

ARTS2752

@cWU`Dc`]hJWg`cZ8YjY`cdaYbh



7ci fgY[·]Cj Yfj]Yk

GhJZZ 7 cbhJWh 8 YhJ] `g

7cbj Ybcfg

BUa Y	9a U]`	5j U]`UV]`]hm	@cWUh]cb	D\cbY
5bhcbmNk]	U"nk]4 i bgk "YXi "Ui			

5W_bck `YX[Ya YbhcZ7ci bhfm

7ci fgY 8YHJ] g

I b]hg cZ7fYX]h * *

Gi a a UfmcZh\Y'7ci fgY

#b 'h\]g'Wci fgY'mci 'k]```YUfb'UVci hh\Y']bZi YbWYg'UbX'dc`]h]Wg'h\Uhg\UdY'UbX'UfY'g\UdYX'VmX]gUghYfgž\ia Ub]hUf]Ub'UWIjcb'UbX'XYj Y`cda Ybh]b'`cWU`WcbhYl hg"'K Y'Yb[U[Y'k]h\'h\Y'X]`Ya a Ug'cZ\i a Ub]hUf]UbUWIjcb'UbX']hg']bhYfZUW'k]h\'XYj Y`cda YbhUbX'W\Ub[Y'UbX'h\Y'k Umg'h\UhdYcd`Y'Yl dYf]YbWJb['h\YgY W\Ub[Yg'fYgdcbX"'

Mci 'k]```XYj Y`cd'mci f'_bck `YX[Y`cZh\Y`]a dcffUbW'cZWcbhYl hzdc`]hJWgz'dck Yfz'UbX`]bhYfYghg`]b XYhYfa]b]b['fYgdcbgYg'hc'Wf]gYg'UbX'X]gUghYfg'U`cb[g]XY'h\Y]f'`cb[Yf'hYfa 'ci hWca Yg"'Mci 'UfY YbWci fU[YX'hc'`YUfb'UbX'XYd`cmX]ZZYfYbhk Umg'cZh\]b_]b['UbX'Xc]b[']b'h\Y'\i a Ub]hUf]Ub'UbX XYj Y`cda YbhgdUW'UbX'k]``Yb[U[Y'k]h\'cf[Ub]gUh]cbU`z'WcbWrdh U``UbX'UbU`mhJWU``ZfUa Yk cf_g'Zcf UddfcUW\]b['\i a Ub]hUf]Ub'UbX'XYj Y`cda YbhWf]gYg'Zfca 'h\Y'dYfgdYWf] Y'cZ`cWU`]bghJhi hJcbg'UbX cf[Ub]gUh]cbgz'Wca a i b]hJYg'UbX'h\Y'ch\Yfg'UZYWhYX'Vmgi W\'Wca d'Yl 'WcbhYl hg"

7 ci fgY @YUfb]b[Ci hWca Yg

- %" ⇒ XYbhjZmh\Y`UWrcfgž`]bhYfYghg`UbX`]XYUg`h\UhYbUV`Y`UbX`WcbghfU]b`\i a Ub]hUf]Ub`UWrjcb`UbX XYj Y`cda Ybh'Uh'h\Y``cWU``Yj Y`
- &" 7 f]h]wU``mUbU`mgY`WU]a g`UVci h`cWU`]gYX`\i a Ub]hUf]Ub`UWh]cb`a UXY`Vm'UWUXYa]Wgž'dc`]whla U_Yfg UbX`UWhi]qhq
- ' " 5 bU`mgY`Uddfcdf]UhY`UWUXYa]WUbX`bcb!UWUXYa]WhYI hg`hc`YI d`cfY``cWU``dc`]hjWg`cZ\i a Ub]hUf]Ub UWhjcb`UbX`]hg`]bhYfZUWY`k]h\`XYj Y`cda Ybh
- ("8YacbglfUlY'kcf_]b['gYbg]h]j]lmUbX'k]h\'UkUfYbYgg'cZ`cWU`dc`]h]Wg']b'X]ZYfYbhWcblYI lig

HYUWX]b['GhfUhY[]Yg'

H\Y`XYg][b`cZh\Y`HYUW\]b['acXY`UbX`WcbHUWi\cifg`fYZYWiVch\`U`Wcaa]haYbhirc`ZJVW`hc`ZJVW`hYUW\]b[fcf`]ZbYYXg`VY`cb`]bY`HYUW\]b['\texi_]b!XYdh\`gWYbUf]c`UbX`WJgY`ghiXmUbU`mgYgžUbX`Yb[U[YaYbhik]h\jUf]cig`UgdYWg`cZV`YbXYX``YUfb]b['z´Ug`cih]bYX`]b`h\Y`IBGK`&\$&)`GhfUhY[mH\YaY`5&.'9XiWJh]cbU`9IWY``YbWY''

 $\label{thm:condition} $$H_Y''W' fgY''W' fgY''V' fgY'' fgY''' fgY'' fgY$

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5ggYgga Ybh

hg 'Ub)

H\]g^]g^bchU^Hi fb]h]b^Ugg][ba Ybh

5XX]h]cbU`XYhU]`g

.

5ggYgga Ybh& @cWU`]g]b[\ia Ub]HUf]Ub UWh]cb!fc`Yd`Umfl fcidŁ

8i Y'XUhY.'%\$#%#&\$&&`\$'.\$\$'DA

5ggYgga YbhcZ[fci d'Wcbhf]Vi h]cb'hc'i bXYfghUbX]b['Wca d'Yl'\i a Ub]hUf]Ub'UWn]cb'fYgdcbgY'hc'Wf]g]g UbX'X]gUghYf"'H\Y'WUgg'k]``XfUk 'cb'Ub'Yl hYbXYX'WUgY'gh XmUbX'U```WUgg'a Ya VYfg'k]```VY'k cf_]b[k]h\]b'[fci dg'Wca a cb'mk cf_]b['hc'UXXfYgg'\i a Ub]hUf]Ub'Wf]gYg'flcWU``[cj Yfba Ybhz'cWU`B; Cgz]bhYfbUh]cbU``Wj]'gcWjYmz'l b]hYX'BUh]cbgz'Wi ghYf'gmghYa '!'\YU'h\z'k UhYfz'dfchYWn]cbz'g\Y'hYfb''

Ghi XYbhg·k]``VY`UggYggYX`VmV&bj Ybcfg·UbX`dYYfg·fY`W&bhf]Vi h]cb·hc``YUfb]b['UbX`fYU`]gh]WUbX`]b! XYdh`i bXYfghUbX]b['cZh\Y`W\U``Yb[Yg`cZYb[U[]b['YZZYWh]j Y`m]b``cWU``\i a Ub]hUf]Ub`UbX`XYj Y`cda Ybh W\U``Yb[Yg"'Ghi XYbhg·fYWY]j Y`[fci d`ZYYXVUW_'UbX`U`[fci d`a Uf_'Zc``ck]b['fc`Y!d`UmYj Ybh'

5XX]h]cbU`XYhJ]`q

H\]g_k]``bchVY'U'ffUX]h]cbU`ffic`Y!d`UmfiVi hfUh\Yf'U'[fci d'dfYgYbhUh]cb"

Mci 'Wub' UXcdh']bbcj Uhjj Y'a YUbg'f]bWi X]b['fc`Y!d`Umž'dUbY`gž']bhYfj]Yk g'cf'ch\Yf'UddfcUW\Yg'!']bWi X]b[a cfY'hfUX]hJcbU`'dfYgYbhUhJcbgŁ'hc'g\Uf]b[']bg][\hg'k]h\'h\Y'fYghcZh\Y'WUgg'k \c'a Umbch\Uj Y'Y! Ua]bYX h\Y'gUa Y'fWUgY'gh Xmficb'k \]W\'mci f'[fci d'\Ug'ZcW gYX"'9UW\'[fci d'k]``'ZcW g'cb'U'X]ZYfYbhgYhJb['UbX k]``'Vf]YZmdfYgYbhi\Y'WcbhY! hž'h\Y'fUb[Y'cZghU_Y\c`XYfgž'cf[Ub]gUhJcbg'UbX'dfcW'ggYg'dfYgYbhž'UbX'k]``\][\`][\hei YghJcbg'cZdck Yfž'hfi ghUbX'h\Y'X]ZZYfYbhJU`']bhYfYghg'dfYgYbh]b'h\]g'gYhhJb["

