"Needed: A Theory of Total Factor Productivity"

1. Introduction

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1. Introduction, *continued*

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2. Index Number Approach

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- ! y_t , p_t
- ! $x_{K,t}$, $w_{K,t}$
- ! $x_{L,t}$, $w_{L,t}$

2. Index Number Approach, *continued*

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2. Index Number Approach, *continued*

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3. Production function approach

- TFP can also be defined with reference to a production function
- This actually leads to for four interpretations of TFP

3. Production function approach, *continued*

$$(5)$$

$$\frac{\partial f(\cdot)}{\partial x_{K,t}} = \frac{w_{K,t}}{p_t}$$

$$\frac{\partial f(\cdot)}{\partial x_{L,t}} = \frac{w_{L,t}}{p_t}$$

3. Production function approach, *continued*

Let μ_t ! $\ln y_t /\ t$ be the instantaneous rate of technological change; we then have:

(8) $$\frac{\partial f(\cdot)}{\partial t} = \mu_t y_t$$

Following Diewert and Morrison (1986), we define the following index of TFP:

(10) $$_{,\,!1} \quad \frac{(\quad _{,\,!1}, \quad _{,\,!1}, \ )}{(\quad _{,\,!1}, \quad _{,\,!1}, \ !\,1)} \quad (\quad _{,}, \quad _{,},$$

3. Production function approach,

3. Production function approach, *continued*

3. Production function approach, *continued*

3. Production function approach, *continued*

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t t

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3. Production function approach, *continued*

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Table 1

imates Parameter

4. Impact of TFP on factor rental prices

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! ϕ_{KT}

4. Impact of TFP on factor rental prices, *continued*

! ϕ_{KT}

4. Impact of TFP on factor rental prices, *continued*

4. Impact of TFP on factor rental prices, *continued*"

$$\frac{\phi_{KT}}{s_{L,t}} \qquad (23) \quad \hat{w}_{L,t} = \mu_t$$

where the hat (^) indicates a relative ch

4. Impact of TFP on factor rental prices,continued

- As long as the technology is progressing, the first term on the right hand side is positive
- If $!_{KT}$ is positive, technological change is anti-labor biased
- It might even be that $t_{KT}/s_{L,t} > \mu_t$, in which case technological change would be ultra anti-labor biased: technological change would then lead to an actual fall in the wage rateÉ
- É even though technological progress would unambiguously increase average labor productivity

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4. Impact of TFP on factor rental prices,continued

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5. Disembodied factor augmenting technological change

A more transparent approach is to assume that technological ...

5. Disembodied factor augmenting technological change, *continued*

5. Disembodied factor augmenting technological change, *continued*

For the rate of technological ch... ...ange ($\mu = \partial \ln y / \partial t$) we get:

5. Disembodied factor augmenting technological change, *continued*

Indeed, one can show

(4.)

$= \ln T$

5. Disembodied factor augmenting technological change,

Table 1

Parameter estimates

	(11)	*(31)*
α_0	8.38851 (3522.9)	8.39259 (4874.6)

6. The decomposition of TFP between labor and capital

6. The decomposition of TFP between labor and capital, *continued*

Making use of (26) ... (27) and (30) in (42) ... (43) we find:

[illegible]

