

# 7ci fgY<sup>·</sup>Cj Yfj ]Yk

# GhJZZ 7 cbhJWh 8 YhJ] g

# 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]hm	@cWUhjcb	D\cbY
B]Wkc`Ugʻ8ci a Ub]g	b"Xci a Ub]g4 i bgk "YXi "Ui	%4%AcbXUm	A cfj Yb 6fck b	\$( &\$' %**' *

GW.cc`7cbhJWh′±bZcfa Uh]cb

## 7ci fgY 8YHJ] g

### 7 fYX]h'Dc]bhg'\*

## Gi a a UfmcZh\Y 7ci fgY

H\]g'Wci fgY'YI d'cfYg'h\Y'XmbUa ]WzX]j YfgYz'UbX'hfci V'YX'Wj ]]nUhjcb'cZ; fYYW'Xi f]b[ 'h\Y'6fcbnYz 5fW\U]Wz'UbX'7`Ugg]\WJ'YfUg'f\Wjf\WJ'\&\\$!' &' '67 \mathbb{L}'' \mathbb{L}' \mathbb{L}' \mathbb{L}'' \mathbb{L}''' \mathbb{L}'' \mathbb{L

## 7 ci fgY @YUfb]b[ 'Ci hWca Yg

- %" 9 I d`U]b h\Y bUhi fY cZ7 `Ugg]\W\`; fYY\_`\]ghcfmž\W\`hi fY 'UbX'gc\\JYhm'\Zca 'h\Y'6fcbnY'5 [ Y hc 5`Y I UbXYf'h\Y'; fYUh
- &" I gY`U`VfcUX`VcXmcZ\_bck`YX[Y`UVci hih\Y`; fYY\_`k cf`X`UbX`]\g`W``hi fYg`\c'i bXYfg\UbX`\h\Y dfc\W\gg\Yg`cZ\]g\cf]\W\``\\Ub[Y
- ' " 5 ft]W `UhY 'h\Y `XmbUa ]Wg cZ; fYY\_'gcW]Yh]Yg 'UbX W `hi fYg 'UWfcgg 'h\Y A YX]hYffUbYUb
- ("I bXYffU\_Y`f][cfcig'UbX`Yb[U[YX`gW\c`Uf`mYbei]fm'UbX`UbU`mg]g'ig]b[`X]ZZYfYbhZcfag'cZ\]ghcf]WU` Yj]XYbWY
- ) " 7 f]h]WU``mbUj ][ UhY`X]ZZYfYbhX]gW]d`]bUfmVcX]Yg`cZ\_bck `YX[ Y`YgdYW]U``mhYl hi U``UbU`mg]gž'UbX UfWXUYc`c[ ]WU``UbX`Ubh\fcdc`c[ ]WU``dYfgdYWfj Yg
- \*" 9b[U[Y'k]h\'Wad`Yl'dfcV`Yag`cZ\]ghcf]c[fUd\mž'd\]`c`c[mž'UbX'UfW\UYc`c[mfY`Uh]b['hc'h\Y UbW]Ybh; fYY\_'k cfX"

## HYUW ]b[ 'GhfUhY[ ]Yg'

< ]ghcfm]g`Ub`UWhji Y`YbXYUj ci fž`]bj c`j ]b[ 'h\Y`Wf]h]WD``UbU`mg]g`cZdf]a Ufmgci fWgž`UbX`h\Y]f`gi VgYei Ybh i gY`hc`fYWcj Yf`UbX`]bhYfdfYhi\Y`dUgh'' < ]ghcfm]g`bchh\Y`dUgg]j Y`a Ya cf]nUh]cb`cZbUa Ygž`XUhYgž`UbX Yj Ybhgž`U`h\ci [\`U`WfhU]b`Ua ci bhcZ\_bck `YX[Y`]g`bYWfggUfmZcf`]bZcfa YX`UggYgga YbhUbX`i gY`cZ df]a Ufmgci fWfg"'H\]g`Wci fgY`dfca chYg`UWhj Y`Yb[U[Ya Ybhk]h\YhUh]cb`cZdf]a Ufmgci fWfg"'K YY\_`mWcbhYbhdfcj ]XYg`Ub`]bhfcXi Whjcb`hc`\_Ymh\Ya Yg`]b`; fYY\_\]bhYfdfYhUh]cb`cZdf]a Ufmgci fWfg"'K YY\_`mWcbhYbhdfcj ]XYg`Ub`]bhfcXi Whjcb`hc`\_Ymh\Ya Yg`]b`; fYY\_\]\]ghcfm"5ggYgga Ybhg`dfcj ]XY`ghi XYbhg`k]h\`h\Y`cddcfhi b]hmhc`Yl d`cfY`hcd]Wg`k\]W\`YgdYWJU``m`]bhYfYgh h\Ya "</p>

H\Y'V'YbXYX'a cXY'cZXY`]j YfmdfYgYblg'cb`]bY'WtblYbhi]b'h\Y'Zcfa 'cZfYUX]b[ gžj ]XYcgž[ U``Yf]Ygžei ]nnYgž UbX'Ui X]c"'H\Y'df]a UfmVYbYZ]hcZh\]g'Zcfa 'cZXY`]j Yfm]g'h\Y'ZYI ]V]`]lmghi XYblg'UfY'UZcfXYX"'K YY\_`m WtblYbhiUg\_g'UfY'WYUf'mXYZ]bYX'UbX'WUb'VY'Wta d'YlYX'UhUbmdc]bhXi f]b[ 'h\Y'k YY\_"'Hi hcf]U'g'Vi ]'X'cb cb`]bY'WtblYbh!']b'WUgg'X]gW gg]cb'dfcj ]XYg'ghi XYblg'k ]h\'h\Y'cddcfhi b]lmhc'fYZ]bY'UbX'W\U``Yb[ Y'h\Y ]bHYfdfYhJhcbg'h\YmZcfa YX'Xi f]b[ 'h\Y'dfYWX]b[ 'k\YY\_"

5ggYgga YbhgʻUfYʻWcgY`mU`][ bYX`k ]h\`cb`]bY`UbX`h hcf]U``WcbhYbh'`G\cfnk f]h]b[ 'Ugg][ ba Ybhgʻdfcj ]XY ghi XYbhgʻk ]h\`h\Y`cddcfh b]hmhcʻfYWY]j YʻfY[ i `Uf`ZYYXVUW\_cb`h\Y]f`dfc[ fYggʻh\fci [\ci hhYfa ž'UbX`\cbY h\Y]f`k f]h]b[ 'g\_]``g"'H\YgY`Ugg][ ba YbhgʻU`gcʻdfcj ]XY`h\Y`cddcfh b]hmZcf`gh XYbhgʻhcʻZcfa 'Uf[ i a YbhgʻUbX hYghh\Ya ']b`h hcf]U`g"'7 f]h]WU``UggYgga YbhcZgci fWygž\_bck `YX[ Y`cZgci bX`\]ghcf]WU``a Yh\cXc`c[ mž'UbX

WfYUłij Y Yb[ U[ Ya Ybhk ]th `\]ghcf]WU`dfcV`Ya gʻUfYʻtYghYX`]bʻthYʻfYa U]b]b[ 'UggYgga Ybhg"'Gi Wk`g\_]``gʻUfY VYghXYj Y`cdYX`k \Ybʻghi XYbhgʻUWlj Y`mdfcXi WY`\]ghcf]WU``Uf[ i a Ybhgʻth\fci [ \ X]gW gg]cbʻUbX`YgdYV]U``m k f]h]b[ "'=bXYYXž'k f]h]b[ 'UbʻUf[ i a YbhUh]j Y`\]ghcf]WU``YggUm]gʻth\Y`WcfY``YUfb]b[ 'UWlj ]hm]bʻth\]gʻWci fgY`—'Ub YggUmVUgYX`cb`df]a Ufmgci fWYʻfYUX]b[ gž]bZcfa YX`VmWci fgY`WcbhYbhUbX`gYWcbXUfmgci fWYʻfYUX]b[ g"

