

# **IEST5007**

Dc`]hJWU``9Wc`c[m'GighU]bUV`Y'8YjY`cdaYbh'/'>ighJWY

HYfa 'C bY''##'&\$&%

## 7ci fgY<sup>-</sup>Cj Yfj ]Yk

## GHJZ7cbHJWh8YHJ]`g

## 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`]lm	@cWUh]cb	D\cbY
8 Ub]Y``F cV]bgcb	X"fcV]bgcb4 i bgk "YXi "Ui	,	F cca " *\$ A cfj Yb 6fck b	\$(+, '&'+-&

## GWXcc``7cbHJWhʻ±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b[ '<ci fg. 'A cbXUm'! : f]XUmž - Ua '! ') da

D\cbY.'Ž\*%'&'-',)'%\*,%

: UI . 'Ž \* %'&'-' , ) ', +\$)

9a U]`. \<u>\U`4 i bgk "YXi "Ui</u>

## 7ci fgY 8YHJ] g

### 7 fYX]hDc]bhg\*\*

### Gi a a UfmcZh\Y 7ci fgY

### 7 ci fgY @YUfb]b[ Ci hWca Yg

- &" 9 I d`U]b`YI hYfbU``dc`]hJWU``YWcbca ]W]bZi YbWYg`cb`h\Y`; `cVU``Gci h\#a U'cf]hmk cf`Xž'UbX`\ck 'h\YgY ]bZi YbWY''gi ghU]bUV`Y`XYj Y`cda Ybh'hmdY`W\U``Yb[ Yg`UbX`]bYei ]hJYg
- ' " 9 I d`cfY`UbX`UggYgg`h\Y`i gY`cZ[`cVU`žYWcbca ]Wž`Y[ U`žUbX`]bgh]hi h]cbU``ZfUa Yk cf\_g`Zcf UXXfYgg]b[ `Ybj ]fcba YbhU``]ggi Yg`]b`h\Y`WcbhYI hcZU`fUb[ Y`cZch\Yf`ghfi WhifU``]bYei U`]h]Yg`UbX W\U``Yb[ Yq
- ("8]gW gg'UbX'Wf]hjei Y'h\Y'YZZYWf] YbYgg'cZ]bghjhi hjcbU``cf'fY[i `UhcfmUddfcUW\Ygž'a Uf\_YhVUgYX UddfcUW\Ygž'hYW\bc`c[mhfUbgZYfž'WJdUWjmvi]`X]b[ž'UbX'ch\Yf'UddfcUW\Yg'Zcf'XYU`]b['k]h\forall 'gi ghJ]bUV`Y'XYj Y`cda Ybh'W\U``Yb[Yg
- ) "I gY'\_bck`YX[Y'cZ'gi gHJ]bUV`Y'XYj Y`cda YbH'VXU``Yb[Yg'UbX'H\Y'dfUW]VU``UbX'Udd`]YX'UHYa dhg hc'fYgc`j Y'h\Ya'hc'UbU`mgY'U'fUb[Y'cZ`cWJ`ž'bUh]cbU`ž'UbX'fY[]cbU``WUgY'ghi X]Yg

### HYUWX]b[ GhfUhY[]Yg

## 5ggYgga Ybh

 $8YHJ]^*YX^*Ugg][ ba Ybh]^bZcfa Uh]^cb^k]^*VY^*dfcj ]XYX^*]^b^*Ub^*Ugg][ ba Ybhg^YYh]^b^*a ccX^*Y^*8i Y^*hc^7CJ = 8!\%gca Y^*WXUb[ Yg^*hc^*h,Y^*dfYgYbHUh]^cb^*a UmVY^*fYei ]fYX"$ 

