

# **ARTS2754**

FYgYUfW. Zcf = dUWi]b; `cVU`8Yj Y`cda Ybh

HYfa "Hk c "# "&\$&%

# 7 ci fgY Cj Yfj ]Yk

## GHJZZ:7cbHJWh:8YHJ]`g

## 7cbj Ybcfg

B Ua Y	9a U]`	5j U]`UV]`] <b>m</b>	@cWUh]cb	D\cbY
@UbU"HUlici f	`"Mhcif4 i bgk "YXi "Ui	•	%%5 A cfj Yb 6fck b 6i ]`X]b[	

### GWXcc``7cbHJWhʻ±bZcfa Uh]cb

GWXcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

d\cbY. '\$&'-',) '% \$+

### 7ci fgY 8YW] g

7 fYX]hDc]bhg\*\*

### Gi a a UfmcZh\Y 7ci fgY

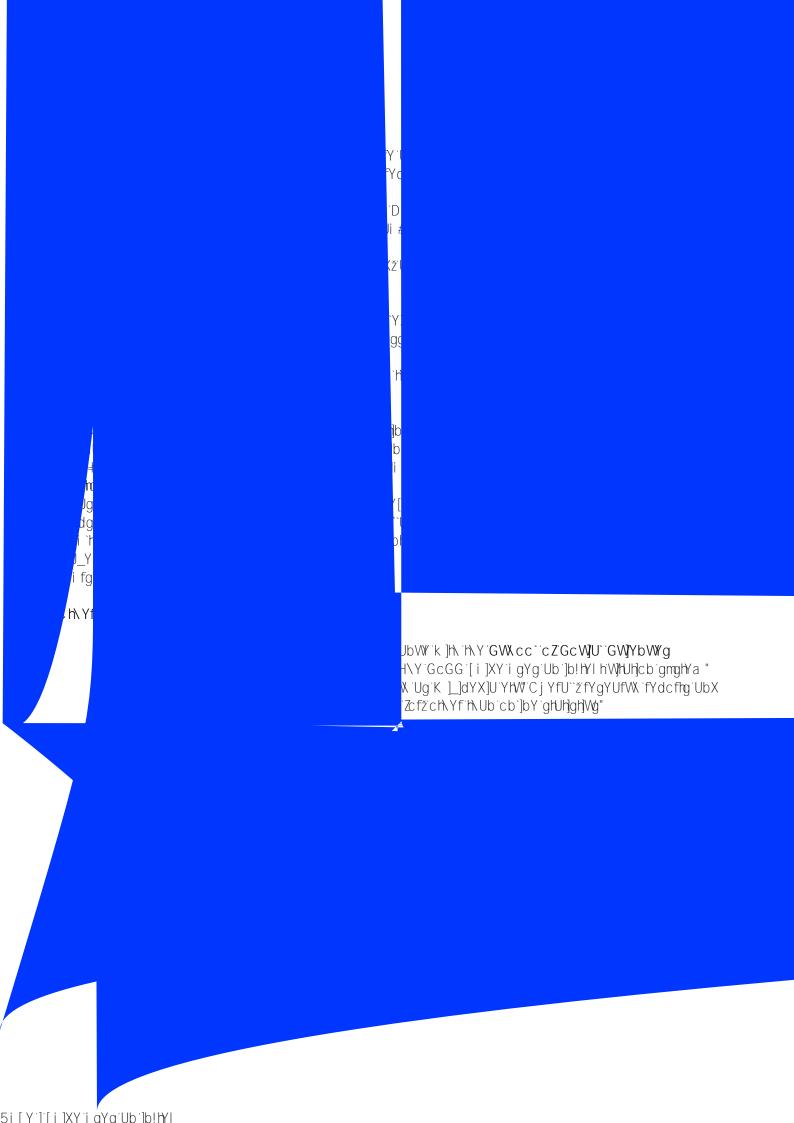
HUW\_`]b[ 'th\Y'k cf\Xfg' [fYUhYgh\W.U`Yb[ Yg'gi \W.'Ug'dcj Yfmz']bYei U`]mz\Ybj ]fcba YbhU``XYghfi \W]cbz i fVUb]gUh]cb'UbX'\W]a UhY']bXi \WX:X]gUghYfg'fYei ]fYg'Ub'i bXYfghUbX]b[ 'cZh\YgY']ggi Yg'Zfca 'X]j YfgY X]g\\[d']bUfmdYfgdY\\[d']j \UfmdYfgdY\\[d']j \UfmfYgYUf\\[d']j \UfmdYfx]g\\[d']j \UfmfYgYUf\\[d']j \UfmfYgY\\[d']j \UfmfYg\\[d']j \UfmfYg\\[d'

