

ARTS2457

7\]bU'-a U[]bYX'UbX'DYfWY]j YX

HYfa 'Hk c''##'&\$&%

7ci fgY 8YHJ] g

7 fYX]hDc]bhg * *

Gi a a UfmcZh\Y 7ci fgY

 $H\g'WcifgY']g'HUi[\h']b'9b[\g']g'UbX'k]h'fYUX]b[g']b'9b[\g']g''$

7ci fgY @YUfb]b['Ci hWca Yg

- % 7 f]h]WU``mfYUX`]a dcfhUbhihYI hg`cZ`]hYfUhi fYžWca dUfUhjj Y``]hYfUhi fYž'Z]`a ž'UbX`7\]bYgY`]bhY``YWhi U`\]qhcfm
- &" 5bU`mgY`X]ZZYfYbh[YbfYg`]b``]MYfUh fYž'Wf]hWU``h\Ycfmž'Zj`a 'UbX'Zj`a 'h\Ycfm
- ' " 5 dd`mfYgYUfWXž'UbU`mhjWU``UbX`YggUmlk f]hjb['g_]``g`hc`U`gY`YWhYX`hcd]W
- ("8YZ]bY`UbX`Wf]h]eiY`h\Y`WcbWfdhcZfcf]YbhU`]ga fi

HYUWX]b['GhfUhY[]Yq'

H\]g`Wci fgY`]g`a UXY`i d`cZ`]hYfUfmž'Zj`a]WUbX`h\YcfYhJWD`Wca dcbYbhg"@YWi fYg`k]```ZcW g`cb`h\Y`hcd]Wg Wcj YfYX`]b`fYUX]b[`Ugg][ba Ybhg`Vi hk]```U`gc`]bWi XY`Ub`UbU`mg]g`UbX`X]gW gg]cb`cZZj`a gž`]b`dUfhJW``Uf Zj`a]W]a U[Yg"`K Y`k]```j]Yk gY[a Ybhg`cZZj`a g`]b`WUgg"`=Zmci 'a]gg`U`WUggž`U```Zj`a g`k]```VY`Uj U]`UV`Y h\fci [\`h\Y`I BGK ``]VfUfm'H\YmWUb`VY`j]Yk YX`VmfYgYfj]b[`U'a UW\]bY`h\YfYžVi h\WbbchVY`Vcffck YX" Gca Y`Zj`a g`WUb`U`gc`VY`j]Yk YX`Cb`]bY"`H\]g`Wci fgY`fYei]fYg`ghi XYbhg`hc`Xc`U`gi VgHUbhJU``Ua ci bhcZ fYUX]b["'=f]g`]a dcffUbhh\Uhmci `Wca a]hmci fgY`Zhc`Xc]b['h\Y`fYUX]b[g`cb`U'fY[i `Uf`VUg]g`Zfca 'h\Y VY[]bb]b[`cZh\Y`Wci fgY"`7`UggYg`VUgYX`cb``]hYfUfmghi X]Yg`UbX`Wf]hJWD``h\YcfmUfY`ZcW gYX`cb`X]gW gg]cbg UbX`WcgY`UbU`mgYg`cZdf]a UfmUbX`gYWcbXUfmhYI hgž'UbX`cb`mVmfYUX]b['h\Ya`UbX`h\]b_]b['fYZYWf]j Y`mcb h\Y]ggi Yg`fU]qYX`]b`WUgg`X]gW gg]cbg`WUb`mci 'YI dYWh'ca UghYf`h\Y`Wci fgY`a UhYf]U`"

5ggYgga Ybh

hc fYZYf hc ch\Yf hYl hg cf Z] a g i gYX]b h\]g Wci fgY"

C fimci ia UmiXYj]gYimci fick biYggUmihcd]W" ≠mci iXcžidiYUgYia U_Yigi fYihciUg_ih\YiiYWhifYfžiY]h\YfiUZhYf WUggži]biWcbgi iHuh]cbi\ci fgžicfij]UiYa U]ik \Yh\Yficfibchh\Yihcd]Wmci idfcdcgYihcik f]hYicbi]giUWWdhUV`Y"

