

# **ARTS2249**

9bj ]fcba YbłU`D\]`cgcd\m

HYfa 'C bY''##'&\$&%

# 7ci fgY<sup>·</sup>Cj Yfj ]Yk

## GhJZZ 7 cbhJWh 8 YhJ] `g

7cbj Ybcfg

## 7ci fgY 8YHJ] g

#### 7 fYX]hDc]bhg \* \*

### Gi a a UfmcZh\Y'7ci fgY

 $H\g`Wci\ fgY`YI\ Ua\ ]bYg`]bZi\ Ybh]U``h\Ycf]Yg`cZbUhi\ fY`UbX`h\Y`Ybj\ ]fcba\ Ybh]b``d\]cgcd\m`UbX`U`fUb[\ Y`cZ\ ]bhYfX]gW]d`]bUfmk\ f]h]b[\ gž\Zfca\ `5f]ghchY`hc`h\Y`dfYgYbh'``H\Y`Wci\ fgY`YI\ d`cfYg`h\Y`Zc``ck\ ]b[\ 'ei\ Ygh]cbg. `=gh\YfY`U`WcbbYW]cb`VYhk\YYb`\ck\ 'bUhi\fY`\Ug`VYYb`WcbW]j\ YX`]b`d\]`cgcd\m`UbX`gW]YbW'`UbX`h\Y`W ffYbh\Ybj\]fcba\YbhU``Wf]g]g3`=g`h\Y`bch]cb`cZbUhi\fY`gh]```U`a\YUb]b[\Z\ ``h\fa\ `]b`h\Y`5bh\fcdc\WbY3``K\Uhi]g`h\Y\X]ZYfYbW'`VYhk\YYb`'bUhi\fY``UbX``W``hi\fY`3`<ck\g\ci\X`\ia\Ubg`i\bX`fgH\bX`h\Y]f`fY``Uh]cbg\]d`hc`'bUhi\fY`3``$ 

H\YgY'ei Ygh]cbg'k ]``VY'UXXfYggYX'\]fca 'U'fUb[Y'cZdYfgdYWh] Ygz'gi W\'Ug. 'D\Ybca Ybc`c[mz'7f]h]\W\\\ H\Ycfmz'; Yfa Ub'F ca Ubh]\M]ga z'Ybj ]fcba Ybh'U`Yh\]\Wz'9\WzYa ]b]ga z'\WcbhYa dcfUfmih\ci [\hiUbX'bcb! K YghYfb'UddfcU\\\Yg''8fUk ]b['cb'h\YgY'X]j YfgY'ffUX]h]cbgz'h\Y'\\Wci fgY'Yl Ua ]bYg'dcgg]\V'Y'U'h\YfbUh]j Yg'\Zcf i bXYfghUbX]b['h\Y'\i a Ub!bUhi fY'X]j ]XY''

### 7 ci fgY @YUfb]b[ Ci hWca Yg

- %" 9j U`i UHY'h\Y'a Yf]hg'UbX``]a ]hUh]cbg'cZ\_Ym'UddfcUW\Yg'hc'9bj ]fcba YbhU``D\]`cgcd\m
- &" 7 f]h]WU``mUddfU]gY`h\Y`fY`Yj UbWY`cZ\]ghcf]WU``UbX`WcbhYa dcfUfmUddfcUW\Yg`hc`bUhi fY`UbX`h\Y Ybj ]fcba Ybhhc`h\Y`WcbhYa dcfUfmWcbhYl h
- ' " 5ggYgg`X]ZZYfYbh\_YmUddfcUWXYg`hc`bUh fY`UbX`h\Y`Ybj ]fcba Ybh]b`d\]`cgcd\mUbX`WcbhYa dcfUfm Ybj ]fcba YbhU``h\ci [\h
- ("G\ck WUgY`\_Ymih\YcfYh]WU``]bei ]fmig\_]``g`cZh\Y`\i a Ub]h]Yg`h\fci [\`Wf]h]WU``UbU`mg]gž'hYl hi U` ]bhYfdfYhUn]cbž'Ub[i a Ybhž'UbX`k f]hhYb`Yl dfYgg]cb

