

“Needed: A Theory of Total Factor Productivity”

1. Introduction

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1. Introduction, *continued*

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2. Index Number Approach

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! y_t, p_t

! $x_{K,t}, w_{K,t}$

! $x_{L,t}, w_{L,t}$

2. Index Number Approach, *continued*

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2. Index Number Approach, *continued*

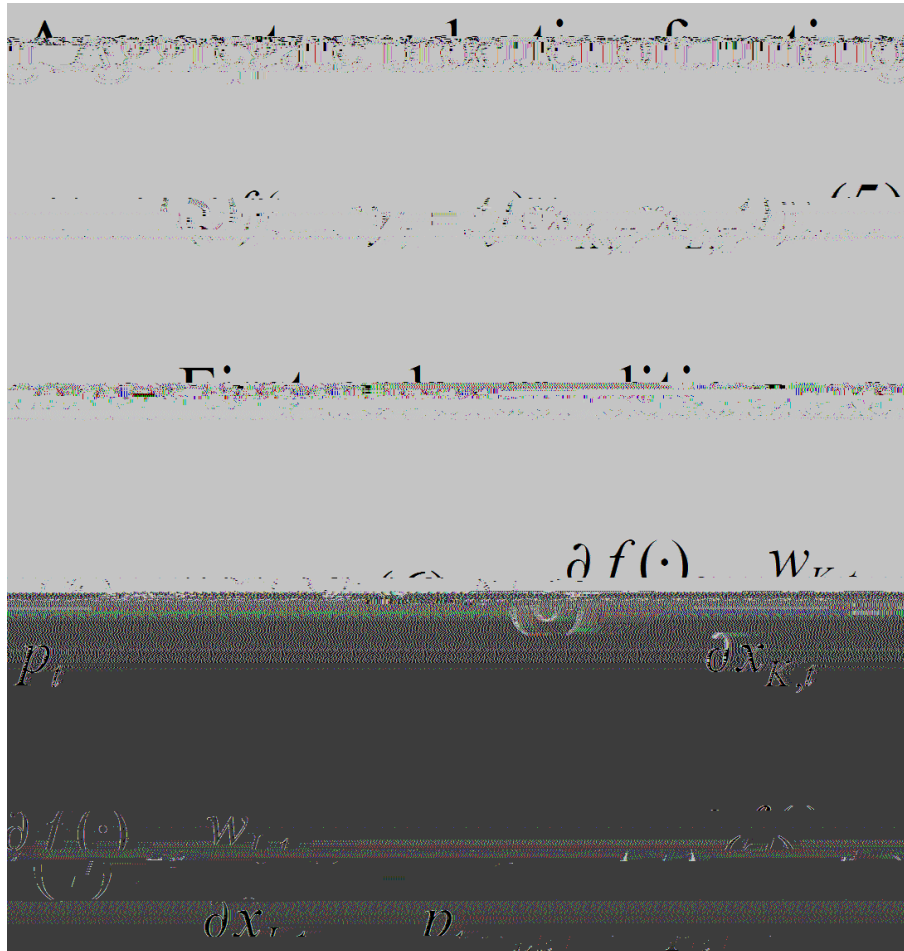
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3. Production function approach

- ⚠ TFP can also be defined with reference to a production function
- ⚠ This actually leads to four interpretations of TFP

3. Production function approach, *continued*



3. Production function approach, *continued*

Let $\mu_t \equiv \ln y_t / t$ be the instantaneous rate of technological change; we then have:

$$(8) \quad \frac{\partial f(\cdot)}{\partial t} = \mu_t y_t$$

Following Diewert and Morrison (1986), we define the following index of TFP:

$$(10) \quad TFP_{t,1} = \frac{f(x_{t,1}, y_{t,1})}{f(x_{t,1}, y_{t,1}, 1)}$$

3. Production function approach,

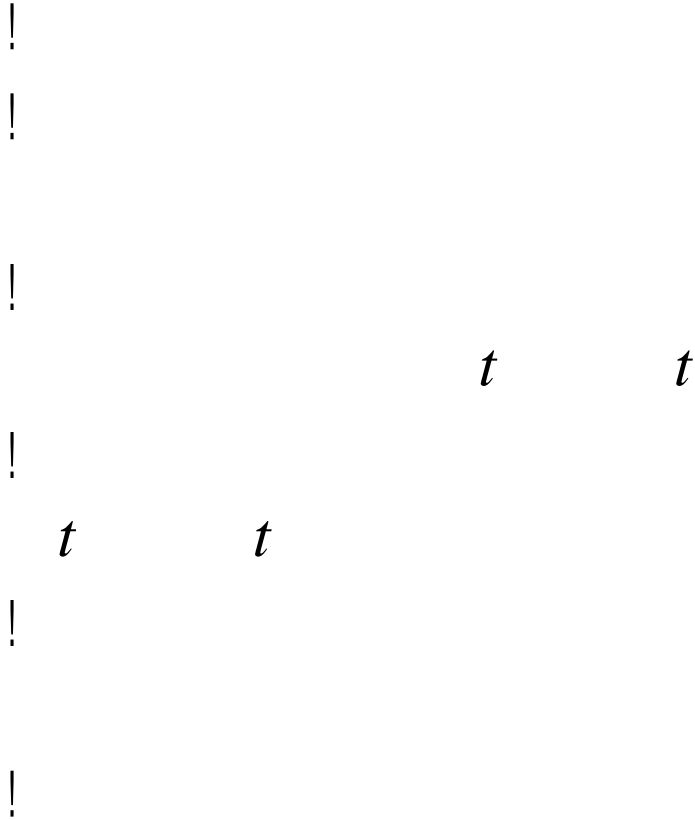
3. Production function approach, *continued*

$$\frac{\partial \ln f(\cdot)}{\partial l} = \frac{\partial \ln f(\cdot)}{\partial \ln y} = \beta + \frac{\partial \ln f(\cdot)}{\partial \ln y} = \ln y$$

$$\frac{\partial \ln f(\cdot)}{\partial l} = \frac{1}{2} \frac{\partial \ln f(\cdot)}{\partial \ln y} = \frac{1}{2} \ln y$$

$$\frac{\partial \ln f(\cdot)}{\partial l} = \frac{1}{2} \frac{\partial \ln f(\cdot)}{\partial \ln y} = \frac{1}{2} \ln y$$