: cf'U```Ugg][ ba Ybhg'< Ufj UfX'cf'5D5']b!hYl hfYZYfYbW]b[']g'dfYZYffYXž'k ]h\`U'Z\```fYZYfYbW'``]ghUhh\Y'YbX'cZ mci f'Ugg][ ba Ybhg'fcb`mfYZYfYbW''h\Y'k cf\_g'h\Uhmci 'i gY'UbX'W]hYŁ"'5`k Umg']bWi XY'U'W]hUh]cb'k \Yb'i g]b[ gca YcbY'Y`gYfig'k cf\_'k \Yh\Yf']h]g'U'dUdYfž'Vcc\_ž'fYdcfhicf'k YVg]hY'': U]`]b['hc'\Uj Y'W]hUh]cbg']g Wcbg]XYfYX'd'U[]Uf]ga "': cf'h\Y'dfYgYbhUh]cbž'd'YUgY'\Uj Y'ga U```]b!hYl hfYZYfYbWfg'k \Yb'mci 'i gY'ch\Yfg k cf\_'X]fYWhmž'UbX'U'g]b[`Y'g`]XY'WcbhU]b]b[`mci f'X]fYWhmi gYX'fYZYfYbWfg'Uhh\Y'YbX'''

### 5ggYgga Ybh HUg\_g

5ggYgga Ybh'HJg_	K Y][\h	8i Y'8UHY	

5ggYgga Ybh& 9ggUm%

GhUfhiXUhY. Bchi5dd`]WUV`Y

@Yb[h\.'% \$\$'kcfXg'fd'ig'fYZYfYbW'gŁ

8 YW] g.

±bX]j ]Xi U`g\cfhYggUm"

5XX]h]cbU`XYhU]`g.

A cfY`XYfU]`g`k ]```VY`dfcj ]XYX`]b`f\Y`Ugg][ ba Ybhg\YYh]b`a ccX`Y"

Gi Va ]gg]cb bchYg. Gi Va ]hUg U D8: cf k cfX XcW]b A ccX Y

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' Hi & fb \end{tabular} $$ h' \end{tabular}$ 

5ggYgga Ybh' . 'A U'cf'fYgYUfW\ fYdcfh

GhUfhXUhY. Bch5dd`]WUV`Y

@Yb[h\."\$\$\$kcfXgfdigfYZYfYbWygŁ

8 YW] g.

 $A \ U\ cf \ fYgYUfW \ fYdcfhUbX \ WUgY \ ghi \ Xmcb \ gdYWZWhcd]WfgLWcj \ YfYX" \ \pm X]j \ ]Xi \ U`k \ f]hhYb \ ZYYXVUW\_ \ dfcj \ ]XYX" \ 5 \ ddfcl " \ $\$\$'k \ cfXg" \ H\ ]g \ ]g \ h\ Y \ ZbU` \ UggYgga \ YbhZcf \ UhhYb XUbVW \ di \ fdcgYg"$ 

#### 5XX]h]cbU`XYhU]`g.

: i ft\Yf`]bZcfa Ut]cb`]g`dfcj ]XYX`]b`t\Y`Ugg][ ba Ybhg\YYh]b`a ccX`Y"

Gi Va ]gg]cb bchYg. gi Va ]hUg U D8: cf k cfX XcW]b a ccX Y dcffU

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhf]b[."H\]g"Ugg][ba Ybh]g"gi Va]hhYX"h\fci [\Thi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"Uf]hmfYdcfhg" \\ \end{tabular}$ 

## 5 https://bxubwww.Fyei]fYaYbhg

Ghi XYbhgʻUfYʻghfcb[`miYbWci fU[YX'hcʻUhhYbX'U```WUggYgʻUbX'fYj ]Yk '`YWhi fY`fYWcfX]b[g"