HYUW ]b[ 'GhfUhY[

lba Yc

### 5ggYgga Ybh

GYY'Wci fgY'A ccX`Y'dU[Y'Zcf'XYHJ]`g'

### 5ggYgga YbhHUg\_g

5ggYgga Ybh'hJg_	K Y][\h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
G\cfh'YggUm	&) ı	% #\$' #&\$&%`%% \$\$`DA	% <u>`</u> (
F YgYUfW\`dcfh7c`]c'	&) ।	&' #\$(#&\$&%`%%\$\$`DA	%ž'' ž'(
9ggUmi	) \$1	' \$#\$(#&\$&%`%%\$\$`DA	%ž'&ž'' ž'(

### 5ggYgga Ybh'8YhJ]`g

5ggYgga Ybh% G\cfhYggUm

GhUfhXUhY. Bch5dd`]WUV`Y

**@Yb[h**\.'%\\$\$'k cfXg

8YHJ]`g.

 $H\[g']g'U'g\cfhYggUm\check{z}'Wca\ d'YhYX'k\]h\[h\]b'h\Y'\[fghZ]j\ Y'k\ YY\_g'cZh\Y'Wci\ fgY'h\Uh']g'df]a\ Uf]'m'U'WcgY'fYUX]b[Yl\YfW]gY'cZUb']a\ dcfhUbhk\ cf_']b'Ybj\]fcba\ YbhU'h\ci\ [\h''$ 

: YYXVUW\_`]g`k f]hhYb`UbX#cf`a Uf\_]b[ 'g\YYhcf'fi Vf]\W

@Yb[h\"\&\\$\"k cfXg

#### 5XX]h]cbU`XYhJ]`g.

8YHJ]YX ]bglfi Whcbg k ]``VY Uj U] UV Y cb A ccX Y .

Hi fb]hjb gYhjb[. `H\]g Ugg][ ba Ybh]g gi Va ]hhYX h\fci [\`Hi fb]hjb UbX gh XYbhg Xc bchgYY Hi fb]hjb g]a ]`Uf]hmfYdcfhg"

5ggYgga Ybh& F YgYUfW dcfhZc`]c

GhUfhXUhY. Bch5dd`]WUV`Y

**@Yb[h**\. '&\$\$\$ k cfXg A ]b]a i a

8YW]`q.

5 'fYgYUfW\`dcfhZc`]c'k\YfY'ghi XYbhg'UXXfYgg']ggi Yg'UbX'ei Ygh]cbg'Zfca 'h\Y]f'k YY\_`mfYUX]b[g'Zcf'h\Y Wti fgY"'H\Y'dcfhZc`]c']g'U'fYWtfX'cZghi XYbhgfidfYdUfUh]cb'Zcf'WUgg'YUW\'k YY\_"

Gh XYbhg·k]``k f]hY`Ug·U·a]b]a i a '\* 'I '' \$\$!k cfX'fYUX]b[ 'gi a a Uf]Yg·UbX'hYI hi U``UbU`mg]g·Ug·k Y```Ug·U

&\$\$!kcfX`fYZYWhjcb`cb`h\Y`fYgYUfW\`dcfhZc`]c`]hgY`Z'

 $H\Y'k\ f]hhYb'Zcfa'\ cZh\Y'dcfhZc`]c']g''Ygg'Zcfa'\ U''h\Ub''Ub'YggUm'UbX'a'\ UmVY'U'a']l'\ cZdfcgY'UbX'Vi'''Yh'dc]bhg''$ 

H\]g`dcfhZc`]c`fYei]fYg`ghiXYbhg`hc`fYZYWhcb`h\Y]f`ckb``YUfb]b[`]b`h\Y`WcifgY`UbX`\ck`UggYggaYbhihUg\_gž]bWiX]b[`h\Y`fYgYUfW\`dcfhZc`]c`]hgY`Zz`aUm\Wcblf]ViHY`hc`h\]g"

: YYXVUW\_`]q`k f]hhyb`UbX#cf`cfU`"`

@Yb[h\]g'Uddfcl "'&\$\$\$ k cfXg

#### 5XX]h]cbU`XYhU]`g.

8YHJ]`YX`]bglfi Wf]cbg'k]``VY`Uj U]`UV`Y`cb'AccX`Y`

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$ 

5ggYgga Ybh' . '9ggUm

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\. &&\$\$ k cfXg

8 YHJ] g.

 $\label{eq:localized-equation} $$H^[g]_g^hY'Z_bU^*UggYgga\ YbhiHUg_'Zcf'UlhYbXUbW'di\ fdcgYg'':\ YYXVUW_']g'k\ f]hHYb'Wca\ a\ Ybhg'UbX'cf'U a\ Uf_]b['gYYh$ 

A U'cf'YggUmcb'U'\_Ym'][ifY'cf'h\YaY']b'h\Y'WcifgY"

&&\$\$ k cfXg

#### 5XX]h]cbU`XYhU]`g.

8 YHU] YX ]bghfi Whjcbg k ] "VY Uj Uj UV Y cb A ccX Y "

## 5 hm/b XUb VW F Yei ]fYa Ybhg

 $\hbox{Ghi XYbhg'UfY'ghfcb[`mYbWci fU[YX'hc'UhhYbX'U```WUggYg'UbX'fYj]Yk ``YWhi fY'fYWcfX]b[g''] }$ 

## 7ci fgY GWX YXi `Y

J]Yk 'WUgg'hja YhUV'Y

H]a YtUV`Y

8 UHY	Hmd Y	7 cbhYbh
K YY_'%'%)': YVfi Ufmi!'%	@YWnifY	H\Y`8]gYbW\Ubha YbhcZBUhi fY
: YVfi Ufm		Zcf'fYUX]b[g'UbX'Zlfh\Yf']bZcfaUh]cb'gYY'accX`Y `YWhifY'cih]bY'UbX'fYUX]b[``]gh

K YY_"%\$."% "5df]`"!"&"	@YWhifY	BUh fY'UbX'F Ygdcbg]V]`]hm
5df]`		
		ZcfifYUX]b[giUbXiZifh\Yfi]bZcfaUh]cbigYYiaccXiY
		`YWnfY`cih]bY`UbX`fYUX]b[``]gh

## FYgci fWfg

DfYgWf]VYX F Ygci fWYg

 $GYY^{T}A ccX^{T}Y^{T}dU[Y^{T}Zcf^{T}XYhU]^{T}g$ 

FYWca a YbXYX FYgci fWYg

GYY'A ccX`Y'dU[Y'Zcf'XYhJ]`g

### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

: YYXVUW\_`Zcf`h\]g`Wci fgY`k]```VY`[ Uh\YfYX`]bZcfa U``mih\fci [\ci h]hg`Xi fUh]cbž'UbX`Zcfa U``m'Uhih\Y`YbX j]U`a m9I dYf]YbW'''`='Ua `U`k Umg`\_YYb`hc`\YUf`Zca `mci `k\Uhik cf\_g`UbX`k\UhiXcYg`bchž'YgdYW]U``m'Ug`h\]g ]g`U`bYk`Wci fgY''8c`bch\Yg]hUhY`hc`WcbhUWfa Y`k]h\`UbmWca a Ybhg`cf`ei Yf]Yg"

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hy'Yd\cby'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cby''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk ]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

## 5WUXYa]W±bZcfaUhjcb

: cf'YggYbhJU`ghi XYbh]bZcfa UhJcb'fY`UhJb['hc.

• fYei Yghg Zcf Yl hYbg]cb/