3. Production function approach, *continued*



3. Production function approach, *continued*

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imates Parameter

Parameter	Estimate
β_1	0.0000
β_2	0.0000
β_3	0.0000
β_4	0.0000
β_5	0.0000
β_6	0.0000
β_7	0.0000
β_8	0.0000
β_9	0.0000
β_{10}	0.0000
β_{11}	0.0000
β_{12}	0.0000
β_{13}	0.0000
β_{14}	0.0000
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β_{20}	0.0000
β_{21}	0.0000
β_{22}	0.0000
β_{23}	0.0000
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β_{26}	0.0000
β_{27}	0.0000
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β_{95}	0.0000
β_{96}	0.0000
β_{97}	0.0000
β_{98}	0.0000
β_{99}	0.0000
β_{100}	0.0000

4. Impact of TFP on factor rental prices

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$$\phi_{KT}$$

4. Impact of TFP on factor rental prices, *continued*

! ϕ_{KT}

4. Impact of TFP on factor rental prices, *continued*"



4. Impact of TFP on factor rental prices, *continued*"

$$\frac{\phi_{KT}}{S_{L,t}} \quad (23) \quad \hat{w}_{L,t} = \mu_t$$

where the hat ($\hat{\cdot}$) indicates a relative change

4. Impact of TFP on factor rental prices, continued

- ✘ As long as the technology is progressing, the first term on the right hand side is positive
- ✘ If $\dot{\theta}_{KT}$ is positive, technological change is anti-labor biased
- ✘ It might even be that $\dot{\theta}_{KT}/s_{L,t} > \mu_t$, in which case technological change would be ultra anti-labor biased: technological change would then lead to an actual fall in the wage rate \dot{w}
- ✘ \dot{w} even though technological progress would unambiguously increase average labor productivity

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4. Impact of TFP on factor rental prices,continued

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5. Disembodied factor augmenting technological change, *continued*

5. Disembodied factor augmenting technological change, *continued*

The result could be obtained in a similar manner with only a few modifications to the model. In particular, we assume that the production function is given by (22)

$$Y = A K^\alpha L^{1-\alpha} \quad (22)$$

where A is a technology parameter that grows at rate g . For the rate of technological change ($g = d \ln A / dt$) we get:

$$g = \frac{1}{\alpha} \left(\frac{d \ln Y}{dt} - \frac{d \ln K}{dt} - (1-\alpha) \frac{d \ln L}{dt} \right) \quad (23)$$

For the rate of technological change ($g = d \ln A / dt$) we get:

$$g = \frac{1}{\alpha} \left(\frac{d \ln Y}{dt} - \frac{d \ln K}{dt} - (1-\alpha) \frac{d \ln L}{dt} \right) \quad (24)$$

5. Disembodied factor augmenting technological change,

Table 1

Parameter estimates

	(11)	(31)
α_0	8.38851 (3522.9)	8.39259 (4874.6)
ϕ_{11}	0.00308 (1.93)	ϕ_{11}
μ_K	0.00182 (1.78)	μ_K
σ^2	1.09	22.670
μ_{2001}	0.008	0.01158
μ_{2002}	0.008	0.01158

6. The decomposition of TFP between labor and capital



6. The decomposition of TFP between labor and capital, *continued*

Making use of (26), (27) and (30) in (42), (42) can be written as

$$\frac{1}{2} \left[\frac{1}{2} (s_{K,t} + s_{K,t-1}) \mu_K \right] \quad (44)$$

so that in view of (38):

$$= \frac{1}{2} (s_{K,t} + s_{K,t-1}) \mu_K$$

so that in view of (38):