# 5ggYgga Ybh

GYj YfU``k YY\_g`VYZcfY`h\Y`YbX`cZhYfa ž`ghi XYbhg`k ]```gi Va ]hiUb`UbbchUhYX`V]V`]c[fUd\mcZgci fWYg`hc`VYi gYX`]b`h\Y]f`fYgYUfW\YggUmg"`+b`UXX]h]cb`hc`h\Y]f`UbbchUhYX`V]V`]c[fUd\mžghi XYbhg`k]```Wca dcgY`U`h\Yg]g ghUhYa YbhiUbX`]bhfcXi Whjcb"`H\]g`UggYgga Ybhk]```dfcj ]XY`ghi XYbhg`k]h\`ZYYXVUW\_`cb`h\Y]f`dfc[fYgg VYZcfY`h\Y`fYgYUfW\YggUm]g`gi Va ]hhYX`Uhih\Y`YbX`cZhYfa "

 $: YYXVUW\underline{'}]g'cZZYfYX']b'h'Y'Zcfa'cZfi Vf]Wg'fk \]WX'UfY'Uj U]'UV'Y'cb'A ccX`YŁ'UbX'Wca a Ybhg'cb'gdYW]Z]WUgdYWhg'cZh'Y'UggYgga Ybh'$ 

#### 5XX]hcbU`XYHJ]q.

D`YUgY`bchY`h\Y`gi Va ]gg]cb`XUhY"`H\Y`Ugg][ ba Ybhg\ci `X`VY[ ]b`k ]h\`h\Y`dUfU[ fUd\!`Yb[ h\`h\Yg]g ghUhYa Ybh'`H\Yb']hg\ci `X`dfcj ]XY`fbb`h\Y`gUa Y`XcW a Ybhz`h\Y`UbbchUhYX`V]V`]c[ fUd\m''±ia i ghdfcj ]XY h\Y`Z\```XYhU]`g`cZh\Y`Z]fgh`]hYa `fVcc\_`cf`Uff]WYžYhWzzZc``ck YX`Vmmci f`Wca a Ybhg`cb`h\Uh`]hYa "`8c`h\YgUa Y`k]h\`h\Y`fYgh'

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tab$ 

5ggYgga Ybh' . F YgYUfWl YggUm

@Yb[h\.'&\$\$\$

#### 8YW]`q.

5 hith Y 'YbX'cZth Y 'hYfa ž'ghi XYbhg'k]``'gi Va]hU'fYgYUfW. 'YggUmcb'cbY'cZUb'Uggcfha YbhcZhcd]Wg'`]ghYX'cb A ccX`Y"

: YYXVUW\_`]g`cZYfYX`]b`h\Y`Zcfa `cZfi Vf]Wg`fk\]W\`UfY`Uj U]`UV`Y`cb`A ccX`YŁ`UbX`Wca a Ybhg`cb`gdYV]Z]W UgdYWg`cZh\Y`UggYgga Ybh'

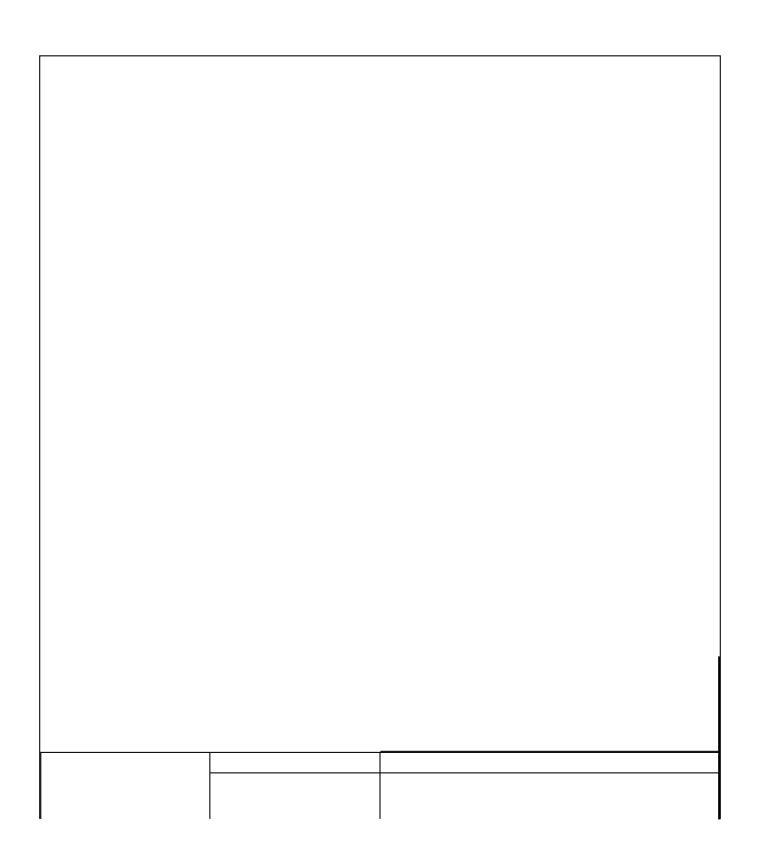
H\]q`]q`h\Y`Z|bU``UgqYqqa YbhZcf`di fdcqYq`cZh\Y`UhhYbXUbWY`fi `Y"

### 5XX]h]cbU`XYhJ]`g.

H\Y`YggUma i ghVY`VUgYX`cb`df]a Ufmgci fWfg": #ia i ghU`gc`VY`VUgYX`cb`a cbc[fUd\g`k f]hhYb`VmYl dYfhg. gdYWJU`]nYX`Vcc\_gžUfhJWYgžVcc\_`W\UdhYfg`cf`k YV`gci fWfg`h\UhfY`UhY`WcgY`mhc`mci f`hcd]W'`D`YUgY`i gY`Uh`YUghi%\$`a cbc[fUd\g`UbX``]ghh\Ya`]b`mci f`V]V`]c[fUd\m''Mci 'g\ci `X`U`gc``]ghU```ch\Yf`gci fWfg`fdf]a Ufm qci fWfžhYl hVcc\_g`YhWz`Vi hH\YmUfY`bchhc`VY`Wci bhYX`]b`mci f``]qhcZ%\$`a cbc[fUd\q"

5b YggUm`]ghk ]``VY dfcj ]XYX VmK YY\_&UbX dcghYX cb A ccX`Y"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b" gYhh]b" . "H\]g" Ugg][ba Ybh]g" gi Va ]hhYX" h\fci [\ 'Hi fb]h]b" UbX" gh XYbhg" Xc" bchgYY" Hi fb]h]b g]a ]`Uf]hmfYdcfhg" \\ \end{tabular}$ 



