'k YY\_`m\ca Yk cf\_']b'dfYdUfUh]cb'Zcf'mci f'K YV'`YWi fYg'UbX'hi hcf]U'" 7 ca d'YhY'k YY\_`mUWlji ]h]Yg']b'A ccX'Y"

@YUfb'h\Y'hcd]WfY`UhYX'j cWUVi `Ufm'UbX'[ fUa a Uf'**VYZcfY**'mci f'h hcf]U'"'5VgYbWY'hc ``YWh fYg'cf'K YVXY`]j Yfm'Ybfc``a Ybhg\ci `X'bchVY'Ub'YI W gY'hc'Wca Y'hc'h hcf]U`g'i bdfYdUfYX"

@Ub[iU[YcZ]bghfiWh]cb]bhihcf]U`g]gGdUb]g\"

## 5ggYgga Ybh HUg\_g

5ggYgga Ybh'hUg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Ygʻ5ggYggYX
K f]h]b[ 'hYgh'	')	K YY_"%\$	%ž <sup>*</sup> &
±b!7`Ugg'Ei]n'	&) ı	K YY_')	%ž <sup>:</sup> &ž <sup>:</sup> (
C fU"	&\$I	: ]bU``]b'k YY_'-	%ž <sup>:</sup> &ž <sup>:'</sup>
9!Dcfh7c`]c`	&\$I	k YY_`m	&ž <sup>.</sup> ' ž <sup>.</sup> (

# 5ggYgga Ybh'8YhJ]`g

5qqYqqa Ybh'% K f]h|b[ hYqh

GhUfhXUhY. K YY\_\*%

@Yb[h\.'%) \$'!'&\$\$'k cfXg"'(\$'a ]bi hYg'Xi f]b['WUgg'h]a Y"

8YW]`q.

G\cfhYggUmf%) \$! &\$\$ k cfXgŁž (\$ a ]bi hYg Xi f]b[ WUgg h]a Y"

#### 5XX]h]cbU`XYhU]`g.

Mci 'k ]```bYYX'hc'gi Va ]h'U'g\cfhik f]hh'b'k cf\_"'=iWci `X'VY'Ub'Ya U]`ž'U'`Yhh'fž'U'X]Ufmcf'V`c[ 'Ybhfmž'YhW'K Y k ]```k cf\_'cb'h\]g'hmdY'cZhYI hg'Xi f]b[ 'h\Y'Wci fgY''

5ggYgga Ybh& ±0!7 `Ugg Ei ]n

GhUfhXUhY. K YY\_')

@Yb[h\." \$ a ]bi hYg"

#### 8 YW] g.

 $H\Y'ei\]n'a\ Um'VY'hYgh]b[\ 'j\ cWUVi\ `Ufmž'[\ fUa\ a\ Ufž'W'\ `hi\ fYž'`]ghYb]b[\ 'zfYUX]b[\ 'UbX\#cf'k\ f]h]b[\ 'g_]``g''`@Yb[\ h\.''\ $a\ ]bi\ hYg'''H\Y'ei\ ]n'a\ Um]bWi\ XY'gYj\ YfU``hmdYg'cZei\ Ygh]cbg'gi\ W\.'Ug'''Z]```]b'h\Y'V`Ub_g''ža\ i `h]d`Y'W\c]WYZg\ cZei\ Ygh]cbg'gi\ W''Ug'''Z]```]b'h\Y'V`Ub_g''Za'''$ 

: YYXVUW\_dfcj ]XYX'j ]U'Fi Vf]Wgžk f]HYb'Wca a Ybhg'UbX']b!WUgg'ZYYXVUW\_"

#### 5XX]h]cbU``XYhJ]`g.

Ei]n'Xif]b['h\Y'gYWcbX'hihcf]U`

Hi fb]hjb gYhjb[. H\]g ]g bchU Hi fb]hjb Ugg][ ba Ybh

5ggYgga Ybh' . CfU`

GhUfhXUhY. K YY\_" ! C b[c]b[

**@Yb[h**\. 'j Uf]Yg

8YHJ]`g.