### 7 ci fgY @YUfb]b[ 'Ci hWca Yg

- % 5 fh]W `UhY 'h\Y 'X]gh]bWha Yh\cXg`cZXc]b[ `fYgYUfW\`]b`XYj Y`cda YbhWcbhYl hgžUbX`Uddfcdf]UhY hYW\b]ei Yg`Zcf` bck `YX[ Y`[ YbYfUh]cb`
- &" GY`YWhUbX`i hj`]gY`Uddfcdf]UhY`a Yh\cXc`c[]WU``UddfcUVXYg`]b`XYj Y`cda YbhfYgYUfVX`
- "8 Ya cbglfUlY 'Ub 'Uk UfYbYgg cZh\Y 'df]bWd Yg cZYh\]WU`fYgYUfW ]b XYj Y`cda Ybh
- ("⇒XYbhjZmUbX`Udd`mh\Y`g\_j``g`UbX`hYW\b]ei Yg`fYei ]fYX`Zcf`Xc]b[`fYgYUfW\`k]h\`dYcd`Y`Zfca `X]ZZYfYbh gcWjc!YWcbca ]WžX]gWjd`]bUfmUbX`dfcZYgg]cbU``VUW\_[ fci bXg`

### HYUWX]b[ 'GhfUhY[]Yg'

HYUWX]b[ 'ghfUhY[ ]Yg'k ]```Ya d\Ug]nY'ghi XYbhg'Vf]b[ ]b[ 'h\Y]f'df]cf``YUfb]b[ 'hc'h\Y'WUggfcca ž'UbX'hYUa k cf\_']b'h\Y'Wc!Wcbghfi Whjcb'cZ\_bck `YX[ Y"'H\]g'fYd`]WUhYg'fYgYUfW. `UbX'dfUWhjWr'Ybj ]fcba Ybhg']b 8Yj Y`cda YbhGhi X]Ygžk \]W\]g']bhYf!X]gW]d`]bUfmž'UbX'k \]W\"YI d`]W]mm]bWi XYg'a i `h]d'Y'j c]WYg'hc'dfcXi WY UWhjcbUV'Y'\_bck `YX[ Yfg\t''@YWhi fYg'k ]```VY']bhYfUWhj Yž'UbX'k ]```]bWi XY'[ fci d'k cf\_ž]bX]j ]Xi U``fYZYWhjcbž UbX'Wf]h]WU``fYZYWhjcb'cb'WcbhYbh''Hi hcf]U`g'k ]```VY'gdUWYg'Zcf'ghi XYbhg'hc'fYZ]bY'h\Y]f'i bXYfghUbX]b[ g" H\YfY'k ]```VY'Ub'Ya d\Ug]g'cb'hYUa 'k cf\_']b'h\Y'hi hcf]U`g''



#### 5ggYgga YbhHUg\_g

5ggYgga Ybh'HJg_	K Y][ \h	8i Y'8UHY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
DYfgcbu`fYgYUfW glUhYa Ybh	(\$1	&, #\$*#&\$&%`\$) .\$\$`DA	%Ž'
F YgYUfW\`dfcdcgU``	* \$1	%&#\$, #&\$&%`\$).\$\$`DA</td><td>%ž<sup>:</sup>&ž<sup>:</sup>(</td></tr></tbody></table>	

