Static Hedging of Non-Exchange Traded Options in South Africa

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Overview

- The risk-neutral measure is often neglected in favour of the real-wor measure due to the pricing of contingent claims.
- The real-world measure is extremely useful in risk management applications.
- We consider a static hedging experiment for vanilla European and European spread call options in a South African context.
- The experiment links the risk-neutral and real-world probability measures, which can help inform trading decisions.

SVJJ characteristic function (continued)

$$P = \frac{-u^2 - iu}{2};$$

$$Q = \quad - \quad _{x;v} \quad _v iu;$$

$$R = \frac{1}{2} \quad ^2_v.$$

Furthermore,

$$_J(u) = e^{-\lambda T(1+iu\mu_J)+\lambda \exp\left\{iu\mu_S+\frac{\sigma_S^2(iu)^2}{2}\right\}\nu};$$

where

$$= \frac{Q + D_1}{(Q + D_1)c - 2 \quad _v P} + \frac{4 \quad _v P}{(D_1 c)^2 - (2 \quad _v P - Qc)^2}$$

$$\times \log \quad 1 - \frac{(D_1 - Q)c + 2 \quad _v P}{2D_1 c} \quad 1 - e^{-D_1 T} \quad ;$$

$$c = 1 - iu \quad _J \quad _v: \quad _v$$

Static hedging approach #1

$$\min_{B} \sum_{i=1}^{n} C(i)B(i);$$

subject to

$$\sum_{i=1}^{n} F(ij)B(i) \geq Y(j); \quad j = 1; 2; \ldots; m;$$

where

$i = 1; 2; \ldots; n$:= the number of instruments in the replicating portfolio;

$j = 1; 2; \ldots; m$:= the price of the underlying asset at some future time;

$C(i)$:= the current price of the i^{th} instrument;

$B(i)$:= the number of units of the i^{th} instrument;

$F(ij)$:= the future price of the i^{th} instrument in state j; and

$Y(j)$:= the future price of the target option in state j:

FTSE/JSE Top40 index

SVJJ daily statistics versus FTSE/JSE Top40

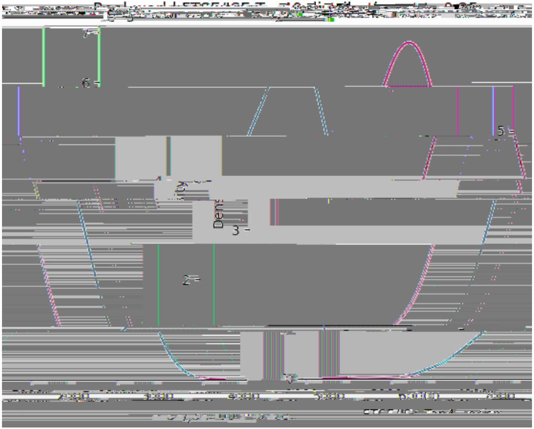
Statistic	FTSE/JSE Top40 index	SVJJ model
Mean	0.0385%	0.0406%
Std dev	1.3290%	1.1410%
Skewness	-0.4369	-0.2418
Kurtosis	9.4344	5.0463
Minimum	-0.1429	-0.0695
Maximum	0.0845	0.0592

Table: SVJJ model daily statistics for the FTSE/JSE Top40

FFT implementation test

Table: MC and FFT European call option prices under SVJJ model with $S(0) = 100;\ r = 0.1;\ v(0) = 0.04;\ \ = 1;\ \ = 0.04;\ _v = 0.05;\ _{x,v} = -0.5;\ \ = 5;\ _S = 0;\ _S = 0.01;\ _J = -0.3;\ v 0 1;$

Real-world FTSE/JSE Top40 distribution at $t = 0{:}25$

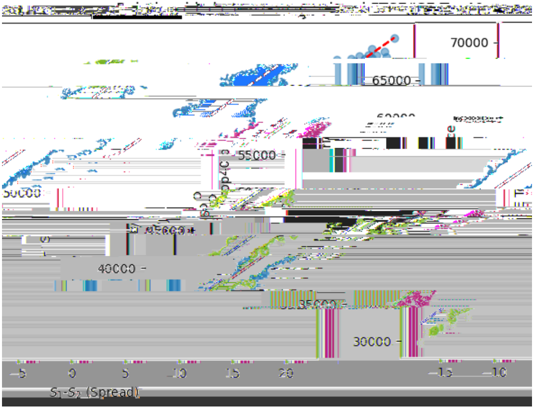


Option quantities based on static hedge #1

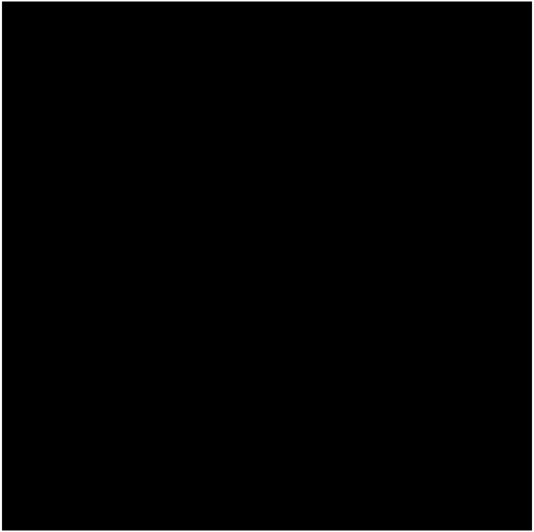
Option quantities based on static hedge #2

Real-world spread distribution at $t = 0.25$

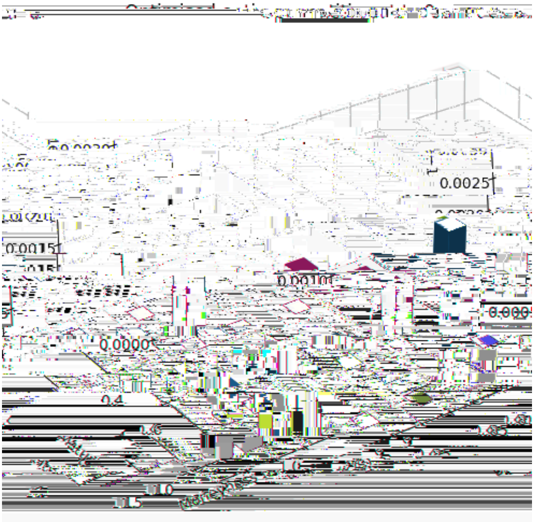
Relationship between spread and FTSE/JSE Top40 pric



Option quantities based on static hedge #1



Option quantities based on static hedge #2



Conclusion

- The hypothesis that the observed data are observed from the SVJJ model is not rejected at a 5% level of signi cance.
- For a vanilla European call option, static hedging gives a simple an e ective way to replicate the written option.
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Thank you