5 'g\cfhcfu``hYgh'`Cb[c]b[ 'UbX`Z]bU``f) 'a ]bi hYgŁ'VmUddc]bha Ybh'

Cfu`ZYYXVUW\_'UbX'a Uf\_g'dfcj ]XYX"

#### 5XX]h]cbU`XYhU]`g.

H\]g"hYgh\Ug"&"Wca dcbYbhg.

- #b WUgg h hcf]U`f%\$1 Ł
- : ]bU`hYgh]b`k YY\_'- ff%\$ı Ł

#### 8YHJ]`g.

**@Yb[h**\.'j Uf]Yg

8YHJ]`g.

 $\label{lem:conditional} Dcfh \hbox{$\mathbb{Z}^*$ is $\mathbb{Z}^*$ or $\mathbb{Z}^*$$ 

: YYXVUW\_'dfcj ]XYX'cb`]bY"

5XX]h]cbU`XYhU]`g.

9j Yfmik YY\_imci ibYYXihciWca diYhYiYl YfWjgYgiZfca imci fi6CC? J  $\pm$ 15A  $\pm$ 15 i5%"H\YgYiYl YfWjgYgiUfYidUfh cZh\YiYYWh fYgžhh hcf]UigžUbXiK 99? @M\ca Yk cf\_"

9LD97H98"H9LHG."J + H5A + B5"5% - @Ž9>"

Mci f'9DCFH: C @€ 'k YY\_`mik cf\_'UbX'fYgci fWfg'UfY']b 'mci f'hYl hifYgch

‰

# FYgci fWyg

### DfYgWf]VYX F Ygci fWfg

9j Yfmik YY\_'mci 'bYYX'hc'Wca d'YhY'Yl YfW]gYg'Zfca 'mci f'6CC?'J  $\pm$ 15A  $\pm$ 35'5%"'H\YgY'Yl YfW]gYg'UfY'dUfhicZ h\Y'`YWhi fYgžhi hcf]U'gžUbX'K 99? @M\ca Yk cf\_"'

9LD97H98"H9LHG."J #15A #35"5%"

Mci f'9DCFH: C @€ ž'k YY\_`mik cf\_ž'UbX'fYgci fWfg'UfY']b'mci f'hYl hUbX'Yl YfW]gY'Vcc\_gž'h\YfYZcfY'mci bYYX'h\YgY'Vcc\_g'cb'XUmcbY'k YY\_'%"6ch\ž'h\Y'X][]hU``hYl hUbX'Yl YfW]gY'Vcc\_ž'WcghU'hchU``cZ` &(Uddfcl 'fl Gł"

%"DifWkUgY'h\Y'X][]hU``Vcc\_g`cb`\hhdg.#g\cd"V`]b\_`YUfb]b["Wca!'@cc\_'Zcf'J +15A +35'5%'5@Z9>"

&"'CbWY'mci'\UjY'h\Y'Vcc\_g'c]b'cif'WUgg'-BHFC8I7HCFM'5'k]h\'h\Y'Zc``ck]b['WcXYg"

HI HCF = 5@D = 65F '7 'Ugg '7 c XY. '7 @ 6 ( %+, )

HI HCF=5@85@5 7 `Ugg 7 c XY. 7 @\* +) &, +, '

 $C \, bW \, [mci \, ] \, bYX \, [h] \, Y \, [w] \, g \, [k] \, [h] \, [h] \, Y \, [w] \, cyg \, [w] \, [h] \, [h] \, Y \, [w] \, g \, [w] \, [h] \,$ 

 $< UfX`Wcd]Yg`UfY`Uj`U]`UV`Y`Uh'h\Y`I BGK `Vcc_'ghcfY"`H\Y`\UfX`Wcdmj Yfg]cb`]bWl XYg`U`WcXY`hc`^c]b`h\Y X][]hU``d`UhZcfa `UbX`U```h\Y`fYgci fWYg"$ 

\hdg.#k k k "Vcc g\cd"i bgk "YXi "Ui #XYhJ]`a"W ]3+19A B C 1 - +, , ( %\* +, &' ) &

→ DCFH5BH! Mci AIGH\UjYh\YgYVcc\_g cb XUm%k YY\_ %"

FYWca a YbXYX FYgci fWYg

GYY ci f'A ccX'Y 7 ci fgY Zcf Z ff\Yf fYgci fWYg"

7 ci fgY 9j U'i Uh]cb UbX 8 Yj Y'cda Ybh

7 ci fgYg UfY dYf]cX]\WU``mfYj ]Yk YX UbX ghi XYbhg' ZYYXVUW\_ ]g i gYX hc ]a dfcj Y h\Ya "'

: YYXVUW\_`]g`[ Uh\YfYX`Zfca `gh XYbhg`i g]b[ `a m9I dYf]YbWY"`±i]g`YbWci fU[ YX`gh XYbhg`Wca d`YhY`h\Y]f

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

# Hi fb]h]b Gi Va ]gg]cb

#mci 'YbWci bhYf'U'dfcV'Ya 'k \Yb'UthYa dhJb[ 'tc'gi Va ]hmci f'Ugg][ ba Ybht\fci [ \'Hi fb]hJbž'd'YUgY
hY'Yd\cbY'9| hYfbU'Gi ddcfhcb'-', ) '''' %cf'Ya U]''h\Ya 'cb'Yl hYfbU'hY'hgi ddcfh4 i bgk "YXi "Ui '"'Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft\"'#c'i'#c'i'
UfY'i bUV'Y'hc'gi Va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZUi 'hk ]h\ 'Hi fb]hJb'mci 'a Um'Udd'm\Zcf'Ub'Yl hYbg]cb\ZVi hmci
a i ghfYhU]b'mci f'hJ\W\_Yhbi a VYf'\Zfca '9| hYfbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fY'Yj UbhXcW a Ybhgthc
]bWi XY'Ug'Yj ]XYbW''hc'gi ddcfhmci f'Yl hYbg]cb'Udd']\WUh]cb'''#c''' Zmci 'Ya U]''9| hYfbU''Gi ddcfhmci 'k ]'`

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

# 5 WUXYa ]W = b Zcfa Uh cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei YghgʻZcfʻYl hYbg]cb/
- `UhY`gi Va ]gg]cbg`[ i ]XY`]bYg/
- fYj]Yk cZa Uf\_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc`]W[Yg/
- YI Ua ]bUh]cb dfcW/Xi fYg/
- qdYWJU``Wcbq]XYfUHjcb`]b`H\Y`Yj YbhcZ]``bYgg`cf`a ]gUXj Ybhi fY/
- gh XYbhYei ]mUbX X]gUV]`]m/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY

\htdg.#k k k "Uffg"i bgk "YXi "Ui #W ffYbhgh XYbhg#UWUXYa ]W]bZcfa Uhcb#dfchcWc`g![i ]XY`]bYg#

.

#### =a U[ Y 7 fYX]h

 $\mathsf{D} \mathsf{Chc} \mathsf{Vm} \underline{\mathsf{F}} \mathsf{CXc} \mathsf{X}\underline{\mathsf{C}} \mathsf{'} \underline{\mathsf{7}} \mathsf{`} \underline{\mathsf{J}} \mathsf{'} \underline{\mathsf{7}} \mathsf{ca} \mathsf{'} \underline{\mathsf{D}} \mathsf{Y} \mathsf{J} \mathsf{Y} \mathsf{\dot{g}}$ 

.

#### 7F = 7CG

7F = 7CG Dfc| XYf 7cXY . \$\$\$-,;

#### 5W\_bck `YX[ Ya YbhcZ7ci blfm

 $K Y : UW\underline{b}ck : YX[Y : h, Y : 6YXY[U : dYcd : Y : k \setminus c : UfY : h, Y : hfUX]h\underline{b}cb : W : ghcX]Ubg : cZh, Y : UbXg : cb : k \setminus JWX : I B G K : Ybg]b[hcb : WUa di g : ]g : cWUhYX"$