$$= \frac{1}{2}(s_{K,t} + s_{K,t-1})\mu_K$$

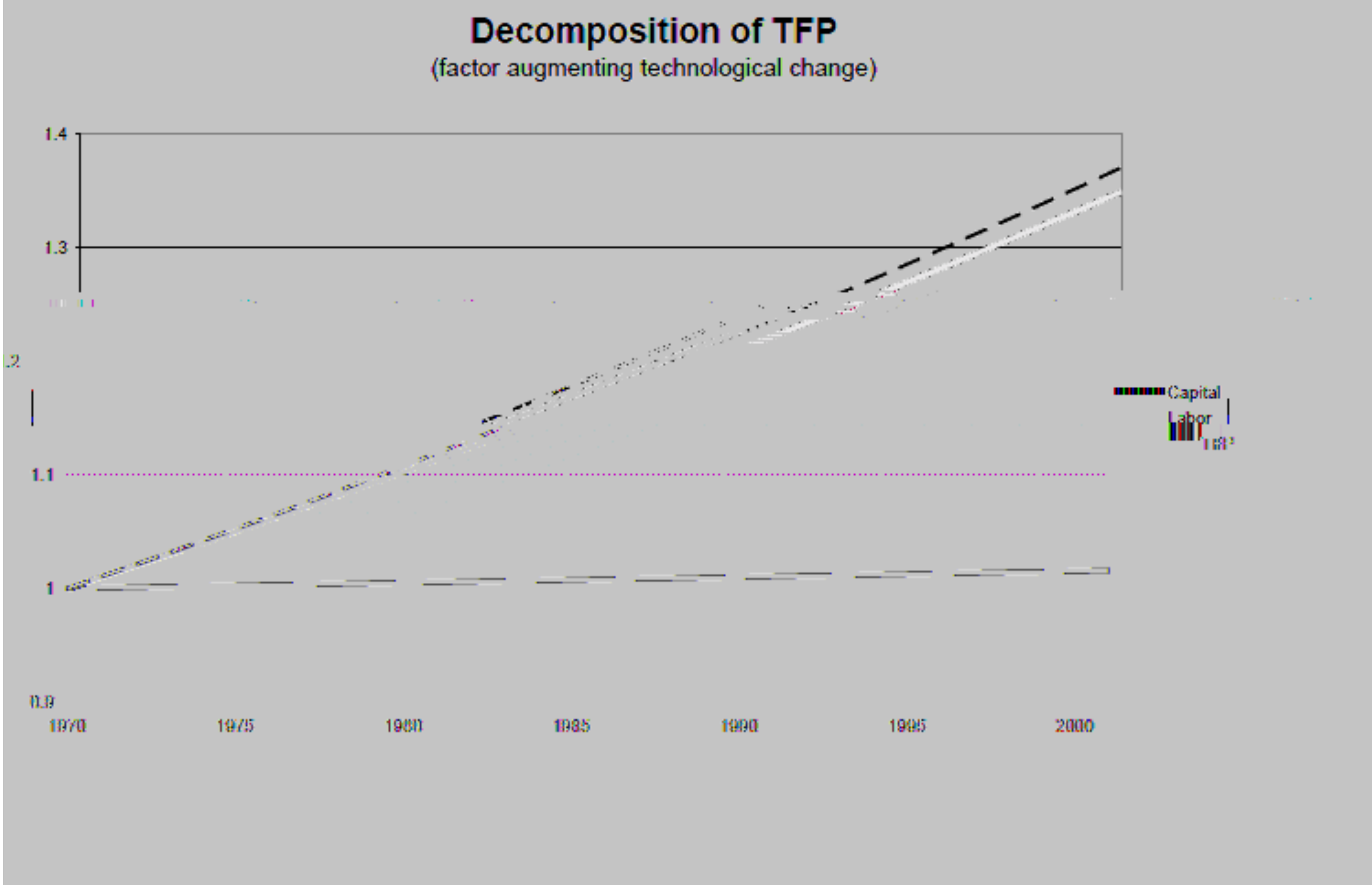
[illegible]

$$= \frac{1}{2}(s_{L,t} + s_{L,t-1})\mu_L$$

so that, in view of (38):

$$= T^K_{t,t-1} T^L_{t,t-1} \qquad (46)$$

Figure 1



Decomposition of TFP
(factor augmenting technological change)
1.4
1.3
1.1
1
0.9
1970
1975
1980
1985
1990
1995
2000
Capital
Labor

7. Factor augmenting technological change and TP flexibility

7. Factor augmenting technological change and TP flexibility, *continued*

Thus:

$$(49)\qquad \tilde{x}_{K,t} \equiv x_{K,t}\, e^{\mu_K t + \frac{1}{2}\lambda_K t^2}$$

$$e^{\mu_L t + \frac{1}{2}\lambda_L t^2}$$

...tion (τ... The instantaneous rates of factor augmenta...

$$\tau_{K,t} \equiv \frac{\partial \ln \tilde{x}_{K,t}(x_{K,t}, t)}{\partial t} = \mu_K + \lambda_K t \qquad (51)$$

7. Factor augmenting technological change and TP flexibility, *continued*

Introducing (49) and (50) into (30), we get:

$$\ln y_t = \alpha_0 + \beta_K \ln x_{K,t} + (1-\beta_K)\ln x_{L,t} + \beta_K \mu_K t + (1-\beta_K)\mu_L t$$