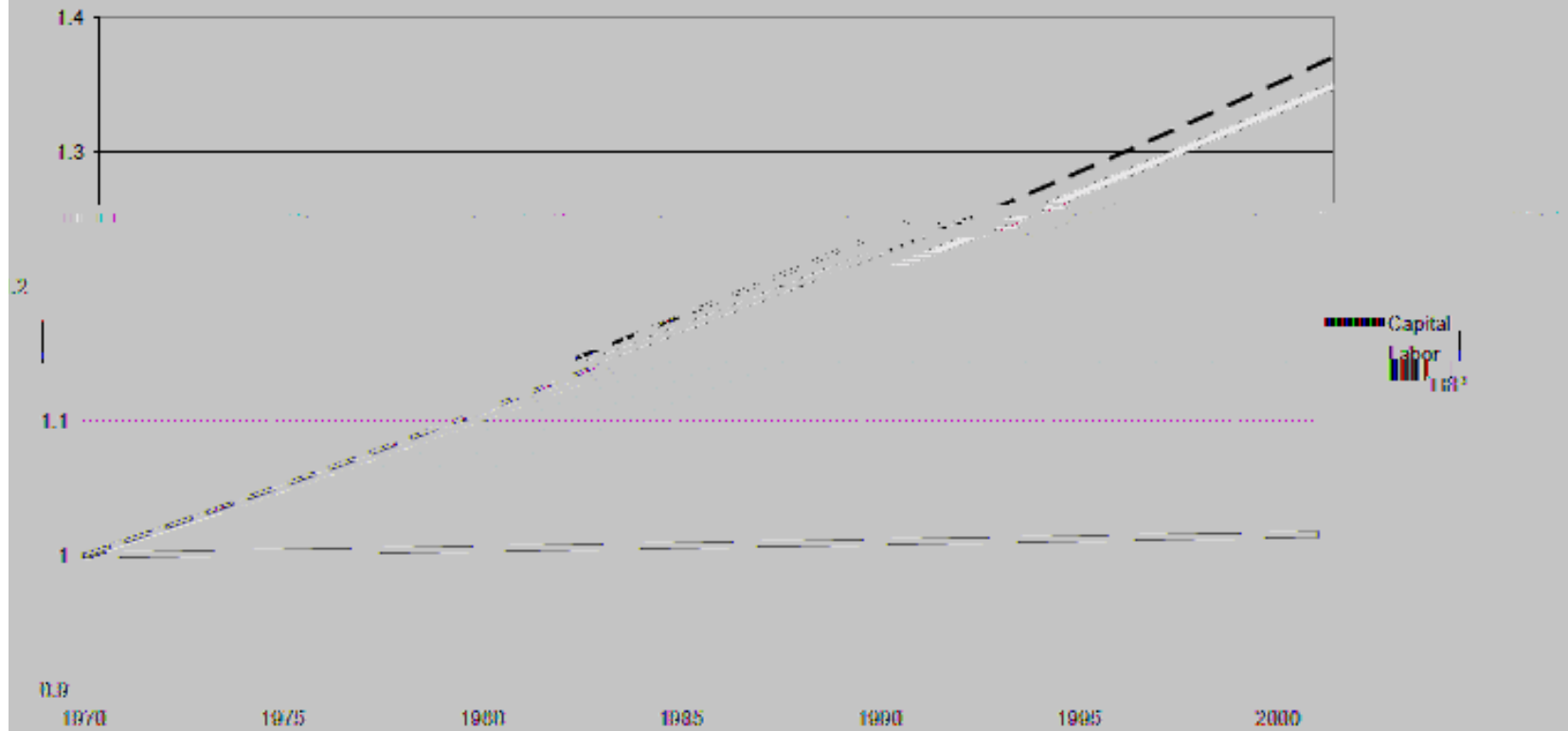
$$= \frac{1}{2} (s_{L,t} + s_{L,t-1}) \mu_L$$

so that in view of (38):

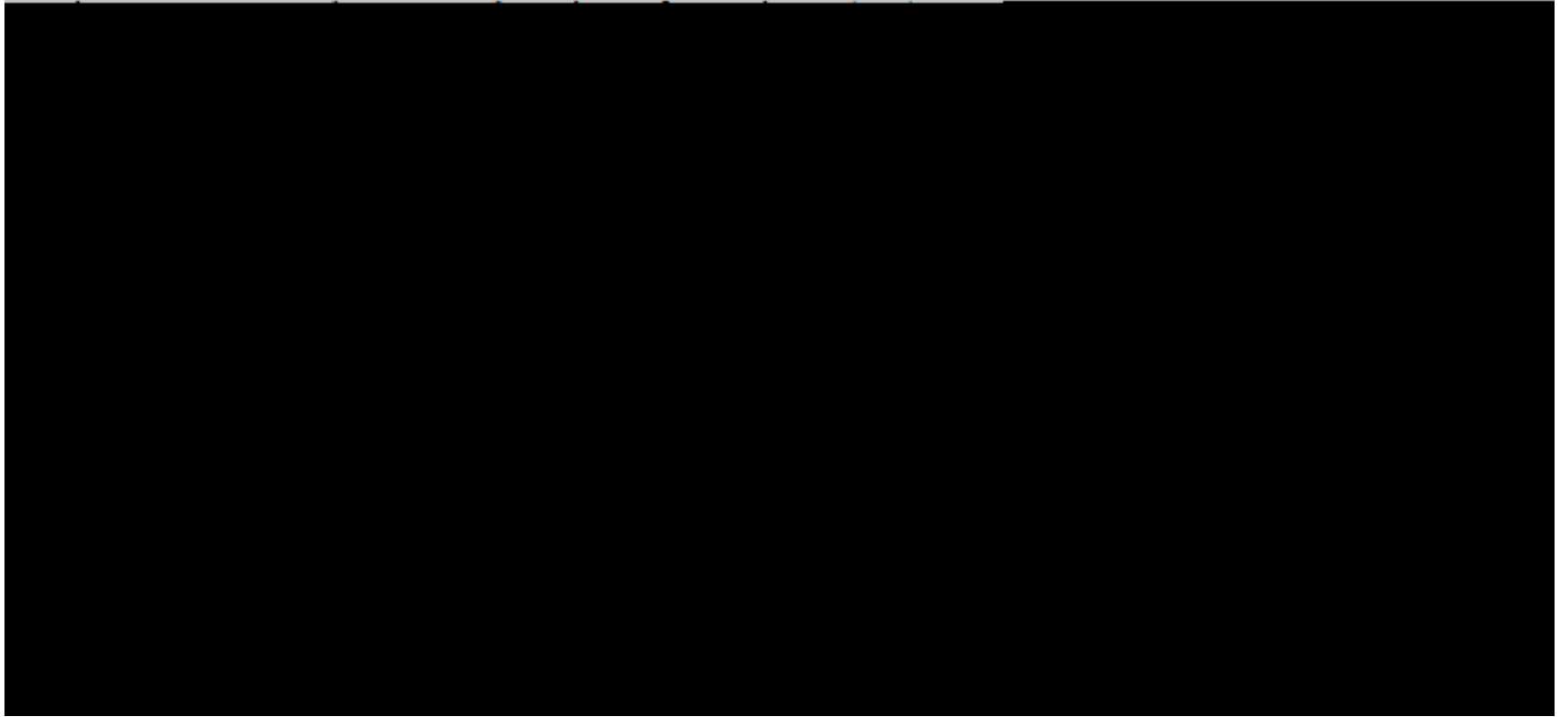
$$= T_{t,t-1}^K T_{t,t-1}^L \quad (46)$$

Figure 1

Decomposition of TFP
(factor augmenting technological change)



7. Factor augmenting technological change and TP flexibility



7. Factor augmenting technological change and TP flexibility, *continued*

Thus:

$$(49) \quad \tilde{x}_{K,t} \equiv x_{K,t} e^{\mu_K t + \frac{1}{2} \lambda_K t^2}$$

$$\frac{\partial \ln \tilde{x}_{K,t}}{\partial t} = \frac{\partial \ln x_{K,t}}{\partial t} + \mu_K + \lambda_K t$$

The instantaneous rates of factor augmentation are $\mu_K + \lambda_K t$.

The instantaneous rate of factor augmentation is $\mu_X + \lambda_X t$.

$$\frac{\partial \ln \tilde{x}_{X,t}}{\partial t} = \mu_X + \lambda_X t \quad (51)$$

7. Factor augmenting technological change and TP flexibility, *continued*

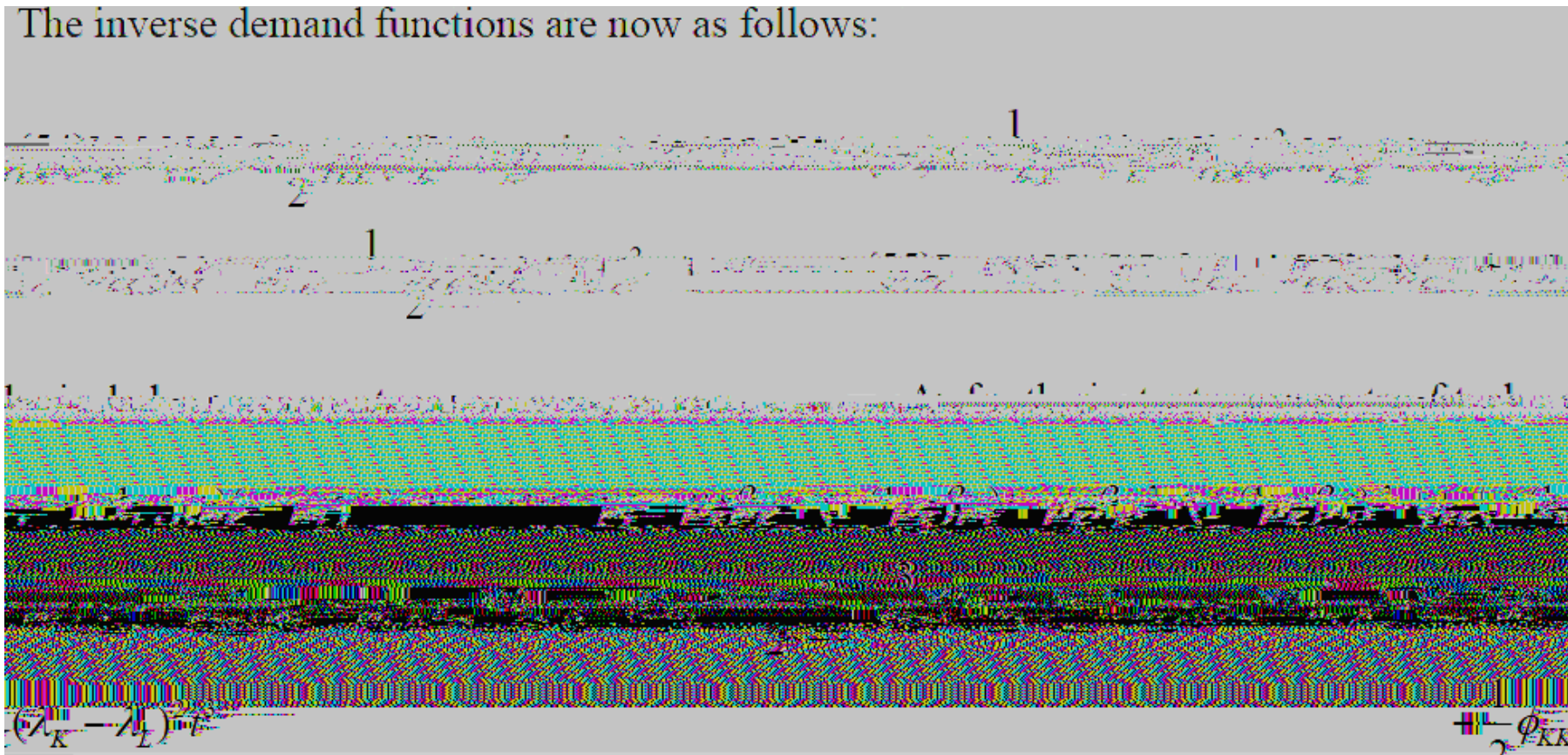
Introducing (49) and (50) into (30) we get:

$$\ln y_t = \alpha_0 + \beta_K \ln x_{K,t} + (1 - \beta_K) \ln x_{L,t} + \beta_K u_{K,t} + (1 - \beta_K) u_{L,t}$$

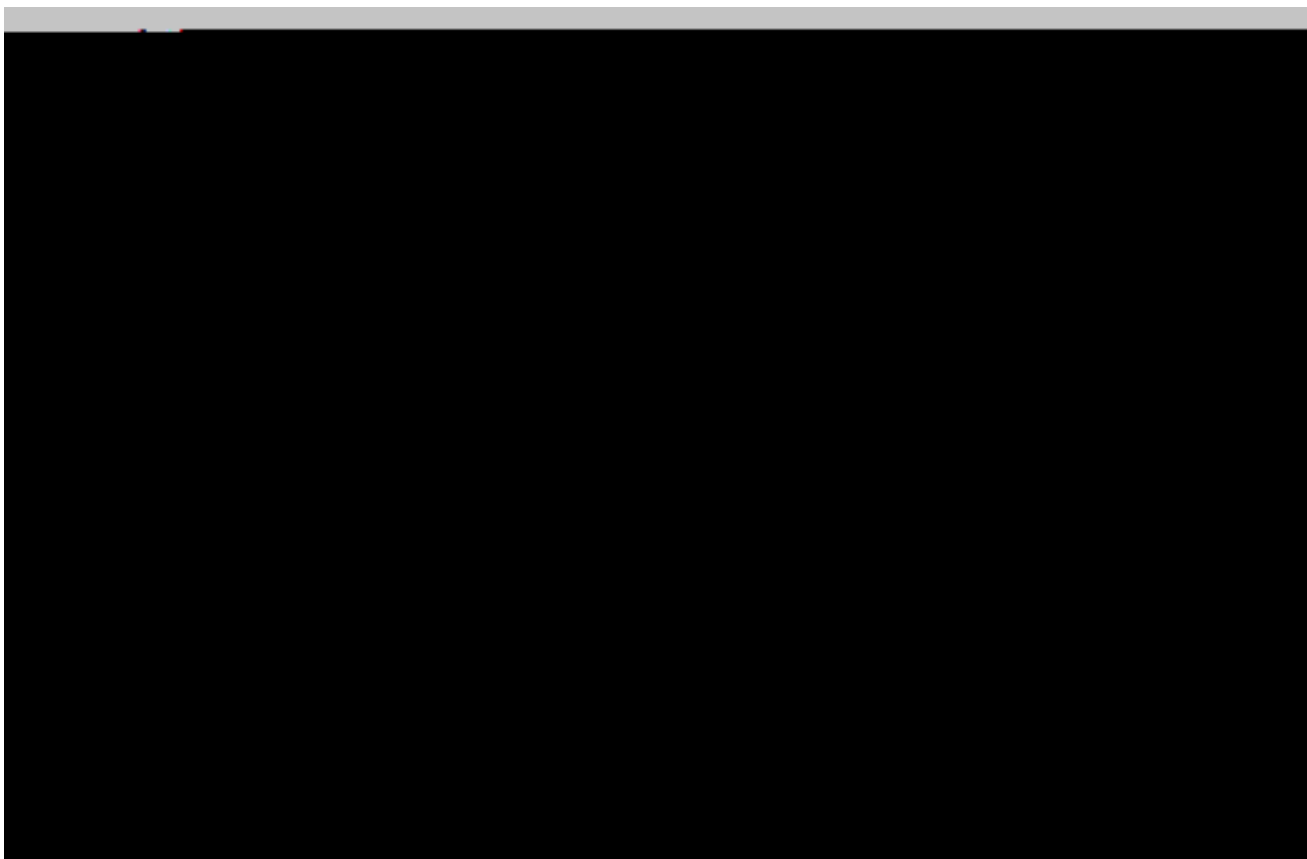
$$-\frac{\phi_{KK}}{\delta} (\lambda_K - \lambda_L) t$$

7. Factor augmenting technological change and TP flexibility, *continued*

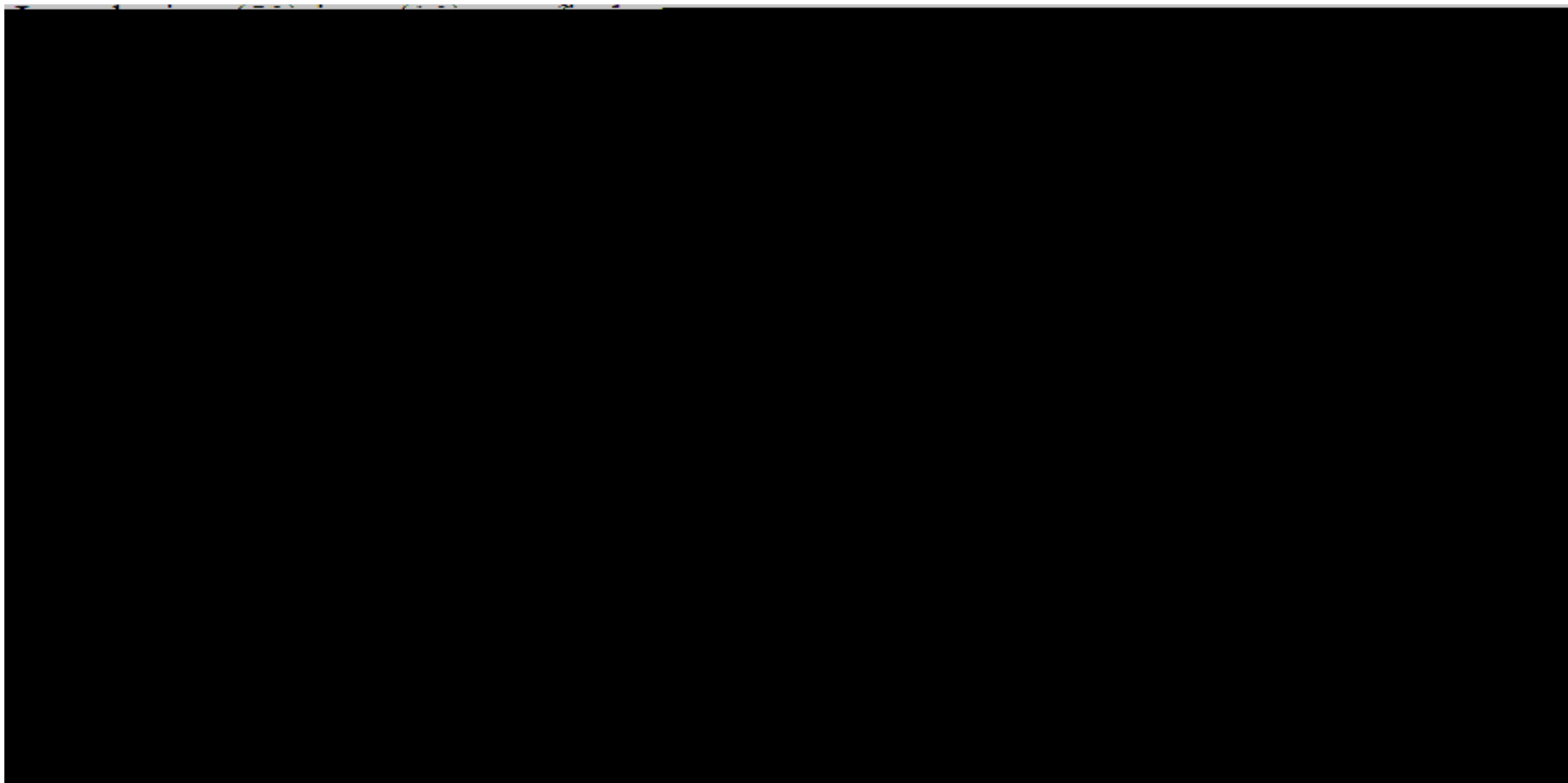
The inverse demand functions are now as follows:



7. Factor augmenting technological change and TP flexibility, *continued*



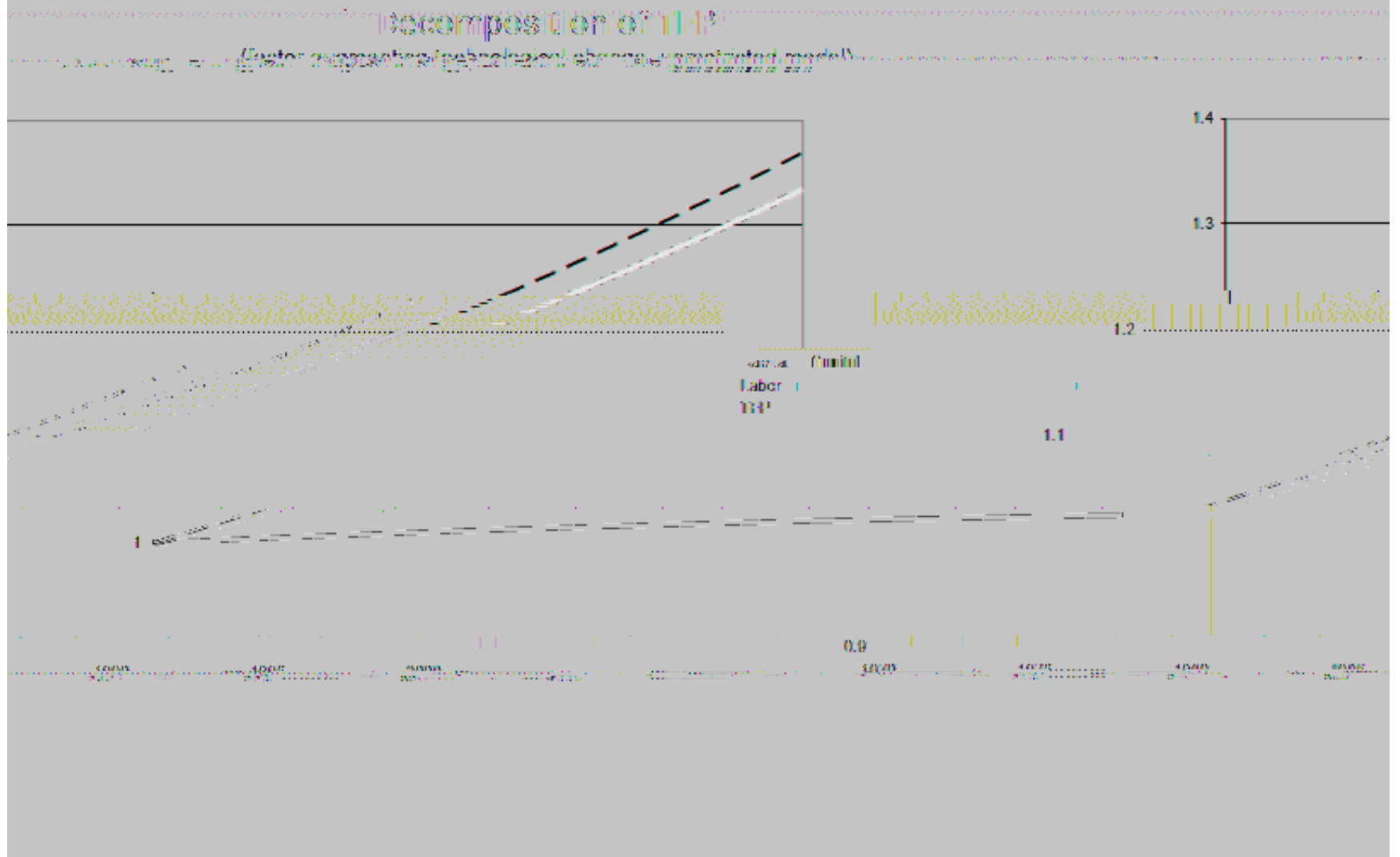
7. Factor augmenting technological change and TP flexibility, *continued*



7. Factor augmenting technological change and TP flexibility, *continued*

$$\begin{aligned}
 & \phi_{KK} (\ln y_{K,t} + \ln y_{L,t} - \ln y_{K,t-1} - \ln y_{L,t-1}) \lambda_{K,t} (2t-1) \\
 & + \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) \mu_K (2t-1) \\
 & + \frac{1}{2} \phi_{LL} (\mu_L - \mu_K) \lambda_{L,t} (4t^2 - 4t + 1) + \frac{1}{2} \phi_{LL} (\lambda_{L,t} - \lambda_{K,t}) \mu_L (2t^2 - 2t + 1) \\
 & + \frac{1}{6} \phi_{LL} (\lambda_{L,t} - \lambda_{K,t}) \lambda_{L,t} (4t^3 - 6t^2 + 4t - 1)
 \end{aligned}$$

Figure 7



8. A parsimonious and yet flexible model

$$E_t \ln x_{t+1} = \beta_{LS} E_t u_{t+1} + (1 - \beta_{LS}) E_t u_t + \phi_{LS} (\ln x_{t+1} - \ln x_{t,P}) (u_{t+1} - u_t) \quad (62)$$

In case the production function becomes...

$$E_t \ln x_{t+1} = \beta_{LS} E_t u_{t+1} + (1 - \beta_{LS}) E_t u_t + \phi_{LS} (\ln x_{t+1} - \ln x_{t,P}) (u_{t+1} - u_t) + \frac{1}{2} \omega_{LS} (u_{t+1} - u_t)^2 \sigma^2 + \frac{1}{2} \lambda \sigma^2 \quad (63)$$

where β_{LS} and ϕ_{LS} become identical to β_{LS} and ϕ_{LS} and ω_{LS} is...

$$E_t \ln x_{t+1} = \beta_{LS} E_t u_{t+1} + (1 - \beta_{LS}) E_t u_t + \phi_{LS} (\ln x_{t+1} - \ln x_{t,P}) (u_{t+1} - u_t) + \frac{1}{2} \omega_{LS} (u_{t+1} - u_t)^2 \sigma^2 + \frac{1}{2} \lambda \sigma^2 \quad (63)$$

8. A parsimonious and yet flexible model, *continued!*

It turns out that the model of equation (62) is equivalent to (11) since there is a one-to-one correspondence between the two sets of parameters:

$$(64) \quad \beta_T = \beta_K \mu_K + (\lambda + \beta_K) \mu_L$$

$$(65) \quad \phi_{KT} = \phi_{KK} (\mu_K - \mu_L)$$

$$(66) \quad \phi_{TT} = \phi_{TT} (\mu_K - \mu_L)^2 + \lambda$$

and expressed in terms of parameters of (11) the coefficient of equation (62) is

$$(67) \quad \phi_{TT} = \phi_{TT} (\mu_K - \mu_L)^2 + \lambda$$

$$(68) \quad \mu_L = \beta_T - \beta_K \frac{\phi_{KT}}{\phi_{KK}}$$

$$(69) \quad \lambda = \phi_{TT} - \frac{\phi_{KT}^2}{\phi_{KK}}$$

8. A parsimonious and yet flexible model, *continued*¹

For TFP we now get:

$$\ln T_{t,t-1} = \beta_K \mu_K + (1 - \beta_K) \mu_L + \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) (\ln x_{K,t} - \ln x_{K,t-1}) \quad (70)$$

TFP is numerically identical to (15)

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$$\ln T_{t,t-1} = (1 - \beta_K) \mu_L + \frac{1}{2} (1 - \beta_K) \lambda (2t - 1) \mu_K$$

$$+ \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) (\ln x_{K,t} - \ln x_{K,t-1}) + \dots$$

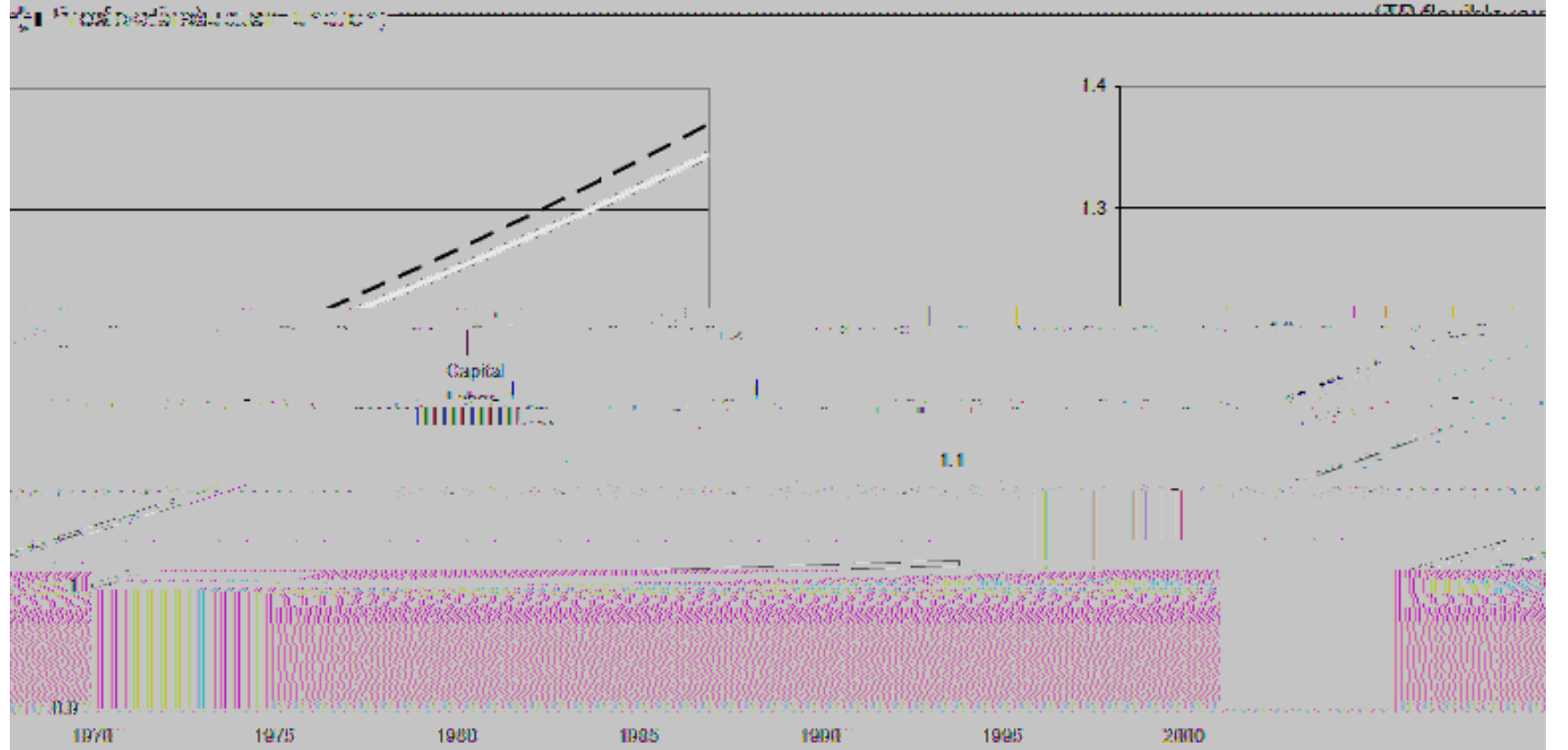
$$- \mu_L) \lambda (3t^2 - 3t + 1)$$

$$+ \frac{1}{2} \phi_{KK} (\mu_K - \mu_L) \mu_L (2t - 1) + \frac{1}{2}$$

Figure 3

Decomposition of TFP

Decomposition



9. The impact of technological change on factor rental prices reexamined, *continued*"

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9. The impact of technological change on factor rental prices reexamined, *continued*"

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9. The impact of technological change on factor rental prices reexamined, *continued*

Let ε_{ij} be the inverse price elasticities of factor demands:

$$(73) \quad \varepsilon_{ij} \equiv \frac{\partial \ln \tilde{w}_i(\tilde{x}_K, \tilde{x}_L, p)}{\partial \ln \tilde{x}_j}, \quad i, j \in \{K, L\}$$

$$(74) \quad \varepsilon_{iK} + \varepsilon_{iL} = 0, \quad i \in \{K, L\}$$

It is well known that:

$$(75) \quad \varepsilon_{KL} = \psi_{KL} S_L$$

$$(76) \quad \varepsilon_{LK} = \psi_{KL} S_K$$

9. The impact of technological change on factor rental prices reexamined, *continued*¹¹

We thus get for the total change in the rental price of an efficiency unit of capital:

$$\left(\frac{\partial \hat{r}}{\partial \hat{z}}\right) = \hat{w}_K = \hat{w}_K - \beta_K = \beta_K = \left(\frac{\partial \hat{r}}{\partial \hat{z}}\right) \hat{r} = \beta_K \hat{r}$$

and similarly for labor:

9. The impact of technological change on factor rental prices reexamined, *continued*"

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9. The impact of technological change on factor rental prices reexamined, *continued*"

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10. Generalization to an arbitrary number of inputs, *continued!*

11. Conclusions

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11. Conclusions, continued

- ✚ We have shown that in the case of a TP-flexible Translog production function TFP can always be interpreted as the outcome of disembodied, factor augmenting technological change
- ✚ Indeed, we have proposed a convenient way to derive the factor-augmenting rates of technological change from the estimates of such a Translog production function
- ✚ We have found that technological change is almost neutral in the case of the United States, so that TFP is overwhelmingly explained by labor

11. Conclusions, *continued*

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Thank you for your attention!

Growth factors 1970-2001

Quantity of capital services:	" #	2.25706
! " # \$ % & ' () * (+ # ,) - (. / - 0 & 1 / . 2 !	" \$	1.70513
3 - & 1 / () * () " % 4 " % 2 !	%	3.76623
! " # \$ % & ' () * () " % 4 " % 2 !	&	2.52563
3 - & 1 / () * (+ # ,) - (. / - 0 & 1 / . 2 (' \$	5.50201
5) % # + (* # 1 %) - (4 -) 6 " 1 % & 0 & % ' 2 !	(1.37071
Capital component of TFP:	T_K	1.01850
Labor component of TFP:	T_L	1.34581
Capital efficiency:	! #	1.06789
Labor efficiency:	! \$	1.50832
7 # ,) - (. 8 # - / 2 !) \$	0.98628
Output per unit of labor:	! ! ! ! ! L	1.48119