FY"WJHJhjcbg. K \Yb'k f]hjb['Ub YggUm'U'k Umg'WjhY h\Y'Vcc_'UbX'dU[Y'bi a VYf'Zfca 'k \]W 'mci 'UfY'ei chjb[cf'gi a a Uf]n]b["'H\Y'YUg]Yghk Umhc'Xc'h\]g']g'di hh\Y']bZcfa Uhjcb']bhc'dUfYbh\YgYg'UZhYf'h\Y'ei chY'cf dUfUd\fUgYžY["'"'===\Wb'hk f]hYZ'gU]X'5\E'g\Ua YZJWYXž'bYfj ci g`m\c'X]b['h\Y'Vfi g\"'+\b'h\UhWUgYž'hc a U_Y']hYUgmZcf'mci ž'XfUk 'U'WjfWY'"'f@' 'Li bž'E WWUfWW E faqWez'dz'%\$, \E''C f']Zmci 'dUfUd\fUgYž'Zcf YI Ua d'Y.'@' 'Li b'hY`g'i g'h\Uh5\E']g'Ub']``]hYfUhY'dYUgUbhž'k\]W 'gYYa g'bchhc'd\UgY'\]g'UvW gYfg'i bXYf h\Y'bYk 'F Ydi V']WUb'[cj Yfba Ybh'H\Ymg]a d'm\Uj Y'\]a 'XfUk 'U'WjfWY'cb'\]g'XYUh\ 'k UffUbh'f@' 'Li bž' E WWUfWW E faqWež'd"'%\$, \E''Mci 'WUb'i gY'Y]h\Yf'<Ufj UfX'cf'7\]WU[c'ghmY.'k\Uh]g'a cgh']a dcfhUbh']g h\Uhmci '[]j Y'h\Y'WjhUh]cb"

 $\label{thm:limit} \begin{tabular}{ll} Hi & fb \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tabular} $$ Hi & fb \end{tabular} $$ h' \end{tabular} $$ I' \end{tab$

5ggYgga Ybh& 9l Ua

8 YW] g.

Ghi XYbhgʻWca dʻYhYʻUbʻYl Uaʻf&ʻ\ci fgŁʻ]bWi X]b[ʻ&\$!'\$`a i `h]dʻYʻWkc]WYʻei Ygh]cbgʻfk Y][\h]b[ʻ, \$i ŁʻUbXʻUb YggUmcZUddfcl "'*\$\$`k cfXgʻ]bʻ9b[`]g\`cfʻ-\$\$ʻ7\]bYgYʻWkUfUWYfgʻfk Y][\h]b[`&\$i Ł''Ghi XYbhgʻfYWY]j YʻU a Uf_ʻUbXʻWUbʻWcbgi `hih\Yʻ`YWhi fYfʻZcfʻZ fh\YfʻZYYXVUW_"'H\]gʻ]gʻh\YʻZ]bUʻʻUggYgga Ybh'ZcfʻUhhYbXUbWY di fdcqYq"

5 hyb XUb WY F Yei]fYa Ybhg

@]ghYb]b['hc'U``cb`]bY``YWi fYg'UbX', \$i 'UhhYbXUbW']b'hi hcf]U`g']g'a UbXUhcfm]b'h\]g'Wci fgY"'±b
@YWi fY#Hi hcf]U`g'mci 'k]``UWij Y`mYb[U[Y'k]h\ 'WcfY'Wci fgY'WcbhYbhžYbUV`]b['mci 'hc'UhhU]b'7@C'%&\Z \Z("
5hYbXUbW'k]```VY'hU_Yb'Uhh\Y'VY[]bb]b['cZYUW\ 'hi hcf]U`"'I bYI W'gYX'UVgYbW'\Zfca 'a cfY'h\Ub'&\si 'cZ
hi hcf]U`g'k]``fYgi 'h']b'U'ZU]`]b['[fUXY"''

5 Whj Y'dufhjWjduhjcb']g'fYei]fYX']b'h hcf]U'g" + Yj Yfmh hcf]U'a YYhjb[žmci 'a UmVY'WU``YX'i dcb'VmbUa Y'hcfygdcbX'hc'Y]h\Yf'mci f'WUgga UhYg'cf'h\Y'dfcZYggcfž'cf'dUfhjWjdUhY']b'[fci d'UWhj]hjYg"' - Zmci 'UfY'WU``YX i dcb'UbX': 5 - @'HC'fYgdcbX'HK - 79']b'Ubma YYhjb[ž']hk]``'VY'Wcbg]XYfYX'Ug'Ub'UVgYbWY"