 $H\Y'cVYWMjY'jg'hc'g\UfY'gcaY'cZmcif'[fcid'X]gWigg]cbgžXYVUhYžh\]b_]b[ž'Wf]hjeiY'UbX'eiYghjcbg']b fY`Uhjcb'hc'U'dUfhjWi`Uf'gYhhjb['k\]W\mci'\UjY'YId`cfYX']b'U'`]hhY'acfYXYhJ)`'h\Ub'ch\Yfg']b'h\Y'WUgg'''H\]g$

hYfa 'hc'U'`ck 'gh' XYbhg'hc'k cf_'hck UfXg'WYUf'mWca a i b]WJhYX'ghJbXUfXg"'

H\]g Ugg][ba Ybh]g gi Va]h\X h\fci [\ Hi fb]hb UbX gh XYbhg Wbb gYY Hi fb]hb g]a]\Uf]hmfYdcfhg"

5XX]h]cbU`XYhJ]`q

H\]g'k]```VY`mci f'Z]bU``UggYgga Ybh'UbX`k]```VY`Ub`cddcfhi b]lmiZcfmci 'hc'dfYgYbh']bg][\hg`Zca 'mci f'ck b 'YUfb]b["'=ik]``U``ck 'mci 'hc'ZcW g'UhYbh']cb'cb'Ub']ggi Y'cZdUfh]W``Uf'WcbWfb'hc'mci 'h\Uh\Ug'VYYb Wcj YfYX']b'h\Y'Wci fgY']b'fY`Uhjcb'hc'U`XYj Y`cda Ybhcf\\ia Ub]hUf]Ub']ggi Y''`Oh\Y'hJhY'g\ci 'X'VY \ia Ub]hUf]Ub'cf'XYj Y`cda YbhWlgY'ghi Xm!'d'YUgY'bchY'XYj Y`cda Ybh'ZcW gYX'X]a Ybg]cbg'UfY'U`gc UWWdhUV'YO'=ia Umd]W_i d']b'Z fh\Yf'XYhU]``Ub'UgdYWicZh\Y'WlgY'ghi Xmk]h\k\\]W\'mci '\Uj Y'Yb[U[YXža][\hwcbg]XYf'h\Y'fc'Y'cZUbmcZh\Y'fUb[Y'cZghU_Y\c`XYfg'cdYfUh]b[']b'`cWb'`Ybj]fcba Ybhgz'cf'a Umd]W_i d'cb'U'dUfh]W``Uf'h\Ya UhJW]ggi Y'UbX'h\Y'dc']hJwg'cZYb\UbW]b['ci hwca Yg']b'WcbhYghYX'Ybj]fcba Ybhg'' H\]g']g'mci f'cddcfhi b]hmhc'Xc'gca Y'cZmci f'ck b'fYgYUfW\'UbX'hc'XYa cbghfUhY']b!XYdh\'i bXYfghUbX]b[UbX'UbU'mg]g'cZU'gWbUf]c'cZ]bhYfYgh'''=Ua 'Ug']bhYfYghYX']b'h\Y'ei Yghjcbg'mci 'fU]gY'UbX'h\Y'UddfcUW\mci 'HU_Y'Ug']b'h\Y'Ubgk Yfg'mci 'XYf]j Y'Zfca 'mci f'fYgYUfW\'.'=Ua 'Uk UfY'ha Y']g'`]a]hYX'UbX'mci fffY'bch']b'Udcg|hcb'hc'Xc'i 'hfU!XYhU]'YX'fYgYUfW\'"

D`YUgY`gi Va]hVmh\Y`YbX`cZk YY_`*`U`Vf]YZUVglfUWik]h\`Uʻh]hYž'Vf]YZWbhYl hz`_Ym]ggi Yg`mci `\cdY`hc Wbj Yfž'UbX`_YmfYZYfYbWfg`gc`h\Uhmci `fYWf]j Y`gca Y`ZYYXVUW_`UbX`Wba a Ybhg`UbX`_bck `mci `UfY`cb`lfUW_UbX`Wb`ghUfhih\]b_]b[`YUf`]Yf`fUh\Yf`H\Ub``UhYf`UVci hih\]g`Ugg][ba Ybh'``=k]```bchbU[`mci `UVci hih\]g`UbX`]h]g`bchfYei]fYX/Vi hd`YUgY`XcbffiYl dYWhh\Y`gUa Y``Yj Y``cZZYYXVUW_`]ZdfYgYbhYX`hc`a Y`UhU``UhYf`dc]bh]b h\Y`Wbi fgY"""

5 hmyb XUb WY F Yei]fYa Yb hg

Ghi XYbhgʻg\ci `X`]bʻa cghiWugYgʻUhhYbX`h\Y`ZUWY&ZUWY`WUgg"``H\cgY`k\c'UfY`cj YfgYUgʻk]```VY`UV`Y`hcʻg][b]bʻhcʻci fʻgYgg]cbg"'H\YmUfY`YI dYWhYX`hc`\Uj Y`gYhh\Ya gY`j Ygʻi d']bʻUbʻUddfcdf]UhY`ei]Yhcb`]bY`UbX ZcWigYX`Ybj]fcba Ybhžʻg\ci `X`\Uj Y`h\Y]f`Wua YfUgʻcbžʻUbXʻg\ci `X'VY`UV`Y`hc`dUfh]WjdUhY`UWhj Y`m'``Ghi XYbhg k\c'UfY`i bUV`Y`hc`dUfh]WjdUhY`UWhj Y`mXYgd]hY`VY]b['Uh'U`X]ghUbWY`k]```\Uj Y`X]ZZJWI`hm`YUfb]b['Zfca 'UbX gUhjqZJWhcf]``Wca d`Yhjb['h\Y`Wci fgY"

7 ci fgY GWX YXi `Y

	5ggYgga Ybh	@cWU`]g]b['\i a Ub]hUf]Ub'UWnjcb'! 'fc`Y'd`Um
K YY_'%\$.'%('Bcj Ya VYf !'% 'Bcj Ya VYf	GYa]bUf	7 \U``Yb[Yg'UbX'cddcfhi b]h]Yg'hc']a dfcj Y XYj Y`cda YbhUbX'\i a Ub]hUf]Ub'gmghYa g" ? Ym]bg][\hg'UbX'`Yggcbg'`YUfbYX"'F Ya U]b]b[ei Ygh]cbg" : YYXVUW_'UbX'Wca a Ybhg'cb'Wci fgY'UbX VYmcbX"""

FYgci fWyg

DfYgWf]VYX F Ygci fWYg

FYUX]b[g'k]``'VY'UjU]UV'Y'h\fci[\'h\Y'@Y[Ubhc'g]hY'UggcW]UhYX'k]h\'h\]g'WcifgY"'

 $D`YUgY`Ybgi fY`h\ Uh'mci `fYUX`U```a \ UbXUhcfmfYUX]b[g`UbX`Wca Y`dfYdUfYX`hc`h\Y`fY`Yj \ UbhgYgg]cbg``\ Uj]b[fYUX`h\Y`fYei]fYX`fYUX]b[g''$

 $: cf`YUWX`gYa]bUf`UXX]h]cbU``fYUX]b[g`k]```U`gc`VY`aUXY`UjU]`UV`Y"``Mci`UfY`YbWcifU[YX`hc`U`gc`g\UfY]bhYfYgh]b[`fYgcifWYg`mci`\UjY`WcaY`UWfcgg`UbX`h\UhaUmVY`cZ]bhYfYghhc`ch\Yfg`]b`h\Y`WUgg"$

BchY'h\UndUfricZmcif``YUfb]b['X]Ufm]g`ghfi WhfYX`Ufci bX`U`Wf]h]WU``fYj]Yk `cZU`_YmWci fgY`fYUX]b[']b'h\Y Zfgh\U`ZcZh\Y`Wci fgY"

FYWca a YbXYX FYgci fWyg

D`YUgY`fYZYf`hc`h\Y`@Y[Ubhc`g]hY`UbX`UbmUXX]h]cbU``fYgci fWYg`h\UhiUfY`i d`cUXYX`hc`A ccX`Y"

5XX]hacbu``a UhYf]u`q`k]```VY`UXXYX`hc`hu_Y`UVWci bhcZfYVWbhdi V`]WUhacbq`UbX`a YX]u`cZ]bhYfYqh'

D`YUgY`a U_Y`gi fY`hc`Wcbgi `hth\Y`Wci fgY`A ccX`Y`g]hY`Yj YfmA cbXUm`]b`UXj UbWY`cZci f`gYgg]cbg`cb`th\Y H\i fgXUm''`=i`]g`]bhYbXYX`th\Uh'Ubmi dXUhYX`a UhYf]U`g`cf`WUf]Z]WUf]cbg`fY`hUg_g`k]``VY`i d`cUXYX`VmA cbXUm `i bW\h]a Y`YUW\`k YY_"

7ci fgY 9j U`i Uh]cb UbX 8Yj Y`cda Ybh

H\]q"Wci fqY"\Uq"VYYb"hchU"\mfYqhfi Wi fYX"Zfca "U"dfYj]ci q"]hYfUh]cb""

D`YUgY`dfcj]XY`Wcbglfi Wfj Y`ZYYXVUW_`UbX`Wca a Yblrg/h\YgY`k]```VY`i gYX`hc`Yb\UbWY`mci f``YUfb]b[``UbX`Yl dYf]YbWY`Xi f]b[`h\Y`Wci fgY`h\]g`mYUf`UbX`k]```U`gc`Wcblf]Vi hY`hc`Yb\UbW]b[`h\Y`Wci fgY`h\]g`mYUf`UbX`k]```U`gc`Wcblf]Vi hY`hc`Yb\UbW]b[`h\Y`Wci fgY`Zcf`h\Y`Z h`fY''

5 WUXYa]W<cbYglmUbX D`U[]Uf]ga

D`U[]Uf]ga i]gii g]b[ih\Yik cfXgicfi]XYUgicZch\YfgiUbXidfYgYbh]b[ih\Ya iUgimci fick b"i±i\WbihU_Yia Ubm Zcfa gžZfca iXYi]VYfUhYiW\YUh]b[ihciU\W]XYbhU`m\Wcdm]b[iZfca iUigci f\Wik]h\ci hiU\w_bck iYX[Ya Ybh'

IBGK [fcidg'd'U[]Uf]ga]bhc'h\Y'Zc``ck]b['WUhY[cf]Yg.

- 7 cdm]b[.'I g]b['h\Y'gUa Y'cf'j Yfmg]a]'Uf'k cfXg'hc'h\Y'cf][]bU'hYI hcf']XYU'k]h\ci hUW_bck 'YX[]b[h\Y'gci fW'cf'i g]b['ei chUh]cb'a Uf_g"'H\]g']bWi XYg'Wcdm]b['a UhYf]U'gž]XYUg'cf'WcbW'dhg'Zfca 'U Vcc_ž'Ufh]WYž'fYdcfhcf'ch\Yf'k f]hHYb'XcW a Ybhž'dfYgYbhUh]cbž'Wca dcg]h]cbž'Uffk cf_ž'XYg][bž XfUk]b['z'W]fWi]hfmž'Wca di hYf'dfc[fUa 'cf'gcZhk UfYž'k YVg]hYž']bhYfbYhž'ch\Yf'Y'YWfcb]WfYgci fWYž'cf Ubch\Yf'dYfgcbfq'Uqq][ba Ybhk]h\ci hUddfcdf]UhY'UW_bck 'YX[Ya Ybh'
- \pm 0Uddfcdf]UhY dUfUd\fUg]b[. '7\Ub[]b['U'ZYk 'k cfXg'UbX'd\fUgYg'k\]Y'a cghmfYhU]b]b['h\Y

5 WUXYa]W + DZcfa Uh]cb

8 i Y'hc'Yj c'j]b['UXj]WY'VmBGK '<YU'h\z'ghi XYbhg'a i ghW\Y<u>W</u>'Zcf'i dXUhYX']bZcfa Uh]cb'fY[UfX]b['cb`]bY 'YUfb]b['Zcf'U``'5fhgz'8Yg][b'UbX'5fW\]hYWi fY'Wci fgYg'h\]g'hYfa 'fJ]U'A ccX`Y'cf'Wci fgY']bZcfa Uh]cb dfcj]XYXŁ"

.

 $\label{eq:decomposition} D`YUgY`gYY. \$\https://dc... $$ Li bgk "YXi"Ui #Ufhg!XYg][b!UfWX]hYWh fY#ghi XYbh`]ZY#Ygci fWYg!gi ddcfh#dfchcWc`g! $$ [i]XY`]bYg`Zcf`YggYbh]U``ghi XYbh']bZcfa Uh]cb`fY`Uh]b[`hc. $$$

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