## 7ci fgY'GWXYXi `Y

## J]Yk 'WUgg'ha YHUV'Y

## H]a YtUV`Y

8UhY	HmdY	7 cbhYbh
K YY_'%'%)': YVfi Ufm!'%- : YVfi Ufm	GYa ]bUf	H\Y'Wci fgY'fi bg']b']bhYbg]j Y'a cXY'Zfca '%\$Ua!) da cb'A cbXUmg'UbX'Hi YgXUmg']b'k YY_g'%l' žk]h\'U'%\ci f'VfYU_'Zcf'`i bW\'Zfca '%da 'YUW\'XUmUbX'g\cfhVfYU_g'Yj Yfm\ci f''H\Y'Wci fgY']g'gW\YXi 'YX'Zcf WUggYg'cb`]bY']b'6\UW_VcUfX'7 c``UVcfUhY'fh\fci [\accX'YE'5B8'ZJWY!fc!ZJWY'g]a i `hJbYci g`m''; ]j Ybh\Y'fYW\bh7CJ=8'ci hVfYU_z'h\Y'ZJWY!fc!ZJWY WUggYg'UfY'i b']_Y`m''\D'h\Y'Yj Ybhh\]g'W\Ub[Ygz'h\Y fcca g'k ci 'X'VY.
		A cb'%\$.\$\$'!'%'.\$\$'EiUXfUb[`Y'%\$(''f7!9%)!%\$(' Ł
		Acb'%(.\$\$'!'%+.\$\$'; c`XghY]b'; \$-'fP!8%*!; \$-Ł
		Hi Y'%\$.\$\$'!'% .\$\$'Ei UXfUb[`Y'; \$(\$'f7!9%)!; \$(\$E
		Hi Y'%(.\$\$'!'%+.\$\$'A Uh\Yk g'&&+'fP!: &' !&&+Ł
K YY_`&.`&&`: YVfi Ufm'!`&* : YVfi Ufm	GYa ]bUf	H\Y'WcifgY'fibg']b']bhYbg]jY'acXY'Zfca'%\$Ua!)dacb'AcbXUmg'UbX'HiYgXUmg']b'kYY_g'%'žk]h\'U'%\cif'VfYU_'Zcf'`ibW\'Zfca'%da'YUW\'XUmUbX'g\cfhVfYU_g'YjYfm\cif''H\Y'WcifgY']g'g\W\YXiYX'Zcf\WUggYg'cb`]b'6\'U\W_VcUfX'7c`'UVcfUhY'fh\fci[\acX'YE'5B8'ZJ\W!hc!ZJ\W'g]ai`HUbYcig`m';]jYbh\Y'fY\Wbh7CJ =8'ciHVfYU_žh\Y'ZJ\W!hc!ZJ\W\'WUggYg'UfY'ib`]_Y`m'=b'h\Y'YjYbhh\]g'\W\Ub[Ygžh\Yfccag'kci'X'VY.
		A cb'%\$.\$\$'!'%'.\$\$'Ei UXfUb[`Y'%\$(' 'f7!9%)!%\$(' Ł
		A cb'%(.\$\$'!'%+.\$\$'; c`XgHY]b'; \$-'fP!8%*!; \$-Ł
		Hi Y'%\$.\$\$'!'% .\$\$'Ei UXfUb[`Y'; \$(\$'f7!9%)!; \$(\$E
		Hi Y'%(.\$\$'!'%+.\$\$'A Uh\Yk g'&&+'fP!: &'!&&+Ł
K YY_" ."%"A UfVX"!") A UfVX	GYa ]bUf	H\Y`Wci fgY`fi bg`]b`]bhYbg]j Y`a cXY`Zfca `%\$Ua!) da cb`A cbXUmg`UbX`Hi YgXUmg`]b`k YY_g`%I` z`k ]h\`U`% \ci f`VfYU_`Zcf``i bW\`Zfca `%da `YUW\`XUm\UbX`g\cfh VfYU_g`Yj Yfm\ci f"`H\Y`Wci fgY`]g`gW\YXi `YX`Zcf WUggYg`cb`]bY`]b`6`UW_VcUfX`7 c``UVcfUhY`fh\fci [\

## FYgci fW/g

### DfYgWf]VYX F Ygci fWYg

 $H \ YfY \ "UfY" bc "fYei" \ ]fYX" hYI \ hg "Zcf" ghi XYbhg" "FYUX] b[g'k] ^"VY "gYh Zcf" YUWX "a cXi "Y"h fci [\"@Y[Ubhc"] b A ccX"Y" "Mci "k] ^"VY "UV"Y hc "UWW gg'h YgY "fYUX] b[g'h fci [\"h\Y'I BGK "] VfUfmik] h mci f"c[] b"$ 

#### FYWca a YbXYX FYgci fW/g

Gca Y'i gYZI ``HYI hgža cghcZk \]WX WUb VY Zci bX']b'I BGK '`]VfUfmž']bWI XY.

DYffYUi `hžH"ž'6f]X[Yž'; "ž/ 'A W7 Ufh\mž'>"`fPXg"L"`f&\$%) L"`FZWDagf'WYWZS` VTaa] aXba'[f[US^WUa'aYk" Fci hYX[Yž'@cbXcb"

DYYhžF "žF cVV]bgžD"ž/ "K UhhgžA ""f9Xg"L""f&\$%\$L""9 aTS^ba (f[US^WUa aYk""F ci hYX[Yž@cbXcb"

 $DYYh\check{z}F"\check{z}/`K Uhhg\check{z}A"`f\&\$\$(L">[TWbSf[a`WUa'aY[We, Wh[da`_WflVwhWab_WfleaU[S^_ahW_Wfe"]FcihYX[Y\check{z}@cbXcb"]$ 

?ch\Uf]ž5"žGU``Y\ž5"ž9g\cVUfž5"ž8YaUf]Už: "ž/ '5\cghUž5"`f9Xg"L"`f&\$\chi`\B'gd\h\de\ 3 baefZ\V\h\Mab\_ W fV[Uf[a` Sdx"`Hi`]\_U'6cc\_g`UbX'5i h\cfgi dZfcbh''9!j Yfg]cb`\\YfY

8f5`]gUž; "ž8Ya Uf]Už: "ž/ '? U``]gž; "`f9Xg"Ł" f&\$%( Ł"`6WYdai fZ, S haUSTg/Sdx XadS` W WbS"`F ci hYX[Yž@cbXcb"

Gca Y'k YVq]hYq'h\Uha UmVY'i qYZ ``]bWi XY.

Gci h\ 7 YbhfY. \hdg.#k k k "gci h\ W bhfY"]bh#

=7 HG8 fHfUXY UbX Gi ghJ]bUV Y 8 Yj Y cda YbrŁ \hdg.#fWgX"]]gX"cf[#

+bh/fbUh/cbU``+bgh/h h/ Zcf'Gi gh/]bUV`Y`8 Yj Y`cda Ybh`\hdg.#k k k "]]gX"cf[#

+bhYfbUhJcbU``+bghhi hY'Zcf'9bj ]fcba YbhUbX'8Yj Y`cda Ybh'\hdg.#k k k "]]YX"cf[#

IB' Yg'h\Uha

Wta a YbhgʻUVci hh\YʻWti fgYʻcfʻUggYgga Ybhgʻʻ

# Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g Hi fb]h]b Gi Va ]gg]cb

⊋mci 'YbWci bhYf'U'dfcV`Ya 'k \Y\$b'Udho'ah\$hjb['hc'zoi]Nj\$ ]hmci f'Ugg][ba Ybhih\f‱[\Yhdi\fghjb\z'd`YUgY\fci[h

Υ

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=iWUbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'i

- i bXYfglUbX`mci f`f][\hg`UbX`fYgdcbg]V]`]h]Yg`Ug`U`gh XYbh'Uhi BGK
- VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hzil BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5WWdhUV`Y'I gY'cZ I BGK '= H'F Ygci fWfg'Dc`]Wh
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWhYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wcaaib]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

## 5 WUXYa ]W + DZcfa Uh]cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei YghgʻZcfʻYl hYbg]cb/
- `UhY 'gi Va ]gg]cbg '[ i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK < YU`h\ "UbX GUZYmdc`]W]Yg/
- YI Ua ]bUh]cb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg'cf'a ]gUXj Ybh fY/
- gh XYbhYei ]hmUbX'X]gUV]']hm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY

\hdg.#k k k "Uffig"i bgk "YXi "Ui #W ffYbhghi XYbhg#UWUXYa ]W]bZcfa Uhlcb#dfchcWc`g![i]XY`]bYg#

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