

LING5026

H\Y'Ghfi WhifY'cZ@Ub[iU[Y

HYfa 'C bY''##'&\$&%

7ci fgY⁻Cj Yfj]Yk

GHJZ7cbHJWh8YHJ]`g

7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] m	@cWUh]cb	D\cbY
A Yb[]gh '5a VYfVYf	a "Ua VYfVYf4 i bgk "YXi "Ui	H65	A 6&(%	f\$&Ł-\$*))*\$)

GWXcc``7cbHJWhi±bZcfa Uh]cb

GWkcc`cZ<iaUb]hJYg'UbX'@Ub[iU[Yg

@c\\U||cb. G\\cc``C\\Z|\\\žA\cfj\Yb'6fck\b'6i]\X]b[\z'@\j\Y``&z'&),

CdYb]b['<ci fg. 'A cbXUm'! : f]XUmž - Ua '! ') da

D\cbY.'Ž*%'&'-',)'%*,%

: UI . 'Ž * %'&'-' ,) ', +\$)

9a U]`. '<u>\U`4 i bgk "YXi "Ui</u>

7ci fgY⁻8YHJ]`g

7 fYX]hDc]bhg**

Gi a a UfmcZh\Y 7ci fgY

 $H\g``WcifgY`]g``Ub``]b!fcXi\Wf]cb`hc```]b[i\g'h]Wg`fh\Y`gW]Ybh]Z]WghXmcZ``Ub[i\U[YŁ`UbX`ZcW]gYg`cb`h\Y$

5ggYgga Ybh

5ggYgga YbhHUg_g

5ggYgga Ybh'tUg_	K Y][\h	8iY'8UHY	Gh XYbh@YUfb]b[Ci hWca Ygʻ5ggYggYX
< ca Yk cf_'DfcV`Ya 'GYhg'	' \$1	Bch5dd`]WUV`Y	&ž˙(
±b!WUgg`hYgh`	' \$ı	\$*#\$(#&\$&%`\$(.\$\$`DA	%ž [*] (
:]bU``91 Ua `	(\$1	: cfa U``9I Ua 'DYf]cX	%ž'&ž''

5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% < ca Yk cf_ DfcV`Ya GYhg

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

G\cfhYl YfW]gYg'hc'VY'Wca d'YhYX'k YY_`mUbX'gi Va]hhYX'cb`]bY"

#!\WUgg^[\text{YbYfU}^cfU^\UbX^k f]\text{htyb}^\text{ZYYXVUW_\text{}}]g^\text{dfcj}]XYX\"

Hi fb]hjb gYhjb[. H\]g]g bchU Hi fb]hjb Ugg][ba Ybh

5ggYgga Ybh& ±b!WUgg hYgh

GhUfhXUhY. Bch5dd`]WUV`Y

8YHJ]`g.

*\$ a]bi hY hYghWcj Yf]b[h\Y WcbhYbhcZ`YWh fYg UbX'hi hcf]U`g"

; $YbYfU``UbX`]bX]j]Xi U``k f]hhYb`ZYYXVUW_`]g`dfcj]XYX"$

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhh]b[."H\]g"ugg][ba Ybh]g"gi Va]hhYX"h\fci [\"Hi fb]h]b "UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a]"uf]hmfYdcfhg" \\ \end{tabular}$

5ggYgga Ybh' . :]bU` 9l Ua

8 YHJ] g.

 $\$ \fg'Y| Ua 'Wcj Yf]b['U``'h\Y'a U'cf'hcd]\\g'cZh\Y'\\circ ifgY"

H\]g`]g`h\Y`Z|bU``UggYgga YbhZcf`UhYbXUbWY`di fdcgYg"

Hi fb]hjb gYhjb[. H\]g Ugg][ba Ybh]g gi Va]hhYX h\fci [\ Hi fb]hjb UbX gh XYbhg Xc bchgYY Hi fb]hjb

g]a]`Uf]hmifYdcfhg"

5 thYbXUbWY F Yei]fYa Ybhg

FYgci fW/g

DfYgWf]VYX F Ygci fWYg

F Yei]fYX 'hYI hVcc_.': fca _]bžJ "'Yh'U'" &\$% "'3`; fcaVgUf[a` fa >S`YgSYW +fZ 3gefc5'[S`!@W LWS'S`V WW [f[a` ž7 Yb[U[Y"

K YVg]hYg. 9h\bc`c[iY/I 7@5'd\cbYh]Wg``UV

5 b Yl hYbg]j Y``]ghcZUXX]hJcbU``Y`YWfcb]WfYgci fWYg']g'dfcj]XYX`]b`A ccX`Y"`Mci 'a i ghWcbgi `h'A ccX`Y fY[i`Uf`mZcfi dXUhYq"

FYWca a YbXYX FYgci fW/g

.

5b YI hYbg]j Y``]ghcZUXX]hJcbU``fYgci fWYg`]g`dfcj]XYX`]b`A ccX`Y'''Mci `a i ghWcbgi `hA ccX`Y`fY[i `Uf`m'Zcf i dXUhYg''

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

7 ci fgYgʻUfYʻdYf]cX]\W\``mfYj]Yk YXʻUbXʻgh XYbhgʻʻZYYXVU\W_`]gʻi gYXʻhc`]a dfcj Yʻh\Ya "

: YYXVUW_`]g`[Uh\YfYX`Zfca `ghi XYbhg`i g]b[`a m9I dYf]YbWY"`±n]g`YbWci fU[YX`ghi XYbhg`Wca d`YhY`h\Y]f gi fj Ymg`VmUVWYgg]b[`h\Y`dYfgcbU`]gYX`k YV``]b_`j]U`h\Y`A ccX`Y`Wci fgY`g]hY"

Gi Va]gg]cb cZ5ggYgga YbhHUg_g Hi fb]h]b Gi Va]gg]cb

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±i\WbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiW\YUh]b[ihciU\W]XYbhU`m\wcdm]b[iZfca iUigci f\Wik]h\ci hiU\w_bck iYX[Ya Ybh'

IBGK '[fci dg'd'U[]Uf]ga ']bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

7 cdm]b[.'i g]b['h\Y'gUa Y'cf'j Yfmg]a]`Uf'k cfXg'hc'h\Y'cf][]bU`hYl h'cf']XYU'k]h\ci h'UW_bck `YX[]b['h\Y gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g'U'gc'Udd`]Yg'hc']a U[Ygž'Ufh'UbX'XYg][b'dfcYWhgž'Ug'k Y``'Ug dfYgYbhUh]cbg'k \YfY'gca YcbY'dfYgYbhg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]h'

± Uddfcdf]UhY 'dUfUd\fUg]b[.'7\Ub[]b['U'ZYk'k cfXg'UbX'd\fUgYg'k\]'Y 'a cghmfYhU]b]b['h\Y'cf][]bU' ghfi WhifY'UbX#cf'dfc[fYgg]cb'cZ]XYUg'cZh\Y'cf][]bU'žUbX']bZcfa UhJcb'k]h\ci hUW_bck 'YX[Ya Ybh''H\]g'U'gc Udd']Yg']b'dfYgYbhUhJcbg'k\YfY'gca YcbY'dUfUd\fUgYg'Ubch\Yf'g']XYUg'cf'k cfXg'k]h\ci h'WfYX]hUbX'hc d]YW]b['hc[Yh\Yf'ei chYg'UbX'dUfUd\fUgYg']bhc'U'bYk'k\c'Yž'k]h\ci h'Uddfcdf]UhY'fYZYfYbW]b['"

7 c``i g]cb. 'k cf_]b['k]th 'cth\Yfg'Vi hdUgg]b['cZZth\Y'k cf_'Ug'U'dYfgcb'g']bX]j]Xi U``k cf_"'7 c``i g]cb'U`gc]bWi XYg'dfcj]X]b['mci f'k cf_'hc'Ubch\Yf'ghi XYbhVYZcfY'h\Y'Xi Y'XUhYžcf'Zcf'h\Y'di fdcgY'cZth\Ya d`U[]Uf]g]b['UhUbmh]a YždUm]b['Ubch\Yf'dYfgcb'hc'dYfZcfa 'Ub'UWUXYa]WhUg_žghYU`]b['cf'UWei]f]b[Ubch\Yf'dYfgcb'g'UWUXYa]Wk cf_'UbX'Wcdm]b[']hz'cZYf]b['hc'Wca d`YfY'Ubch\Yf'dYfgcb'g'k cf_'cf'gYY_]b[

- i bXYfgHUbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbhUhI BGK
 VY'Uk UfY'cZd`U[]Uf]ga ž'Wcdmf][\hz'I BGK 'Gh XYbh7cXY'cZ7cbXi WhUbX'5VWWdhUV`Y'I gY'cZ IBGK =7 HF Ygci fWrg Dc`]Wh

5 WUXYa]W + DZcfa Uh]cb

: cf'YggYbhJU`'gh XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei YghgʻZcfʻYl hYbg]cb/
- `UhY 'gi Va]gg]cbg '[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK < YU`h\ "UbX GUZYmdc`]W]Yg/
- YI Ua]bUh]cb dfcWXi fYg/
- gdYWJU``Wcbg]XYfUhJcb`]b`h\Y`Yj YbhcZ]``bYgg'cf'a]gUXj Ybh fY/
- gh XYbhYei]hmUbX X]gUV]]hm/
- UbX'ch\Yf'YggYbh]U`'UWUXYa]W]bZcfa Uh]cbž'gYY

\hdg.#kkk"Ufng"i bgk"YXi "Ui #W ffYbhghi XYbhg#UWUXYa]W]bZcfa Uh]cb#dfchcWc`g![i]XY`]bYg#

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