### 5ggYgga Ybh'8YHJ]`g

5ggYgga Ybh% DYfgcbu fYgYUfW gHJHYa Ybh

@Yb[h\. 8ž\$\$\$

8 YW] g.

 $DYfgcbU``fYZYWf]cb'ci h]b]b[ 'h\Y'ghi XYbhg'dcg]h]cbU`]hmž'UddfcUW\ hc'Yh\]WU``fYgYUfW\ 'dfUWf]WY ž'UbX'h\Y df]bW]d`Yg'k \]W\ k]```[i ]XY'h\Y]f'WcbXi Wh]b'h\Y''Z]Y`X''f\%z) $$! &z$$$'k cfXgŁ"'K f]hhYb'ZYYXVUW_'k]```VY dfcj ]XYX'k]h\]b'hk c'k YY_g'cZgi Va ]gg]cb"$ 

#### 5XX]h]cbU`XYhU]`g.

D`YUgY`Wcbgi `hA ccX`Y`Zcf`Z fh\Yf`]bZcfa Uh]cb"'

 $\label{eq:higher} \begin{tabular}{ll} High \$ 

5qqYqqa Ybh& F YqYUfWX dfcdcqU`

@Yb[h\." ž\$\$\$!' ž)\$\$

8 YW] g.

±0X]j ]Xi U``fYgYUfW\`dfcdcgU``fl \$\$\$!') \$\$`k cfXgŁ'h\Uh]XYbhJZYg`U`gi ]hUV`Y`fYgYUfW\`ei YghJcb`UbX`XYg][ bg gi ]hUV`Y`a Yh\cXg`hc`Ubgk Yf`h\Y`ei YghJcb`]bWi X]b[ `U`X]gW gg]cb`cZ`Yh\]WU``Wcbg]XYfUhJcbg"`K f]hhYb ZYYXVUW\_`k]``VY`dfcj ]XYX`k]h\]b`hk c`k YY\_g`cZgi Va ]gg]cb"`H\]g`]g`h\Y`Z]bU``UggYgga YbhZcf`h\Y`di fdcgYg cZh\Y`UhYbXUbW'`dc`]Wh'

#### 5XX]h]cbU`XYhJ]`g.

D`YUgY`Wcbgi`hAccX`Y`ZcfacfY`]bZcfaUhjcb"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b gYh]b[."H\]g Ugg][ba Ybh]g gi Va]hhYX"h\fci [\h]h]b UbX"ghi XYbhg `Wbb"gYY"Hi fb]h]b g]a]`Uf]hm fYdcfhg" \\ \end{tabular}$ 

## 5 https://bxubwww.Fyei ]fya Ybhg

GYa ]bUf`UthYbXUbWY`]g`Wca di `gcfmZcf`5FHG&+) ("'Mci 'a i gha U]btU]b`, \$i 'UthYbXUbWY`Zcf`th\Y`thYfa fg` gYa ]bUfg`Zcf`5FHG&+) ("'H\]g`a YUbg`mci 'WUb'a ]gg`th c`cZth\Y`gYa ]bUfg"`

## 7ci fgY GWX YXi `Y

#### J]Yk WUgg'ha YHUV'Y

### H]a YHJV`Y

8UHY	HmdY	7 c bhYbh		
K YY_'%'' %'A Um'!'( '>i bY				
K YY_'&.'+'>i bY'!'%% >i bY	6`YbXYX	7 ci fgYʻ±btfcXi Wgcb.∵K \UhjgʻF YgYUfWXʻZcf 8 Yj Y`cda YbhUbXʻGcWjU`ʻ7 \Ub[ Y		
K YY_'' .'%) '>i bY'!'% >i bY	6`YbXYX	Dck Yfž'Dcg]hjcbU`]hmUbX'F YZYI ]j ]hm]b'F YgYUfVX'Zcf GcVJU`'7 \Ub[ Y		
K YY_'(.'&%'>i bY'!'&) >i bY	6`YbXYX	FYgYUfWX '9h\]Wg		
K YY_').'&, '>i bY'!'&'>i `m	6`YbXYX	D`Ubb]b[ 'mci f'F YgYUfW\ 'Dfc'YWh		
K YY_'*.") '>i `m!'- '>i `m	FYUX]b[	FYUX]b['kYY_'!'8YjY`cd]b['mcif'fYgYUfWkeiYgh]cb"'		
	5ggYgga Ybh	DYfgcbU`FYgYUfW.GHJHYaYbhf5ggYggaYbh'Æ! 8iY'cb'HI9G85M!''\$'>ibY'&\$&\$'!'A -8B ≑ < H'''		
K_YY+.^\&^>i `m!^\%* '>i `m_6`YbXYX		8 Yg][b]b[mcifFYgYUfW\AYh\Xc`c[m		
K YY_', .'% '>i `m!'&' '>i `m	6`YbXYX	8 UHJ 7 c "YWJcb A YH\cXg		
K YY_''&*'>i `m!'' \$'>i `m	6`YbXYX	8 UHU 7 c``YWNjcb`A Yh\cXg		
K YY_'%\$.'&'5i [i gh!'* 5i [i gh	6`YbXYX	DUfhjWjdUhcfmUbX`5Wfjcb`FYgYUfW.		

### FYgci fW/g

#### DfYgWf]VYX F Ygci fWYg

D`YUY`gYY`VY`ck 'UbX`Wcbgi `HJhA ccX`Y"'

#### FYWca a YbXYX FYgci fW/g

9UWX k YY\_=\Uj Y gY`YWYX Wca di `gcfmfYUX]b[ 'h\Uha i ghVY fYUX df]cf 'rc YUWX 'hi rcf]U`" =b 'UXX]h]cbžh\YfY UfY 'U'bi a VYf'cZ[ccX'hYl h/cc\_q'h\Uh\Wb'VY'i gYZ\`[i]XYg'Zcf'h\]g'Wci fgYž'UbX'Ug'U'fYZYfYbWY'Zcf'mci f Zi hi fy fygyufvx dfc ywlq":A UbmcZh\y k yy\_`mfyuX]b[g'Ufy Zfca 'cby'cZh\ygy'hYl h\cc\_g" F Uh\yf'h\Ub  $\label{eq:control_bound} \begin{picture}(100,0) \put(0,0){\line(1,0){100}} \put(0,0){\line(1,0){10$ 

#### HYI h/cc\_g

%" D VeVSdUZ S` V 8[WVi ad] [`  $6 WhWab_W fl 8$ " < Ua a Y ht ilde vert 7" Hk ma Ub ilde Ub X ilde A " ; fU ilde Ua ilde z ilde 5 V]b[Xcb.F ci hYX[Y'f&\$%] Ł"

Hž6WeS[S` & BaffNV 6aNNMab\_WED96W88UZ (12398)[8]TUN/(F?"S[cHYfž@cbXcb.G5; 9.f&\$\$\* L"5jU]`UV`Y`]b`AU]b `]VfUfm'G''', "-\$\$-%+&#&

" " D WeWSdUZ Xad6 WhWab\_ W f, 3ž

%ž

 $\label{thm:condition} $$HYY^{n}_gHYX^{n}_i$ if bu^{n}_gY^{n}_jY$ 

,

H\YfY`UfY`a Ubmcb`]bY`dcffU`g`f\Uh`\Ug`XYj Y`cda YbhfYgYUfW\`WcbXi WfYX`VmXYj Y`cda YbhU[ YbW]Ygž f\YgY`]bWi XY`fUXX]f\cbU``cbYg`k ]``VY`a UXY`Uj U]`U`VY`cb`A ccX`YŁ`

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

; YbYfU``mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\W)``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh'' =b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUhjj Y'gi Va ]gg]cb'XYHJ]`g''

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

# 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i=hWJbihU\_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiWXYUh]b[ihciUWW]XYbhU`mWcdm]b[iZfca iUigci fWYik]h\ci hiUW\_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

• i bXYfgHJbX'mci f'f][\hg'UbX'fYgdcbg]V]`]h]Yg'Ug'U'gh XYbh'Uhil BGK

# 5 WUXYa ]W + DZcfa Uhjcb

: cf'YggYbhJU`gh XYbh]bZcfa UhJcb fY`UhJb[ hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY 'gi Va ]gg]cbg '[ i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK < YU`h\ UbX GUZYmdc`]W[Yg/
- YI Ua ]bUh]cb dfcW/Xi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a ]gUXj Ybh fY/
- gh XYbhYei ]hmUbX'X]gUV]`]hm/
- UbX'ch\Yf'YggYbhJU`'UWUXYa ]W]bZcfa UhJcbž'gYY