		C b!`]bY``YWhifY`h\UriWcj Yfg`h\Y`\]ghcfmUbX`df]bW]dU`h\YaYg`cZh\Y`dYf]cX`	
K YY_'+.'&- 'A UfWX'!'&	Hi hcf]U`	H\Y`XYUh\cZGcWfUhYg.`KUg`GcWfUhYg`[i]`hm3	
5df]`	Cb`]bY'5Wfj]hm	K YV fYUX]b[g'UbX'ch\Yf']hYa g'fY`UhYX'hc'h\Y h\Ya Yg'cZGYI 'UbX'G`Uj Yfm]b'5bW]Ybh; fYYWY	
		C b!`]bY``YWfi fY`h\Uh'Wcj Yfg`h\Y`\]ghcfm'UbX'df]bWjdU` h\Ya Yg`cZh\Y`dYf]cX`	
K YY_', .') '5df]`!'- '5df]`	Hi kcf]U`	GYI 'UbX'GYI i U`]hm'K \Uhk YfY'h\Y'fi 'Yg'[cj Yfb]b[gYI 3'A cfY'VfcUX`mž'k \Uhifc`Yg'k YfY'a Yb'UbXk ca Yb'YI dYWYX'hc'd`Um']b'7`Ugg]WU`'5h\Yb]UbgcW]Yhm3	
	Cb`]bY'5WMjj]hm	5 Whjj jhjyg:fy`UhyX`hc`h\Y`fjgY`cZA UWYXcb`UbX 5`YI UbXYf`h\Y`; fYUh Cb!`jbY``YWhi fY`h\Uh'Wcj Yfg`h\Y`\jghcfm'UbX`dfjbWjdU`	
		h\Ya YgʻcZh\YʻdYf]cX'	
K_YY_``%&'5df]`!'%* 5df]`	Hi hcf]U`	5`YI UbXYf'h\Y'; fYUh' <ck '[="" fyuhk="" ubxyf3<br="" ug'5`yi="">K \Uhk YfY'\]g'U]a g'Ug'Wcbei Yfcf3</ck>	
	C b`]bY'5Wf]j ]hm		
		C b!`]bY``YWn fY'h\Uh'Wcj Yfg'h\Y'\]ghcfm'UbX'df]bW]dU` h\Ya Yg'cZh\Y'dYf]cX'	
K YY_'%\$.'%-'5df]`'!'&' 5df]`	Hi hcf]U`	< Y``Yb]gh]WK cf`X.`K \Uhik YfY`h\Y``Y[UW]Yg`cZ 5`YIUbXYffg`Wcbei Yghg3	
	C b`]bY 5 Wfj ]hm	5 Wfj ]hjYg:fY[ UfX]b[ 'h\Y'; fYY_g:UbX'k \Uh'h\Yma YUb hc:h\Y'a cXYfb:k cf:X	
		C b!`]bY``YWfi fY`h\Uh'Wcj Yfg`h\Y`\]ghcfm'UbX'df]bWjdU` h\Ya YgʻcZh\Y'dYf]cX'	

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

5```fYgci fWyg'UWWygg]V`Y'cb!`]bY

FYWca a YbXYX FYgci fWYg

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

## Hirtholdbisjoyca ]gg]cb

⊋mci YbWci bhYf`U'dfcV`Ya k\Yb`UhYa dhjb['hc'gi Va ]hmci f`Ugg][ ba Ybhh\fci [\'Hi fb]hjbž'd`YUgY
hY`Yd\cbY`9| hYfbU``Gi ddcfhcb'-',,)'''' %cf`Ya U]``h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui "'`Gi ddcfh\ci fg`UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg`UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg`fl \*) 'XUmg`U'mYUfŁ'''⊋mci UfY'i bUV`Y'hc'gi Va ]hmci f`Ugg][ ba YbhXi Y`hc'U'ZJi `hk ]h\'Hi fb]hjb`mci 'a UmUdd`mZcf'Ub'Y! hYbg]cbžVi hmci a i ghfYhJ]b`mci f`hj\\Yhbi a VYf'Zfca '9| hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf`fY`Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj ]XYbWY'hc'gi ddcfhmci f'Yl hYbg]cb'Udd`]\WJhjcb'''⊋mci 'Ya U]`'9| hYfbU``Gi ddcfhmci 'k]``

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

# 5 WUXYa ]W + DZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb fY`UhJb[ hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va ]gg]cbg '[ i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK < YU`h\ UbX GUZYmdc`]W[Yg/
- YI Ua ]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a ]gUXj Ybh fY/
- gh XYbhYei ]hmUbX'X]gUV]`]hm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY