

# **SOCW7851**

7 ca a i b]hm8 Yj Y`cda Ybh

HYfa "Hk c "#" &\$&%

# 7 ci fgY Cj Yfj ]Yk

## GHJZZ:7cbHJWh:8YHJ]`g

### 7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`]hm	@cWUhjcb	D\cbY
>cmVV^K i	o o	W¢bgi `HJHjcb	F cca '%() ž A cfk Yb 6fck b 6i] X]b[	\$('(&%%)-

## GWXcc`7cbHJWhi±bZcfa Uhjcb

GWXcc``cZGcWJU``GWJYbWYg

Fcca '%) -

A cfj Yb 6fck b 7 &\$

Ya U]`. 'gcgg4 i bgk "YXi "Ui

d\cbY. '\$&'-',) '% \$+

### 7ci fgY 8YW] g

### 7 fYX]hDc]bhg\*\*

### Gi a a UfmcZh\Y 7ci fgY

I g]b[ 'Uˈf][ \hg 'VUgYX 'UddfcUW\žh\Y 'Wti fgY 'YI Ua ]bYg'h\Y 'bUhi fY 'cZWta a i b]hmXYj Y`cda Ybhi]b 'Ub ]bhYfbUhjcbU``UbX 'bUhjcbU``WtbhYI hzXfUk ]b[ 'cb'UʻfUb[ Y`cZh\YcfYh]WU``UbX`WUgY 'ghi Xma UhYf]U`g''`H\Y Wti fgY 'Wtj Yfg'U`Vf]YZ\]ghcfmcZWta a i b]hmXYj Y`cda YbhYh\Y 'W\Ub[ ]b[ 'bUhi fY 'cZWta a i b]hmk cf\_žh\Y WtbWYdhg'cZW 'hi fY']b 'fY`Uhjcb'hc 'Wta a i b]hmk cf\_jb 'XYj Y`cd]b[ 'gcWjYhjYgžUbX'h\Y 'XmbUa ]Wg'cZ Wta a i b]WUhjcbž'|bWi X]b[ 'Wfcgg'W 'hi fU` Wta a i b]WUhjcbž'k cf\_]b[ 'k ]h\ ']bhYfdfYhYfgž'\i a Ub'f][ \hg XcW a YbhUhjcb'UbX'UXj cWUwh'8]ZZYfYbh]XYc`c[ ]WU``UddfcUW\Yg'hc 'Wta a i b]hmk cf\_'UfY YI Ua ]bYX'UbX'Ub UbU`mg]g'cZh\Y']a dUWhg'UbX'ci hWta Yg'h\Uhih\YgY'UddfcUW\Yg'a ][ \hi\Uj Y'cb'Wta a i b]hjYg']g'YI d`cfYX'' 5`hYfbUhjj Y'a cXY`g'cZfYgYUfW\Z'Wtbgi `hUhjcb'UbX'bYYXg'UbU`mg]gž'd`Ubb]b[ 'UbX'gi ghU]bUV`Y'gYfj ]WW XY`]j YfmUfY'YI Ua ]bYX']b'cfXYf'hc 'UXXfYgg'W\Ub[ ]b[ 'UbX'Ya Yf[ ]b[ 'bYYXg''

### 7ci fgY @YUfb]b[ 'Ci hWca Yg

- %" ⇒XYbhjZmh\Y``]b\_g`VYrk YYb`Wca a i b]lmiXYj Y`cda YbhUbX`U`f][ \hg`VUgYX`ZfUa Yk cf\_`Zcf`GcWJU` XYj Y`cda Ybh
- &" @cWUHY 7 ca a i b]lmi8 Yj Y`cda Ybh]b`k ]h\]b`U`VfcUXYf`ZfUa Yk cf\_cZWca a i b]lmik cf\_
- '" =XYbhZmh\Y]a dUWhcZ; YbXYfUbX7i`h fYcb7ca a i b]hm8YjY`cda YbhdfcYWhq
- ("⇒XYbhZmh\Y'fc`Y'cZUXjcWWmjb'Wcaaib]lmxYjY`cd

#### HYUWX]b[ 'GhfUhY[]Yq'

H\]g`]g`U`dcgh[[fUXi UhY`Wci fgYz`UbX`ghi XYbhg`UfY`Yl dYWYX`hc`i bXYffU\_Y`U`g][b]Z]WJbh`Ua ci bh`cZ
]bXYdYbXYbhfYgYUfW\]bhc`h\Y`]ggi Yg`i bXYf`Wcbg]XYfUhjcb"`H\YfY`]g`Ub`Yl dYWfUhjcb`h\Uhighi XYbhg`k]```i gY
Ub`UbU`mhj\WJ``UddfcUW\`hc`h\Y]f`k cf\_z`UbX`g\ck 'h\Y`UV]`]hm`hc`Udd`mh\Ycfm'UbX`hc`h\]b\_`Wf]hj\WJ``m'`9j ]XYbW\
cZh\]g`a i ghVY`dfcj ]XYX`]b`U``k f]hhYb`k cf\_`gi Va ]hhYX`Zcf`UggYgga Ybh'

5 b 'UXi 'h'UddfcUWk 'hc 'YUfb]b[ ']g'hU\_Yb/h\ Uh]g'gh XYbhg'UfY 'fYgdcbg]V'Y 'Zcf'h\ Y'dUWk 'UbX'Ua ci bhcZ fYUX]b[ 'UbX'YI hfU 'fYgYUfWk 'h\ YmXc'Ug'k Y``'Ug'UhYbXUbWk 'UhWUggYgž'UbX'Zcf'fU]g]b[ 'k ]h\ 'h\ Y'WccfX]bUhcf ]ggi Yg'cf'dfcV'Ya g'k ]h\ 'h\ Y]f'`YUfb]b[ "'Gh XYbhg'UfY 'U'gc'YI dYWkYX'hc'hU\_Y'U'dUfh]W]dUhcfmUddfcUWk']b WUggYg'gc'h\ Uh\_bck 'YX[ Y']g'g\ UfYXž'ei Ygh]cbYXž'XYVUhYX'UbX'Uddfcdf]UhYX']bX]j ]Xi U``mUbX']b'[ fci dg" GYa ]bUfg'UfY'dfY! fYwcfXYX'UbX'Uj U]`UV'Y'Zcf'ghfYUa ]b[ #Xck b`cUX'Zfca 'A ccX'Yž'UbX'h\ YfY']g'U j c'i bhUfmcbY!\ci f'hi hcf]U``YUWk 'k YY\_'hc'\Y'd'gh XYbhg'k ]h\ 'Ubmei Ygh]cbgž'Wca a Ybhg'UbX'cddcfhi b]hmZcf X]gW gg]cb'hc'h\ Y' k YY\_fg'hcd]W'DUfh]W]dUh]cb']g'bch[ fUXYX"'< ck Yj Yfž']h']g'U'Wci fgY'YI dYWfUh]cb'h\ Uh fYei ]fYX'fYUX]b[ g'k ]```VY'fYUX'VmU``'dUfh]W]dUbhg'YUWk 'k YY\_"

# 5ggYgga Ybh

# 5ggYgga Ybh'HUg\_g

5ggYgga Ybh'hUg_	K Y][\h	8iY'8UhY	Gh XYbh@YUfb]b[ Ci hWca Yg 5ggYggYX
A ]X!HYfa 'F YZYW¶j Y'9ggUm	&) ।	&, #\$*#&\$&%`%%\$\$`DA	

7\ccgY`Zfca`th\Y``]ghcZei Ygh]cbg`cf`WfYUhY`mci f`ck b`fd`YUgY`Wcbgi`thk]h\`th\Y`Wci fgY`Wcbj YbYf`]Z
W\ccg]b[`mci f`ck b`ei Ygh]cbt"`H\Y`dUdYf`]g`i d`hc`' \$\$\$`k cfXg`]b`UXX]h]cb`hc`fYZYfYbWYg`CF`mci `\Uj Y`th\Y
cdh]cb`cZ\Uj ]b[`U`th\_Y`\ca Y`hYgh]bghYUX"`H\Y`th\_Y`\ca Y`hYghk]``[c`Zcf`Z]j Y`XUmgž`UbX`mci `bYYX`hc`k f]hY
th\fYY`g\cfh`YggUmg`f%\$\$\$`k cfXg`dYf`g\cfh`YggUmt"`H\Y`ei Ygh]cbg`k ]```VY`fY`YUgYX`cbY`XUmVYZcfY`thYgh
ghUffg"`K f]hhYb`ZYYXVUW\_UbX`U`a Uf\_`k]``VY`dfcj ]XYX`k]h\]b`&`k YY\_g`cZgi Va ]gg]cb"

 $\label{thm:limit} \begin{tabular}{ll} Hi fb]h]b"gYH]b[."H\]g"Ugg][ba Ybh]g"gi Va]hYX"h\fci [\Thi fb]h]b"UbX"ghi XYbhg"WUb"gYY"Hi fb]h]b"g]a]"Uf]hm fYdcfhg" \\ \end{tabular}$ 

### FYgci fW/g

### DfYgWf]VYX F Ygci fWYg

FYei]fYXfYUX]b[g'k]``VY'UjU]`UV`Y'cb'AccX`Y"`

## FYWca a YbXYX FYgci fWYg

 $\label{eq:final_problem} F\ Y\ W\ ca\ a\ Y\ b\ X\ Y\ T\ Y\ T\ U\ g\ k\ ]\ ```VY\ U\ U\ U\ U\ Y\ C\ b\ A\ C\ C\ X\ Y\ ''$ 

•

### 7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

Ghi XYbhZYXVUW\_cZYj U`i Uhjcb`]g`Wfi WjU`hc`XYj Y`cd`UbX`]a dfcj Y`h\]g`Wti fgY`Z fh\Yf`]b`gi VgYei YbhmYUfg" A m9I dYf]YbWf`gi fj Ymk]``VY`WtbXi WhYXhck UfXg`h\Y`YbX`cZh\Y`hYfa "`5hK YY\_'' žh\YfYk]``VY`U``Wti fgY\YU`h\`W\Y\W\_``k]h\`h\Y`WUgg"`H\]g`k]```VY`Ub`cddcfhi b]hmZcf`ghi XYbhg`hc`dfcj]XY`ZYYXVU\\\_UbX gi [[Ygh]cbg`cb``YWhi fYžWtbhYbhz`fYUX]b[gžhi hcf]U``UWhjj]h]Yg`UbX`ch\Yf`h\]b[g`k\]W\`k]``Yb\UbWf``YUfb]b[YI dYf]YbWf"

## Gi Va ]gg]cb cZ5ggYgga YbhHUg\_g

### Hi fb]h]b Gi Va ]gg]cb

Image: Ybwei bhyfu'dfcv'ya k \yb'uhhya dhjb[ 'hc'gi va ]hmci f'Ugg][ ba Ybhh\fci [ \ 'Hi fb]h]bž'd'YUgY

Hy'Yd\cby'9| hy'fbu''Gi ddcfhcb'-', ) '' ' ' %cf'Ya U]''h\Ya 'cb'Y! hy'fbu'hy'hgi ddcfh4 i bgk "YXi "Ui "''Gi ddcfh
\ci fg'UfY', .\$\$Ua '-'%\$.\$\$da 'cb'k YY\_XUmg'UbX'-.\$\$Ua '-').\$\$da 'cb'k YY\_YbXg'fl \*) 'XUmg'U'mYUft"'=Z'mci
UfY'i bUV'Y'hc'gi va ]hmci f'Ugg][ ba YbhXi Y'hc'U'ZJi 'hk ]h\ 'Hi fb]h]b'mci 'a UmUdd'mZcf'Ub'Y! hy'bg]cbž'Vi hmci
a i ghfYhJ]b'mci f'h]W\_Yhbi a VYf'Zfca '9! hy'fbU''Gi ddcfhfU'cb[ 'k ]h\ 'Ubmch\Yf'fy'Yj UbhXcW a Ybhgt'hc
]bWi XY'Ug'Yj ]XYbW' 'hc'gi ddcfhmci f'Y! hy'bg]cb'Udd']WUh]cb"'=Zmci 'Ya U]''9! hy'fbU''Gi ddcfhmci 'k ]'`
Ui hca Uh]WJ'`mfYW']j Y'U'h]W\_Yhbi a VYfZ'Vi h]Zmci 'hY'Yd\cbY'mci 'k ]``bYYX'hc'gdYW]Z]WJ'`mUg\_'Zcf'cbY''
Hi fb]h]b'U'gc'dfcj ]XYg'i dXUhy'g'cb'h\Y]f'gmghYa 'ghUh' g'cb'Hk]hyf''

