

# **ARTS3633**

DfcZYgg]cbU``>UdUbYgY'6

HYfa 'Hk c''##'&\$&%

# 7ci fgY<sup>·</sup>Cj Yfj ]Yk

## GhJZZ 7 cbhJWh 8 YhJ] `g

## 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
M a ]_c '< Ug\]a chc				

## 5ggYgga Ybh

#### 5ggYgga YbhHUg\_g

5ggYgga Ybh'hUg_	K Y][\h	8i Y'8UhY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
K f]thYb HYgh	&) ।	\$&#\$+#&\$&%`\$' .' \$`DA</td><td>%<u>`</u>`)</td></tr><tr><td>F YgYUfVX 'Dfc YVVi</td><td>) \$1</td><td>D`YUgY`gYY`UXX]h]cbU``XYhJ]`g VY`ck"</td><td>%ž'&ž' ž'(ž')</td></tr><tr><td>GdYWJU``DfcYWi</td><td>&) ı</td><td>Bch5dd`]WUV`Y</td><td>%ž.' ž.(</td></tr></tbody></table>	

#### 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% K f]hYb HYgh

GhUfhiXUhY. \*\$&#\$+#&\$&%\*\$&.%\$\*DA

@Yb[h\.', \$'a ]bi hYg

8 YHJ]`g.

Ghi XYbhgʻWca dʻYhYʻUʻhYghifWU"ʻ%\ci fŁʻ]bWi X]b[ˈg\cfhl·UbXʻ`cb[YfʻUbgk Yfʻei Ygh]cbg"`Ghi XYbhgʻfYWY]j YʻU a Uf\_`UbX`Wub`Wcbgi `hih\Yʻ`YWfi fYfʻZcfʻZ fh\YfʻZYYXVUW\_"

#### 5XX]h]cbU`XYhJ]`g.

 $Ghi XYbhg'k ] ``Wca d`YhY'U'a ] X!hYfa ']b!WUgg'hYghicZh\Y]f'\_bck `YX[Y'cZh\Y'`YWhi fY'WcbhYbhg'UbX'fYUX]b[g'' : YYXVUW_k ] ``VY'[]j Yb'j ] U'bi a Yf]WU`[fUXY'UbX'cfU`Wca a Ybh''$ 

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYhh]b[. Hi]g Ugg][ba Ybh]g gi Va]hhYX hi fci [in Hi fb]h]b UbX ghi XYbhg Xc bchgYY Hi fb]h]b g]a] [in Hi]hmfYdcfhg"$ 

5ggYgga Ybh& FYgYUfW Dfc YWh

GhUfhXUhY. K YY\_'-

@Yb[h\. j Uf]ci g

8 YHJ] g.

Gh XYbhg`dfYdUfY`Zcf`Ub`UVghfUWhUbX`\UbXci hg`Zcf`%\$i ž'dfYgYbh'U`dUdYf`Zcf`&\$i `fWU"%)`a ]bi hYgŁž'UbX k f]hY`U`dUdYf`Zcf`&\$i `fWU"`&\$\$\$! 'jŁ": YYXVUW\_`j ]U`]bX]j ]Xi U``Wca a Ybhgž'fi Vf]WŽ'UbX`]b!WUgg X]gW gg]cb"H\]g`]g`h\Y`Z]bU``UggYgga Ybh'HUg\_`Zcf`UhhYbXUbW'`di fdcgYg"

#### 5XX]h]cbU`XYhU]`g.

DfYgYbhUhjcbijgihciVYi\YiXijbiK YY\_i-icfiK YY\_%\$"

8 i Y XUhY Zcf'h\Y fYgYUfW\ dUdYf ]g cbY k YY\_'UZhYf ]hg dfYgYbhUh]cb""

.

 $\label{eq:higher} \mbox{Hi fb]h]b "gYhh]b[ . "H\]g"]g"bch"U"Hi fb]h]b"Ugg][ ba Ybh$ 

5ggYgga Ybh' . 'GdYWJU` Dfc YWh

GhUfhXUhY. K YY\_&

@Yb[h\. j Uf]ci g

8YHJ]`g.

Ghi XYbhgʻWca d`YhYʻj Uf]ci gʻUWhj ]hjYgʻk YY\_`mcfʻV]k YY\_`mcbʻh\Y]fʻck bʻWkc]WYʻUbXʻgi Va ]hiUʻdcfhzcʻ]c ]bWi X]b[ 'fYdcfhgʻUbXʻci fbU`g'': YYXVUW\_j ]Uʻfi Vf]WUbX`]bX]j ]Xi UʻʻWca a Ybhg''

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYh]b" . "H\]g"Ugg][ba Ybh]g"gi Va ]hhYX"h\fci [\ 'Hi fb]h]b"UbX"gh XYbhg"Xc"bchgYY"Hi fb]h]b g]a ]"Uf]hmfYdcfhg" \\ \end{tabular}$ 

## FYgci fWyg

#### DfYgWf]VYX F Ygci fWYg

f&\$\$-Ł 5```ch\Yf`YggYbh]U``a UhYf]U`g`UfY`hc`VY`Uj U]`UV`Y`cb`h\Y`Wci fgY`A ccX`Y"

### FYWca a YbXYX FYgci fWyg

GYY'h\Y'Wci fgY'A ccX`Y"

### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\Y`Wci fgYg`]g`dYf]cX]WJ``mfYj ]Yk YX`UbX`ghi XYbhg'`ZYYXVUW\_`]g`i gYX`hc`]a dfcj Y`]hg`WcbhYbhg`UbX UWij ]hjYg'': YYXVUW\_`]g`[ Uh\YfYX`Zfca `ghi XYbhg`i g]b[ `a m9| dYf]YbWY''=n]g`YbWci fU[ YX`h\Uhghi XYbhg Wca d`YhY`h\Y]f`gi fj Ymg`VmUWWgg]b[ `h\Y`dYfgcbU`]gYX`k YV``]b\_`j ]U`h\Y`A ccX`Y`Wci fgY`g]hY''

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK