# Summary

# Welfare Change the New Goods Problem

Consumer's cost function:

C(u,p) ∑ min $_q$ {p⌠q ; f(q) ∅ u}

e(u)MC /P << 0.826 0 Td [(mi)-1(n.m1)Tj /C2_0 7a737529 -17C2_0 1 Tf

# Welfare Change the New Goods Problem

Valid measures of utility change over the two periods under consideration are the following Hicksian *equivalent and compensating variations*:

$Q_E(q^0,q^1,p^0) \sum C(f(q^1),p^0)\ 4\ C(f(q^0),p^0)$ ;

$Q_C(q^0,q^1,p^1) \sum C(f(q^1),p^1)\ 4\ C(f(q^0),p^1)$ .

**Note**: Samuelson (1974):

$Q_S(q^0,q^1,p) \sum C(f(q^1),p)\ 4\ C(f(q^0),p)$ .

Hence there is an entire family of cardinal measures of utility change: one measure for each reference price vector p.

# Welfare Change the New Goods Problem

The observable Bennet (1920) variation is the arithmetic average of the Laspeyres and Paasche variations:

$$V_B(p^0,p^1,q^0,q^1) \sum \tfrac{1}{2}(p^0 + p^1) \lceil (q^1\ 4\ q^0)$$
$$= p^0 \lceil (q^1\ 4\ q^0) + \tfrac{1}{2}(p^1\ 4\ p^0) \lceil (q^1\ 4\ q^0)$$
$$= V_L + \tfrac{1}{2} \sum (p_n\ 4\ p_n{}^0)(q_n{}^1\ 4\ q_n{}^0)$$

Bennet variation is equal to the Laspeyres variation $V_L$ plus a sum of N Harberger (1971) consumer surplus triangles of the form $(1/2)(p_n{}^1\ 4\ p_n{}^0)(q_n{}^1\ 4\ q_n{}^0)$. Also:

$$V_B(p^0,p^1,q^0,q^1) = V_P\ 4\ \tfrac{1}{2} \sum$$

# Welfare Change the New Goods Problem

Recap:

Hicksian equivalent variation can be approximated by $V_L$

Hicksian compensating variation can be approximated by $V_P$

Hicks (1942) obtained the Bennet quantity variation $V_B$ as an approximation to the arithmetic average of the equivalent and compensating variations.

## Welfare Change the New Goods Problem

So far, no economic justification for taking the average of $V_L$ and $V_P$.

Diewert and Mizobuchi (2009

# Welfare Change the New Goods Problem

Then, for normalized prices, we have the following exact equality:

$$V_B(p^0,p^1,q^0,q^1) = \tfrac{1}{2}\, Q_E(q^0,q^1,p^0) + \tfrac{1}{2}\, Q_C(q^0,q^1,p^1)$$

i.e., the observable Bennet variation is *exactly equal* to the arithmetic average of the unobservable equivalent and compensating variations.

Hence, a strong justification from an economic perspective for using the Bennet quantity variation. Also, it has strong justification from an axiomatic perspective (Diewert, 2005)

# Welfare Change the New Goods Problem

Now benefits of the introduction of a new good (or service) to a consumer who cannot purchase the good in period 0 but can purchase it in period 1.

Assume (as per Hicks 1940) that there is a shadow price for the new good in period 0 that will cause the consumer to consume 0 units of the new good in period 0.

Let the new good be indexed by the subscript 0 and let the N dimensional vectors of period t prices and quantities for the continuing commodities be denoted by $p^t$ and $q^t$ for $t = 0,1$.

Period 0 shadow price for commodity 0 is not observed but we make some sort of *estimate* for it, denoted as $p_0^{0*} > 0$ and. The period 0 quantity is observed and is equal to 0; i.e., $q_0^0 = 0$.

# Welfare Change the New Goods Problem

<u>Adapting Proposition 9 of Diewert (2005):</u>

If a superlative index number is chosen for P and Q, $V_B$ approximates $V_E$ to the second order for $q^0=q^1$ and $p^0=p^1$.

The U.S. uses the superlative Fisher Quantity Index for GDP, so:

$$V_E^F \sum ½\ p^0 \int q^0(1+P^F)(Q^F-1) – ½\ (p^0 + p^1) \int (q^1\ 4\ q^0) = V_B$$

Re-arranging:

$$Q^F – [(p^0 + p^1) \int (q^1\ 4\ q^0)]/[p^0 \int q^0(1+P^F)] +1$$

Note that the numerator is 2 x $V_{B.}$

# Welfare Change the New Goods Problem

$2V_B = (p^0 + p^1)⌠(q^1\ 4\ q^0)$

$= 2p^1⌠(q^1\ 4\ q^0)\ 4\ (p^1\ 4\ p^0)⌠(q^1\ 4\ q^0) + 2p_0{}^1q_0{}^1$ $\mathbf{4\ (p_0{}^1\ 4\ p_0{}^{0*})q_0{}^1}$

Assuming that the approximation holds exactly, then substitute into:

$Q^F = [(p^0 + p^1)⌠(q^1\ 4\ q^0)]/[p^0⌠q^0(1+P^F)] +1$

If $Q^F$ omits the new good in period 0, and we assume that $P^F$ (the aggregate GDP deflator between adjacent periods) is unaffected by the introduction of the new good, then the (approximate) amount missing from $Q^F$ is:

$\mathbf{(p_0{}^{0*}}\ 4\ \mathbf{p_0{}^1)q_0{}^1\ /}\ [p^0⌠q^0(1+P^F)]$

which can simply be added to $Q^F$ if $p_0{}^{0*}$ is known or can be estimated. $P^F$ will typically fall with the inclusion of the new good, so this is a lower bound on the amount to add.