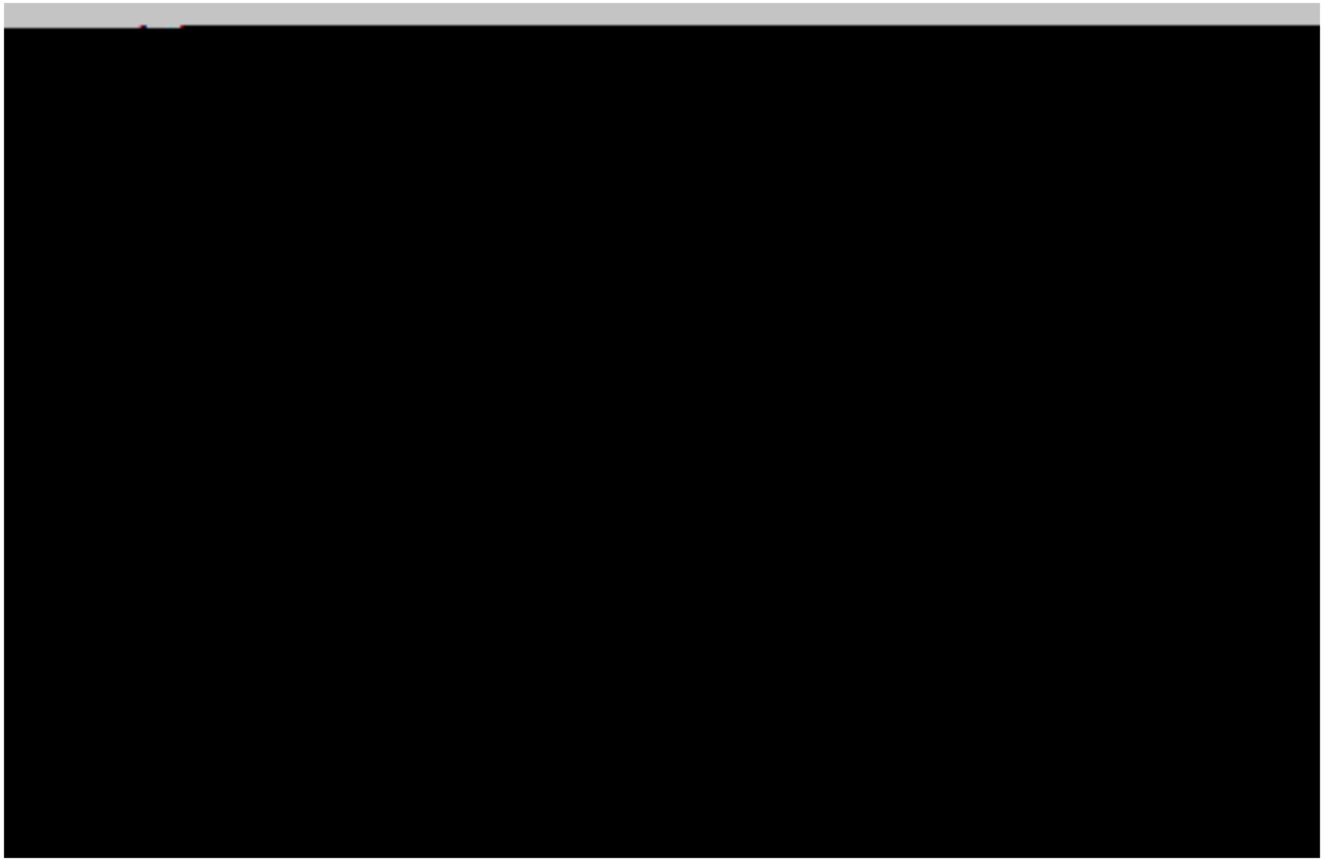
[illegible]

$$+\frac{1}{8}\phi_{KK}(\lambda_K - \lambda_L)^2 t^2$$

7. Factor augmenting technological change and TP flexibility, *continued*

The inverse demand functions are now as follows:

7. Factor augmenting technological change and TP flexibility, *continued*



7. Factor augmenting technological change and TP flexibility, *continued*

[redacted]

7. Factor augmenting technological change and TP flexibility, *continued*

$$+\frac{1}{2}\phi_{KK}(\mu_K - \mu_L)\mu_K(2t-1)$$

$$+\frac{1}{\;}\phi(\mu - \mu)\lambda(4t^2 - 4t + 1) + \frac{1}{\;}\phi(\lambda - \lambda)\mu(2t^2 - 2t + 1)$$

$$+\frac{1}{\;}\phi(\lambda - \lambda)\lambda(4t^3 - 6t^2 + 4t - 1)$$

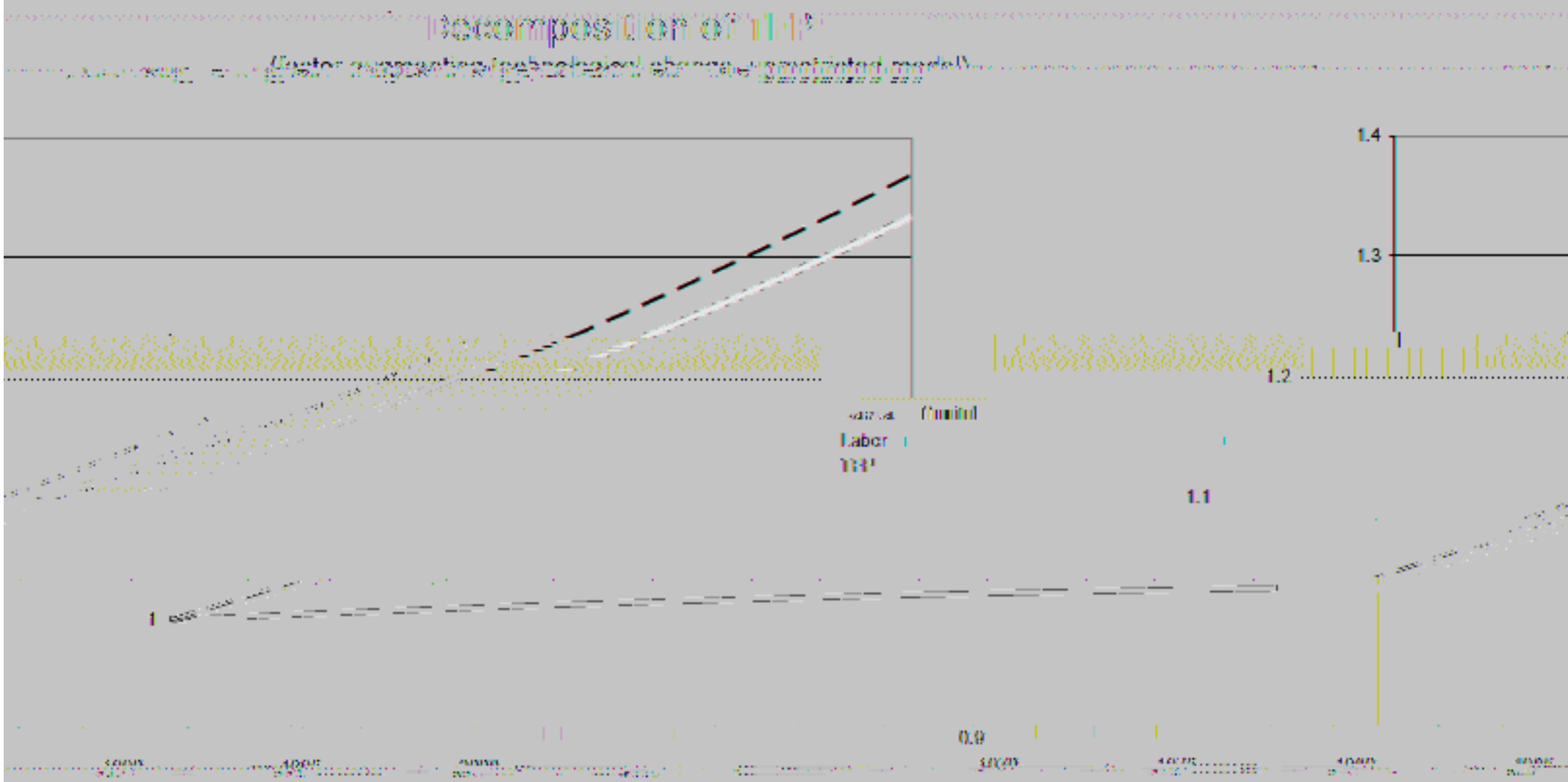
Table 1

Parameter estimates

	(31)	(53)		(11)
	8.30250	8.38044		8.38851
	[illegible]	[illegible]	[illegible]	[illegible]
λ_L			0.00077	
			(2.02)	
λ_S			0.00075	
			(3.29)	
LL	226.70	224.09	227.89	

μ_{2001}	0.01116	0.01158	0.01008
$s_{K,2001}$	0.28007	0.28452	0.28454
$\psi_{KL,2001}$	2.01263	1.75267	1.66797

Figure 2



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8. A parsimonious and yet flexible model

case the production funct

$$(63) \quad u_t = \beta_K u_K + (1-\beta_K) u_L + \phi_{KK}(\ln x_{K,t} - \ln x_{L,t})(u$$

8. A parsimonious and yet flexible model, *continued*

It turns out that the model of equation (62) is equivalent to (11), since there is a one-to-one [illegible]

[illegible]

(64) $\beta_T = \beta_K \mu_K + (1 - \beta_K)\mu_L$

(65) $\phi_{KT} = \phi_{KK}(\mu_K - \mu_L)$

(66) $\phi_{TT} = \phi_{KK}(\mu_K - \mu_L)^2 + \lambda$

[illegible]

(67) [illegible]

(68) $$\mu_L = \beta_T - \beta_K \frac{\phi_{KT}}{\phi_{KK}}$$

(69) $$\lambda = \phi_{TT} - \frac{\phi_{KT}^2}{\phi_{KK}}$$

8. A parsimonious and yet flexible model, *continued*

For TFP we now get:

$$\ln T_{t,t-1} = \beta_K \mu_K + (1-\beta_K)\mu_L + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)(\ln x_{K,t} - \ln$$

(70)

f TFP is numerically identical to (15). This es

$$\ln T^{K}_{t,t-1} = (1-\beta_K)\mu_L + \frac{1}{2}(1-\beta_K)\lambda(2t-1)$$

$$- \mu_L)\lambda(3t^2 - 3t + 1)$$

$$+\frac{1}{2}\phi_{KK}(\mu_K - \mu_L)\mu_L(2t-1)+$$

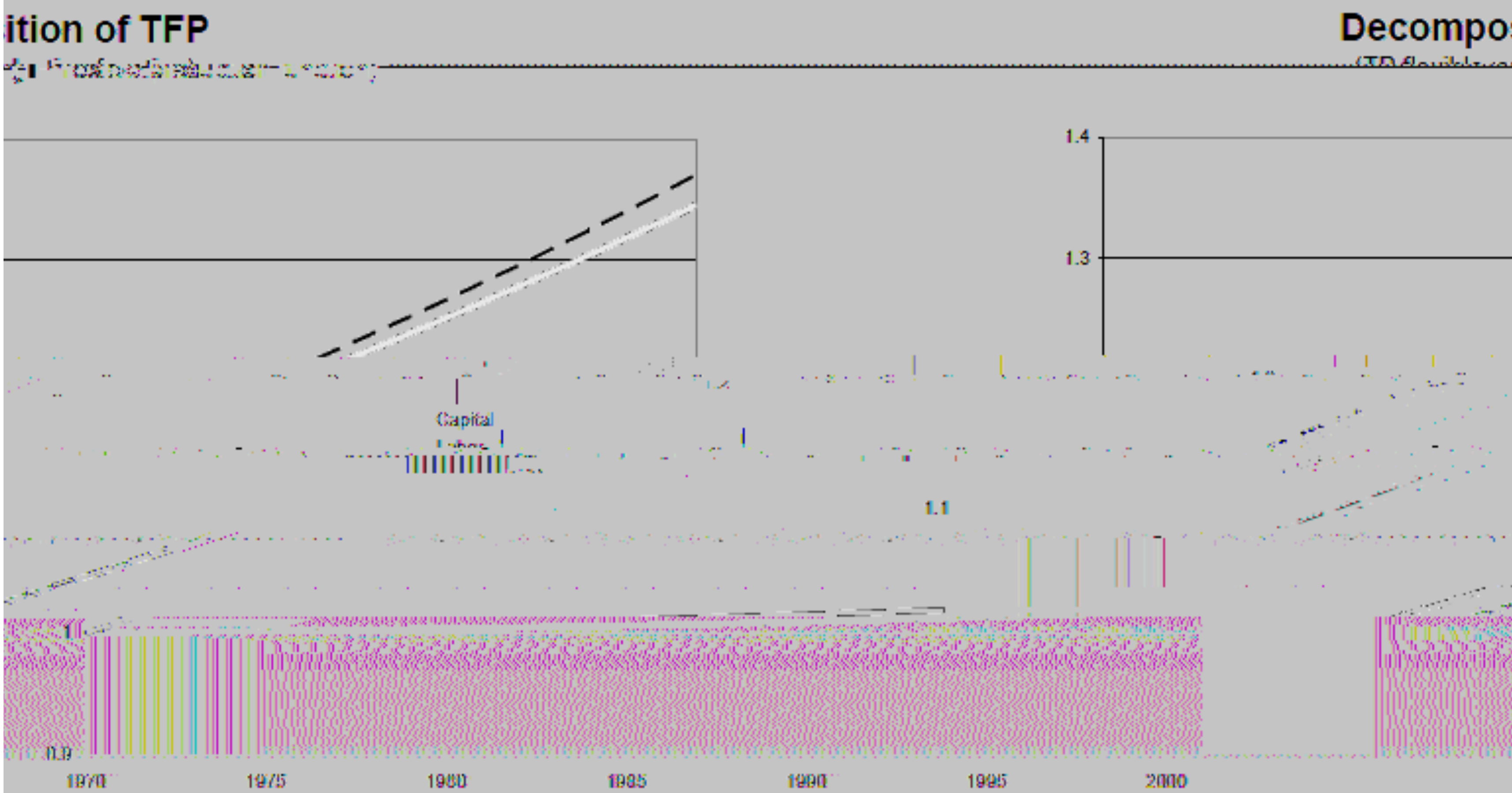
$$

Table 1

Parameter estimates

	(11)	(31)	(53)	(62)	
	8.38851	8.39259	8.38944	8.38851	α_0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Figure 3



ition of TFP
Decompos
1.4
1.3
Capital
1.1
1
0.9
1970
1975
1980
1985
1990
1995
2000

9. The impact of technological change on factor rental prices reexamined, *continued*"

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9. The impact of technological change on factor rental prices reexamined, *continued*"

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Let ε_{ij} be the inverse price elasticity of factor demand:

$$(73) \quad \varepsilon_{ij} \equiv \frac{\partial \ln \tilde{w}_i(\tilde{x}_K, \tilde{x}_L, p)}{\partial \ln \tilde{x}_j}, \quad i, j \in \{K, L\}$$

[illegible]

$$(74) \quad \varepsilon_{iK} + \varepsilon_{iL} = 0, \quad i \in \{K, L\}$$

It is well known that:

$$(75) \quad \varepsilon_{KL} = \psi_{KL} s_L$$

$$(76) \quad \varepsilon_{LK} = \psi_{KL} s_K$$

We thus get for the total change in the rental price of an efficiency unit of capital:

[illegible]

(80) $\hat{w}_K =$ [illegible]

and similarly for labor:

[illegible]

9. The impact of technological change on factor rental prices reexamined, *continued*"

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9. The impact of technological change on factor rental prices reexamined, *continued*"

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10. Generalization to an arbitrary number of inputs, *continued*

11. Conclusions

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! **five different**

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! *t* *t*

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11. Conclusions, continued

- We have shown that in the case of a TP-flexible Translog production function TFP can always be interpreted as the outcome of disembodied, factor augmenting technological change
- Indeed, we have proposed a convenient way to derive the factor-augmenting rates of technological change from the estimates of such a Translog production function
- We have found that technological change is almost Harrod-neutral in the case of the United States, so that TFP is overwhelmingly explained by labor

11. Conclusions, *continued*

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Thank you for your attention!

Growth factors 1970-2001

Quantity of capital services:!	"#	2.25706
! "#$%&%'()*(+#,)-(./-0&1/.2!	"$	1.70513
3-&1/()*()"%4"%2!	%	3.76623
! "#$%&%'()*()"%4"%2!	&	2.52563
3-&1/()*(+#,)-(./-0&1/.2(	'$	5.50201
5)%#+(*#1%)-(4-)6"1%&0&%'2!	(	1.37071
Capital component of TFP:	T_K	1.01850
Labor component of TFP:	T_L	1.34581
Capital efficiency:	!#	1.06789
Labor efficiency:	!$	1.50832
7#,)-(.8#-/2!	)$	0.98628
Output per unit of labor:	! ! ! ! !$_L$	1.48119