	F YUX]b[<pre>% &* Xi f]b['th\Y 6Y]ZJ</pre>
	6 YbXYX	":]ZmiZj Y`8 Umg`Uh`DY_]b["`fl G5`% *' & `%) ' a]bg"fJ]Yk 'Yl WYfdhg`UbX`X]gW gg&" "GUbX`DYVV`Yg"`fl G5`% **&'% *`a]bg"`fJ]Yk Yl WYfdhg`UbX`X]gW gg&"
K YY_'(.'&%'>i bY'!'&) >i bY	@YWhi fY	%"·7\]bU·UbX·h\Y·9i fcdYUb·9b`][\hYba Ybh &"··H\Y·Gi VU`hYfb
	Hi lcf]U`	<pre>< ck 'UfY'"cf]YbhU""WXUfUWfYfg'UbX'h\Y'Cf]YbhVY]b[i gYX'VmA cbhYgei]Yi 'UbX'; c`Xga]h\'Ug'k Y``Ug']b h\Y'7\Uf']Y'7\Ub'UbX': i 'A UbWXi 'Z]`a g3'K\Uhk YfY h\Y'\]ghcf]WU''VUW_[fci bXg'ci hcZk\]WX'h\YgY WXUfUWfYfg'Yj c`j YX3'K\UhghYfYchmdYg'UfY'dfYgYbh]b'h\Y'<c``mk 'a="" 'hc'h\y'k="" 'xc'h\ymx]zzyf'zfca="" :="" ccx'xyd]wfjcbgž\ck="" dy_]b["3<="" fgi="" fyg="" i="" pre="" ubwxi="" ubł'ufy'xyd]wfyx']b'"))'8="" um\wfhu]b'7\]b'gy'z[[="" umg'uh="" wx'ug'df]bwy'8i="" yf'zfca="" yuwx'ch\yf'ubx'k\m3'="g'h\YfY'Ubm\WJffmcj"></c``mk></pre>
	F YUX]b[FYUX]b[g. A cbhYgei]Yi z > WffdMe BWdeS` We'fl'H\Y DYfg]Ub'@YhhYfg"L'fl%+&%L/C`]j Yf'; c`Xga]h\zFZW 5[f[] W aXfZWI adV'fU_U'"H\Y'7\]bYgY'@YhhYfg"L fl%+*&L"
	6`YbXYX	J]Yk."H\YAUg_cZ:iAUbWki"flG5%'&tUbX "7\Uf]Y7\Ub.AifXYfCjYfBYkMcf_"flG5%(\$t
K YY_') . '&, '>i bY'! '&'>i `	m @YWhifY	%":H\Y:Gi VU:hYfb:UbX:h\Y:K Ygh
	Hi hcf]U`	K\ma][\hgcaY'K YghYfb'fYUXYfg'hcXUmgh]``h\]b_5fh\ifGa]h\'g'%h\'WbhifmVcc_'5Z[`WeW5ZScbUfWd[ef[Ue'j U`i UV`Y']b'i bXYfghUbX]b['7\]bU3K\UhXcYg'@'Libff %8%'*E\UjYhc'Xc'k]h\cf]YbhU`]ga3 <ck'xcyg'cf]ybhu`]ga'aub]zygh]hgy`z]bh\y'xyd]wf]cbg'cz7\]bu'ubx'h\y'7\]bygy'dycd`y']b''h\y'; ccx'9ufh\'3'<ck'xcyg'h\y'%)*'z[a'jyfg]cbcz"h\y'byk'myuffg'guwf]z]wy''x]zzyf'zfca'h\y'%&(g\cfhghcfmn\iz'="" th="" vm@'lib3'5fy'h\yfyugdywfg'czh\y'z]a'k\]w\'hybx'hc'"cf]ybhu`]gy"<=""></ck'xcyg'cf]ybhu`]ga'aub]zygh]hgy`z]bh\y'xyd]wf]cbg'cz7\]bu'ubx'h\y'7\]bygy'dycd`y']b''h\y';>

	&""H\Y"DF7"Ug"h\Y""7YblfY". "k\Uh"]g""fYU"""\]ghcfm3

Gi Va]gg]cb cZ5ggYgga YbhHUg_g

Hirtholdbisjoyca]gg]cb

⊋mci YbWci bhYf`U'dfcV`Ya k\Yb`UhYa dhjb['hc'gi Va]hmci f`Ugg][ba Ybhh\fci [\'Hi fb]hjbž'd`YUgY
hY`Yd\cbY`9| hYfbU``Gi ddcfhcb'-',,)'''' %cf`Ya U]``h\Ya 'cb'Y! hYfbU`hY`hgi ddcfh4 i bgk "YXi "Ui "'`Gi ddcfh\ci fg`UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY_XUmg`UbX'-.\$\$Ua '-').\$\$da 'cb'k YY_YbXg`fl *) 'XUmg`U'mYUfŁ'''⊋mci UfY'i bUV`Y'hc'gi Va]hmci f`Ugg][ba YbhXi Y`hc'U'ZJi `hk]h\'Hi fb]hjb`mci 'a UmUdd`mZcf'Ub'Y! hYbg]cbžVi hmci a i ghfYhJ]b`mci f`hj\\Yhbi a VYf'Zfca '9| hYfbU``Gi ddcfhfU'cb['k]h\'Ubmch\Yf`fY`Yj UbhXcW a YbhgŁ'hc
]bWi XY'Ug'Yj]XYbWY'hc'gi ddcfhmci f'Yl hYbg]cb'Udd`]\WJhjcb'''⊋mci 'Ya U]`'9| hYfbU``Gi ddcfhmci 'k]``

5 WUXYa]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

5 WUXYa]W + DZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb fY`UhJb[hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va]gg]cbg '[i]XY`]bYg/
- fYj]Yk 'cZa Uf_g/
- I BGK < YU`h\ UbX GUZYmdc`]W[Yg/
- YI Ua]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a]gUXj Ybh fY/
- gh XYbhYei]hmUbX'X]gUV]`]hm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa]W]bZcfa UhJcbž'gYY