# Welfare Change the Free Goods Problem

Consider a consumer whose preferences over N market goods and services and M commodities that are available to the household with no visible charge.

Utility function f(x,z): where x Ø $0_N$ and z Ø $0_M$ are vectors which represent the consumption of market commodities and of free commodities respectively.

We assume that f(x,z) is defined over the nonnegative orthant in $R^{N+M}$ and has the following properties:

(i) continuity,

(ii) quasiconcave in x and y, and

(iii) f(x,z) is increasing if all components of x increase and increasing if all components of z increase.

# Welfare Change the Free Goods Problem

Define two cost functions that are dual to f. The first cost function is the consumer’s *regular cost function* that is the solution to the following cost minimization problem which assumes (hypothetically) that the household faces positive prices for market and free goods and services so that $p >> 0_N$ and $w >> 0_M$:

$$C(u,p,w) \sum \min_{x,z} \{p \lceil x + w \lceil z: f(x,z) \varnothing u, x \varnothing 0_N, z \varnothing 0_M\}.$$

The *conditional cost function* minimizes the cost of market goods and services needed to achieve utility level u, conditional on having the vector $z \varnothing 0_M$ of free goods and services at its disposal:

$$c(u,p,z) \sum \min_{x} \{p \lceil x: f(x,z) \varnothing u, x \varnothing 0_N\}.$$

# Welfare Change the Free Goods Problem

Decompose the first cost function into a two-stage minimization problem using the second cost function:

$$C(u,p,w) \sum \min_{x,z} \{p⌠x + w⌠z : f(x,z) \emptyset u;\ x \emptyset 0_N,\ z \emptyset 0_M\}$$

$$= \min_{z} \{c(u,p,z) + w⌠z : z \emptyset 0_M\}.$$

Suppose $z^* \emptyset 0_M$ solves this cost minimization problem and suppose further that $c(u,p,z^*)$ is differentiable with respect to the components of z at $z = z^*$.

Then the first order necessary conditions for $z^*$ to solve the cost minimization problem imply that:

$$\subseteq_z c(u,p,z^*) = 4w\ .$$

# Welfare Change the Free Goods Problem

With $z = z^*$, we can find an x solution which we denote by $x^*$; i.e., $x^*$ is a solution to:

c(u,p,z*) ∑ min $_x$ {p⌠x: f(x,$z^*$) ∅ u, x ∅ $0_N$}.

It can be seen that ($x^*$,$z^*$) is a solution to the regular cost minimization problem defined by so that:

C(u,p,w) ∑ min $_{x,z}$ {p⌠x + w⌠z: f(x,z) ∅ u, x ∅ $0_N$, z ∅ $0_M$}
= p⌠$x^*$ + w⌠$z^*$.

Thus the imputed marginal valuation prices w ∑ 4$⊆_z$c(u,p,$z^*$) ∅ $0_M$ are appropriate prices to use when valuing the services of free goods in order to construct cost of living indexes or measures of money metric utility change.

# Welfare Change the Free Goods Problem

*Marginal willingness to sell function* for free good m:

$W_m(u,p,z) \sum c(u,p,z4e_m)\ 4\ c(u,p,z)$ ; m = 1,...,M.

where $e_m$ is a unit vector of dimension M with a 1 in component m and zeros elsewhere for m = 1,...,M.

Survey methods could be used in order to obtain approximate measures for these marginal willingness to sell functions.

Let W(u,p,z) denote the vector $[W_1(u,p,z),..., W_M(u,p,z)]$.

It can be seen that W(u,p,z) is a discrete approximation to the marginal valuation price vector $w \sum 4\subseteq_z c(u,p,z)$

# Welfare Change the Free Goods Problem

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Attempts to find prices for “free” goods include:

Brynjolfsson and Oh (2012)

*“We develop a new framework to measure the value of free services using the insight that even whenpe (*

# Welfare Change the Free Goods Problem

# Summary

Defined a framework for measuring welfare change.

Drew on the work of Hicks (1941-42), Bennet (1920) and Diewert and Mizobuchi (2009).

Derived an explicit term that is the value of a new good on welfare change.

That is, we get a measure of the contribution to welfare of a new good, and hence the extent of welfare change mismeasurement if it is omitted from statistical agency collections.

Showed how to work out a lower bound on the addition to real GDP growth from the introduction of a new good.

We then re-work the theory allowing for there to be “free” goods (with an implicit or imputable price).