; YbYfU`mž'UggYgga YbhiHug\_g'a i ghVY'gi Va ]hhYX'Y`YVWfcb]\WJ``mij ]U'Y]h\Yf'Hi fb]h]b'cf'U'A ccX`Y Ugg][ ba Ybh''=b']bgHJb\W'g'k \YfY'h\]g']g'bchdcgg]V`Yž']hk ]```VY'gHJhYX'cb'mci f'\\circ i fgY'g'A ccX`Y'g]hY'k ]h\U`hYfbUh]j Y'gi Va ]gg]cb'XYHJ]`g"

cfi]bZcfa Uhjcbicbi\ckihcigi Va ]hUgg][ ba Ybhgicbi]bYij ]UiA ccXiY.i\hdg.#gh XYbhi bgk "YXi "Ui #\ck! gi Va ]hUgg][ ba Ybha ccXiY

## 5 WUXYa ]W<cbYghmUbX D`U[]Uf]ga

D`U[]Uf]ga `]g`i g]b[ 'h\Y`k cfXg`cf`]XYUg`cZch\Yfg`UbX`dfYgYbh]b[ 'h\Ya `Ug`mci f`ck b"`±hWJb`hU\_Y`a Ubm Zcfa gžZfca `XY`]VYfUhY`W\YUh]b[ 'hc`UWW]XYbhJ``mWcdm]b[ 'Zfca `U`gci fWY`k ]h\ci hUW\_bck `YX[ Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga ]bhc'h\Y'Zc``ck]b[ 'WUhY[cf]Yg.

 $7 \text{ cdm}b[...i g]b[...h\Y.gUa Y.cf.j Yfmig]a] \Uf'k cfXg'hc'h\Y.cf][]bU''hYl h'cf']XYU'k]h\ci h'U\V_bck `YX[]b[...h\Y.cf][]bU''hYl h'cf']XYU'k]h'cf']$ 

- i bXYfgfUbX mci f'f][\hg'UbX'fYgdcbg]V]`]hJYg'Ug'U'gh XYbhUhl BGK
- VY`Uk UfY`cZd`U[]Uf]ga ž`Wcdmf][\hz'l BGK `Gh XYbh7cXY`cZ7cbXi WhUbX`5VWWdhUV`Y`l gY`cZ I BGK `=\forall H`F Ygci f\W\g`Dc`]\m\m
- VY 'Uk UfY 'cZh\Y 'ghUbXUfXg 'cZVY\Uj ]ci f'Yl dYWYX 'cZYj YfmcbY ']b 'h\Y 'l BGK 'Wca a i b]hm
- `cWUHY 'gYfj ]WYg'UbX `]bZcfa UH]cb'UVci hI BGK 'UbX'I BGK '@]VfUfm

Gca Y'cZh\YgY'UfYUg'k]``VY'ZJa]`]Uf'hc'mci ž'ch\Yfg'k]``VY'bYk"'; U]b]b['U'gc`]X'i bXYfgHJbX]b['cZU``h\YfY`UhYX'UgdYWng'cZ9@+69'k]``\Y`d'mci 'a U\_Y'h\Y'a cghicZmci f'ghi X]Yg'UhI BGK "'\\hd.##qi VYWfi |XYg"`]VfUfm'i bqk "YXi "Ui #\`]qY#JVci hY`]qY

# 5 WUXYa ]W + DZcfa Uhjcb

: cf'YggYbhJU`ghi XYbh']bZcfa UhJcb'fY`UhJb['hc.

- fYei Yghg Zcf Yl hYbg]cb/
- `UhY`gi Va ]gg]cbg`[i ]XY`]bYg/
- fYj ]Yk 'cZa Uf\_g/
- I BGK '< YU'h\ 'UbX'GUZYhm'dc']W]Yg/
- YI Ua ]bUh]cb dfcWYXi fYg/
- gdYWJU``Wcbg]XYfUHjcb']b'H\Y'Yj YbhcZ]``bYgg'cf'a ]gUXj Ybh fY/